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PRACTICAL CONSIDERATIONS FOR LC DIFFERNTIAL OSCILLATORS DESIGN

Practical design of LC differential oscillators faces several challenges – achievement of sufficient output power, reliable start-up of the oscillator, low power consumption and etc. This paper attempts to give analytical expressions for these quantities with the aim to facilitate the design of LC differential oscillators. The paper presents derivation of the Van der Pol differential equation for LC differential oscillators. The condition for reliable start-up is formulated using the Van der Pol equation. Analytical expression for the output amplitude based on a simple oscillator model is also derived. Comparison between the analytical and simulated results is done and it shows a very good agreement.

1. Van der Pol equation of the LC differential oscillator.

In this section the Van der Pol equation for differential LC oscillators is derived. A simple LC differential oscillator is given on Fig.1. It consists of the differential pair Q_1, Q_2 and the LC tank which includes the capacitor C and the two inductances $L/2$. The two series resistances $R_l/2$ represent the resistive losses in the inductors and allow for taking into account the quality factor Q_L of the inductors. The resistance R_C accounts for the quality factor Q_C of the capacitor.

For low output amplitude $u_{OUTm}=u_C$ ($u_{OUTm} \leq 0.6V$ approx.) when the transistors Q_1 and Q_2 (Fig.1) are not in deep saturation, we can neglect the base currents and we can write the relations (first Kirhoff's law for the collector nodes):

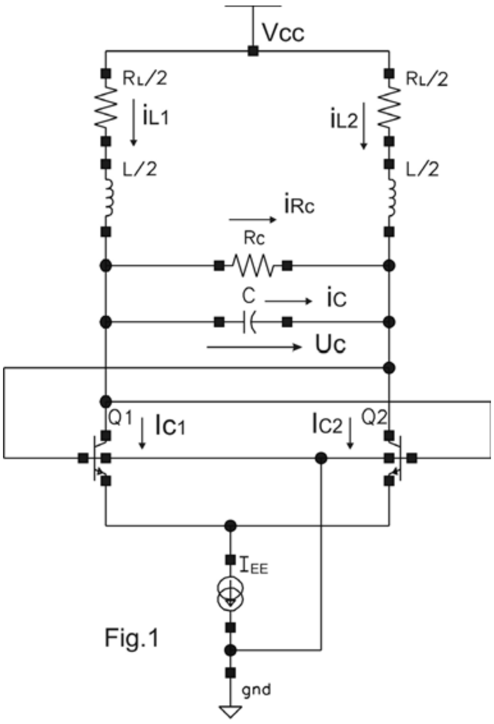

Fig.1

$$i_{L1} = I_{C1} + i_C + i_{Rc} \qquad (1)$$

$$i_{L2} = I_{C2} - i_C - i_{Rc} \qquad (2)$$

also:

$$i_C = C\frac{du_C}{dt} \qquad (3)$$

$$i_{Rc} = \frac{u_C}{R_C}. \qquad (4)$$

From these relations we can find:

$$i_{L1} - i_{L2} = I_{C1} - I_{C2} + 2i_C + 2i_{Rc} \qquad (5)$$

and

$$\frac{d}{dt}(i_{L1} - i_{L2}) = \frac{d}{dt}(I_{C1} - I_{C2}) + 2\frac{di_C}{dt} + 2\frac{di_{Rc}}{dt}. \qquad (6)$$

Now if we substitute (5) and (6) in the equation (7) (second Kirhoff's law for the tank loop)

$$\frac{R_L}{2} i_{L1} + \frac{L}{2}\frac{di_{L1}}{dt} + u_C - \frac{L}{2}\frac{di_{L2}}{dt} - \frac{R_L}{2} i_{L2} = 0 . \tag{7}$$

we obtain:

$$LC\frac{d^2u_C}{dt^2} + (R_L C + \frac{L}{R_C})\frac{du_C}{dt} + (1+\frac{R_L}{R_C})u_C + \frac{R_L}{2}(I_{C1} - I_{C2}) + \frac{L}{2}\frac{d}{dt}(I_{C1} - I_{C2}) = 0 . \tag{8}$$

Now we can use the well-known relation for the differential pair:

$$I_{C1} - I_{C2} = -I_{EE}\, th(\frac{u_C}{2V_T}) , \tag{9}$$

where u_C in our case is both the input of the differential pair and the output of the oscillator, I_{EE} is the tail current, and V_T is the termal potential. Differentiating this equation gives:

$$\frac{d}{dt}(I_{C1} - I_{C2}) = -I_{EE}\frac{1}{ch^2(u_C/2V_T)}\frac{1}{2V_T}\frac{du_C}{dt} . \tag{10}$$

