This especially concerns such sensors and such applications, when sensor detector ought to be in direct contact with measured surface, what is very often found in medical applications.

From among flip chip processes for humidity silicon sensor, flip chip solder bonding seems to be the best solution. For silicon gas sensor with backside contacts, thermocompression bonding process is the most suitable since enables, as well thermocompression wire bonded connections to produce as wireless connections i.e. thermocompression flip chip bonding to realize. For the sensors on flex substrate packed, the underfilling process is needed to relieve the thermal mismatch between bonding materials and reliability of flip chip joints to enhance.

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ANALYSIS OF PIEZORESISTORS ARRANGEMENTS ON THE SURFACE OF CIRCULAR MEMBRANE IN PRESSURE SENSORS

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A theoretical analysis concerning metrological properties of thick film piezoresistors placed on circular edge-clamped membrane is described. The change of resistance of a thick film piezoresistor, caused by the deflection of the membrane, related to the distance from the membrane centre and the width of the piezoresistor, is evaluated. Analytical expressions for different piezoresistors arrangements are presented and discussed.

1. Introduction

About 1940 the construction of pressure sensor with etch-foil metal resistors glued on circular membrane was elaborated. In 1970s this construction was adopted in miniaturized silicon pressure sensors: the membrane was etched in silicon monocrystal and metal-foil resistors were replaced with diffused semiconductor piezoresistors which gauge factor GF is about hundred times greater than for metals. The output signal increased because of an immense change in the resistivity of strained semiconductor – whereas in metals GF is related with so called "geometrical piezoresistivity" caused by change of dimensions.

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However, semiconductor piezoresistors exhibit significant sensitivity to temperature; and in late 1970s an attempt to reduce the temperature sensitivity of miniaturized pressure sensors was made: to the alumina membrane were bonded thick film piezoresistors which exhibit low temperature coefficient of resistance (TCR) and very good stability.

2. Properties of thick film piezoresistors and their operating on membrane

Roughly speaking, thick film piezoresistors are about 10÷20 μm thick and planar dimensions are 1mm×1mm or more, so the diameter of circular membrane is about 10 mm at thickness of 0.1÷0.2 mm. The gauge factor as high as 35 is achievable, due mainly to change of resistivity, and for a piezoresistive strip of uniform cross-section GF is defined as (1):

$$\mathrm{GF} = \frac{\frac{\Delta R}{R_0}}{\varepsilon} = \frac{\frac{\Delta\rho}{\rho_0}}{\varepsilon} + 1 + 2\nu\,, \tag{1}$$

where R_0 and ρ_0 denote the resistance and resistivity of non-strained piezoresistor, respectively; ΔR and $\Delta\rho$ are the changes of resistance and resistivity caused by the strain ε; ν is a Poisson's ratio ($0.2 \le \nu \le 0.4$).

The value of GF is a sum of resistivity-change effect (GF_ρ) and geometrical effect (GF_g); for thick film piezoresistors GF is approximately equal to GF_ρ . Moreover, a slight difference in value of GF for strain ε_l parallel to supply current I (GF_l) and for strain ε_t perpendicular to I (GF_t) is observed; for thick film piezoresistors this difference may be roughly assumed as negligible. The change of resistivity $\Delta\rho$ caused by the strain ε can be expressed as (2):

$$\Delta\rho = \rho_0 \cdot \mathrm{GF} \cdot \varepsilon \tag{2}$$

In thick film piezoresistors, the effect of change of resistivity is isotropic, i.e. independent of the direction of the strain ε ; moreover, the effects of both strains ε_l and ε_t are additive.

For a circular edge-clamped membrane, bended by a pressure p uniformly distributed over the top surface, the strains: radial – ε_r , and tangential (i.e. circumferential) – ε_t , caused by the stresses σ_r and σ_t are written using the formulae (3) and (4):

$$\varepsilon_r = \frac{3p}{8h^2E}(1-\nu^2)(3r^2-a^2) \tag{3}$$

$$\varepsilon_t = \frac{3p}{8h^2E}(1-\nu^2)(r^2-a^2) \tag{4}$$

where h denotes the thickness of the membrane, E is the Young's modulus, a is the radius of the membrane, and r is the polar coordinate of a point on the top surface (Fig.1). These equations are valid assuming linear dependence between stress and pressure, and that the deflection in the central point of membrane is less than $h/2$.

The equations (3) and (4) imply that ε_r and ε_t are related to r parabolically; the value of $\varepsilon_r(r)$ is negative for $r < a/\sqrt{3}$, in the central area of top surface, whereas the value of $\varepsilon_t(r)$ is negative on the whole top surface.