If we now substitute (9) and (10) in (8) we come to:

$$LC\frac{d^2u_C}{dt^2} + [R_L C + \frac{L}{R_C} - \frac{L}{4V_T} I_{EE}\frac{1}{ch^2(u_C/2V_T)}]\frac{du_C}{dt} + (1+\frac{R_L}{R_C})u_C - \frac{R_L}{2} I_{EE}\, th(\frac{u_C}{2V_T}) = 0 . \tag{11}$$

Now if we use the Taylor series representation of $th(x)$ and $1/ch^2(x)$ in the vicinity of $x=0$:

$$th(x) \approx th(0) + \frac{1}{1!}th'(0)x = x , \tag{12}$$

where the first two terms of the series are used and

$$1/ch^2(x) \approx 1/ch^2(0) + \frac{1}{1!}(1/ch^2(x))'_{x=0}\, x + \frac{1}{1!}(1/ch^2(x))''_{x=0}\, x^2 = 1 - x^2 , \tag{13}$$

where the first three terms of the Taylor series are used. If we substitute (13) and (14) in (11) we come to:

$$LC\frac{d^2u_C}{dt^2} + [R_L C + \frac{L}{R_C} - \frac{L}{4V_T} I_{EE}(1 - \frac{u_C}{2V_T}^2)]\frac{du_C}{dt} + (1+\frac{R_L}{R_C} - R_L\frac{I_{EE}}{4V_T})u_C = 0 . \tag{14}$$

Equation (15) is a form of Van der Pol differential equation which has generally the following form:

$$\frac{d^2y}{dx^2} - \mu(1-y^2)\frac{dy}{dx} + y = 0 . \tag{15}$$

Equation (15) can be transformed to the form of (16) with proper substitutions, which are lengthy and are not given in the paper.

2. Oscillator start-up condition

Now we can use equation (15) to derive the oscillator start-up condition. In the beginning of oscillator start-up the output voltage u_C is very small, close to zero. Therefore we can neglect the term u_C^2 and the equation transforms to :

$$LC\frac{d^2u_C}{dt^2} + [R_L C + \frac{L}{R_C} - \frac{L}{4V_T} I_{EE}]\frac{du_C}{dt} + (1+\frac{R_L}{R_C} - R_L\frac{I_{EE}}{4V_T})u_C = 0 . \tag{16}$$

This is an ordinary linear differential equation with constant coefficients. From the theory of this type of differential equations, in order that it has periodic solution with increasing amplitude, it is necessary that the coefficient before the first derivative du_C/dt is negative. (This is true if the coefficient before the second derivative is positive and it is positive (LC).) This means that we can write the start-up condition the following way:

$$R_L C + \frac{L}{R_C} - \frac{L}{4V_T} I_{EE} < 0 \qquad (17)$$

and after rearranging

$$I_{EE} > 4V_T\left(\frac{R_L C}{L} + \frac{1}{R_C}\right) = 4V_T\left(\frac{1}{R_L Q_L^{\,2}} + \frac{1}{R_C}\right), \qquad (18)$$

where Q_L is the quality factor of the inductors for $w_0 = 1/\sqrt{LC}$. Relation (18) describes the start-up condition of the oscillator. It shows that the requirements for reliable and fast start-up are contrary to the requirements for low power consumption and there is always some trade-off between them.

The validity of (18) has been investigated with simulations of the circuit given on Fig.1 with parameters: R_L*=3Ohm*, *L=2nH*, R_C*=600Ohm*, *C=2pF*. The circuit was implemented in 0.35um HBT technology of Austriamicrosystems with HBT npn bipolar transistors with area=10umx0.4um. As simulation tool the Spectre simulator from Cadence environment was used. All circuit parameters were chosen with the aim to design 2.4GHz VCO (Voltage Controlled Oscillator) for the Bluetooth standard. The values of R_L and L model a real technology given inductor with 1nH inductivity and quality factor $Q \approx 10$ at 2.4GHz. The values of C and R_C model the technology given MOS varactors with $C_{max}=1.75pF$ and quality factor $Q=18$ at 2.4GHz.

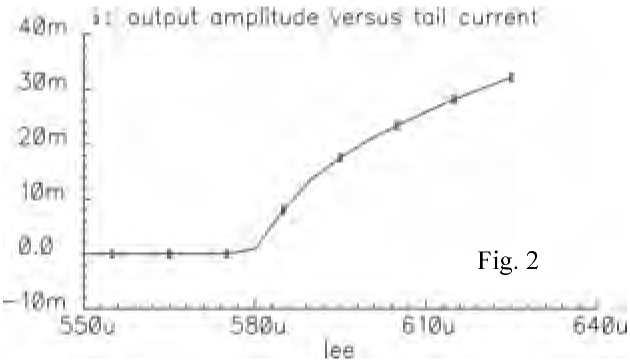

Fig. 2

The results from simulation showing the output amplitude versus the tail current are given on Fig.2. We can see that the oscillations stop at tail current $I_{EE}=580uA$. The analytical expression (18) gives $I_{EEmin}=490uA$. The difference of approximately 100uA between the analytical and experimental results is due to the fact that the output resistances r_O seen from the collector nodes of the two transistors Q_1 and Q_2 are not accounted for in the analytical model. They should be included in parallel with R_C and this way they will increase the tail current threshold with $4V_T/2r_O$ which gives $r_O=500Ohm$ approximately. These output resistances consist of the collector-emitter resistances and the base-emitter resistances of the two transistors where the later is much smaller and its effect dominates. Estimate of the base-emitter resistance from the model parameters is difficult to make because the transistors are modeled with the VBIC (Vertical Bipolar Inter-Company) model which does not give explicitly the base-emitter resistance. The intrinsic and extrinsic base resistances of the model are $r_{BX}=155Ohm$ and $r_{BI}=65Ohm$ respectively which gives in total approximately 200Ohm.

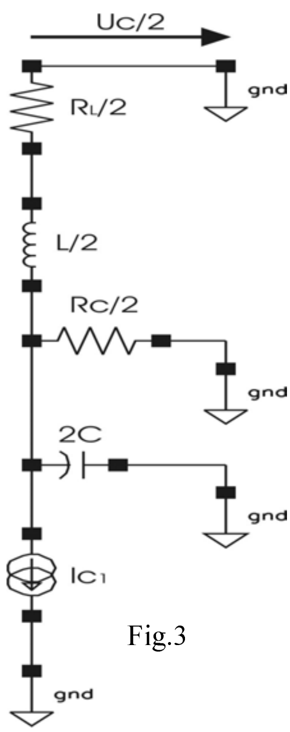

Fig.3

3. Oscillator amplitude

The symmetry in the circuit implies that the voltage potential at the axis of symmetry is zero and we can divide the circuit of Fig.1 into two equal circuits (Fig.3) and continue the analysis of only one of them. The total output voltage will be twice the output voltage of the half-circuit since the potentials at the collector nodes of the transistors Q_1 and Q_2 are differential – when one goes up with some Δu the other goes down with the same Δu.

Now if we transform the series resistance of the inductance into a parallel one for the oscillation frequency $w_0 = 1/\sqrt{LC}$ with the relation:

$$R_L' = R_L\left(Q_L^{\,2} + 1\right) \approx R_L Q_L^{\,2} \qquad (19)$$

we come to the circuit of Fig.4. The current source I_{C1} has rectangular pulse shape with amplitude I_{EE}, frequency w_0 and duty cycle of one half (Fig.5). If we represent this pulse sequence with Fourier series it will have an average value and odd harmonics of the frequency w_0. Since the quality factor of the *RLC* tank is relatively high ($Q_{tank}\approx 6$) only the first harmonic will produce significant output voltage. The first harmonic of this sequence has amplitude of $(2/\pi)I_{EE}$ and then one half amplitude of the output voltage will be:

$$\frac{u_C}{2} = \frac{2}{\pi} I_{EE} \left(\frac{R_L}{2} Q_L^{\,2} \,||\, \frac{R_C}{2}\right) \tag{20}$$

and the total output voltage is then:

$$u_C = \frac{2}{\pi} I_{EE} (R_L Q_L^{\,2} \,||\, R_C) = \frac{2}{\pi} I_{EE} R_P\,, \tag{21}$$

where R_P is the total parallel resistance of the tank. The validity of (21) is proved with simulations of the circuit on Fig.1. The results from the simulations together with a plot of (21) are given on Fig.6. We can see that up to the output voltage of approximately 800mV (tail current of approximately 6mA) the formula (21) gives fair results. When the tail current is further increased the output voltage is no longer linearly dependant on the tail current but enters saturation. This is explained with the fact that the transistors Q_1 and Q_2 enter in deep saturation regime of operation and the collector currents I_{C1} and I_{C2} no longer can be approximated with rectangular pulse waveforms. Also the base currents of the transistors which were neglected before start to play role. These two effects change the assumptions under which the formula (21) was derived and disqualify its validity.

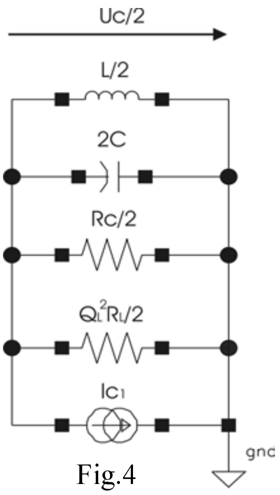

Fig.4

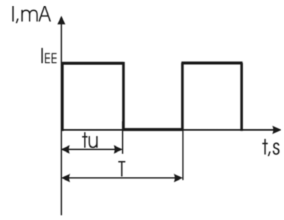

Fig.5

One method of increasing the output voltage is presented on Fig.7. The idea is to translate the base DC potentials sufficiently below the collector DC potentials. This way we can allow for significantly bigger amplitude of the output signal without transistors Q_1 and Q_2 entering into saturation. The capacitances are $C_1=C_2=1pF$ and the resistances are $R_1=R_3=2k$ and $R_2=R_4=4k$. The simulated and calculated output voltages of the circuit are given on Fig. 8. We can see that the validity of (21) extends significantly. In this case twice the parallel resistance of R_1 and R_2 is included again in parallel with the tank resistance R_P in the formula (21). The impedance of the capacitances C_1 and C_2 are negligible at these frequencies compared to $R_1||R_2= R_3||R_4$. and is not accounted for.

3. Conclusions

In this paper the Van der Pol differential equation for LC differential oscillators is derived. Based on this equation the start-up condition of the oscillator is formulated. The simulated and calculated results for the start-up condition show good agreement. Further improvement the precision of the start-up condition necessitates a better modeling of the differential pair output resistance. The paper also derives analytical expression for the output amplitude in the so called current limited regime of the LC differential oscillator.

Method for extending the current limited regime towards higher output voltages is presented. The analytical expression for the output amplitude is checked versus simulation results and it proves to be an excellent approximation. A method for increasing the output amplitude is also presented.

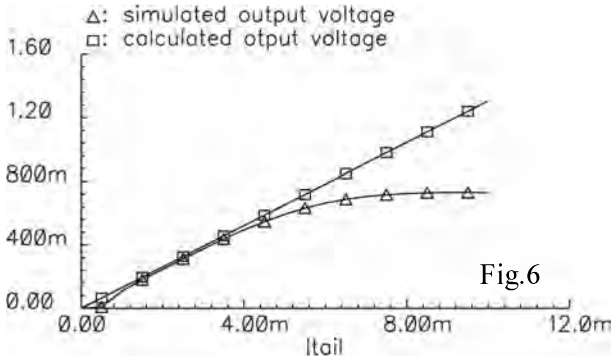

Fig.6

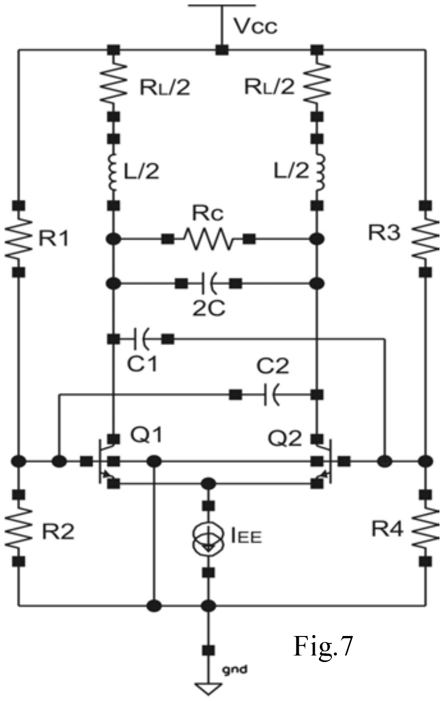

Fig.7

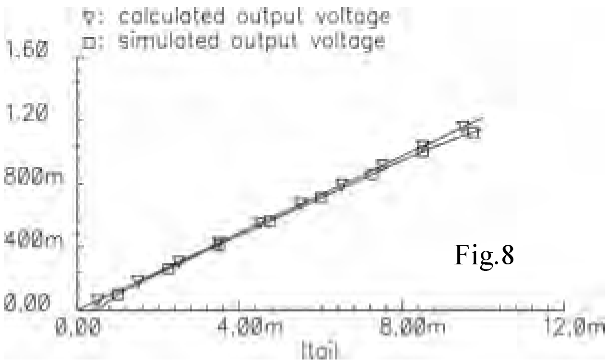

Fig.8

The results in this paper have been used to design 2.4GHz VCO for Bluetooth standard using 0.35um HBT BiCMOS technology of Austriamicrosystems. The VCO shows 800mV_{pp} output amplitude, 2.38-2.68GHZ tuning range, phase noise 123dBc/Hz at 1MHz offset from the carrier, 26mW power consumption. (See *"Fully-Integrated VCO with low phase noise", Nachev R., Ivanov K. and etc., Proceedings of the International Scientific and Applied Science Conference ELECTRONICS 2004*).

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