When the strain $\varepsilon_r(r)$ is applied to a given point of the membrane top surface, the change of resistivity $\Delta\rho_r(r)$ can be calculated as (5):

$$\Delta\rho_r(r) = \rho_0 \cdot \mathrm{GF} \cdot \varepsilon_r(r) = \rho_0 \cdot \mathrm{GF} \cdot \frac{3p}{8h^2E}(1-\nu^2)(3r^2-a^2) \tag{5}$$

Analogically, when the strain $\varepsilon_t(r)$ is applied to a given point of the membrane top surface, the change of resistivity $\Delta\rho_t(r)$ can be calculated as (6):

$$\Delta\rho_t(r) = \rho_0 \cdot \mathrm{GF} \cdot \varepsilon_t(r) = \rho_0 \cdot \mathrm{GF} \cdot \frac{3p}{8h^2E}(1-\nu^2)(r^2-a^2) \tag{6}$$

Because the effects caused by $\varepsilon_r(r)$ and $\varepsilon_t(r)$ for thick film piezoresistors are additive, the overall change of resistivity $\Delta\rho(r)$ is given by the expression (7):

$$\Delta\rho(r) = \rho_r(r) + \rho_t(r) = \rho_0 \cdot \mathrm{GF} \cdot \frac{3p}{4h^2E}(1-\nu^2)(2r^2-a^2) \tag{7}$$

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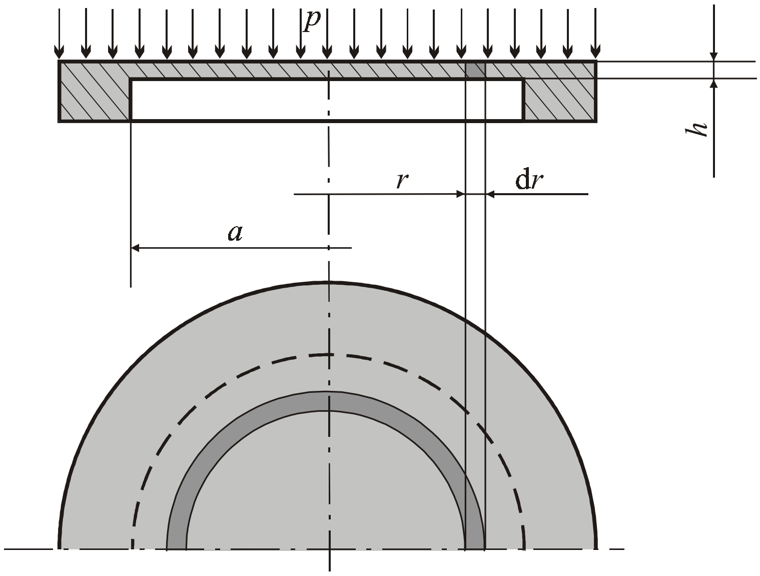

Fig. 1. Circular edge-clamped membrane under measured pressure p

From the equation (7) one can see that $\Delta\rho(r)$ is related to r parabolically; the value of $\Delta\rho(r)$ is negative for $r < a/\sqrt{2}$, in the central area of the top surface of the membrane.

The formula for resistivity $\rho(r)$ in a given point of the top surface has the functional form (8) :

$$\rho(r) = \rho_0 + \rho_0 \cdot \mathrm{GF} \cdot \frac{3p}{4h^2 E}(1-\nu^2)(2r^2 - a^2) \tag{8}$$

The form of the expression (8) can be simplified using substitutions for its constant components:

$$\mathrm{C} = \rho_0 \cdot \mathrm{GF} \cdot \frac{3\mathrm{p}}{4\mathrm{h}^2\mathrm{E}}(1-\nu^2) = \rho_0 \cdot \mathrm{C}' \quad , \quad \mathrm{C}_1 = \rho_0 - \mathrm{C} \cdot \mathrm{a}^2 = \rho_0 \cdot (1 - \mathrm{C}' \cdot \mathrm{a}^2) \tag{9}$$

The short formula for resistivity $\rho(r)$ can be written in form (10):

$$\rho(r) = 2\mathrm{C}\, r^2 + \mathrm{C}_1 \tag{10}$$

or in Cartesian coordinates (11):

$$\rho(x, y) = 2\mathrm{C}\, x^2 + 2\mathrm{C}\, y^2 + \mathrm{C}_1 \tag{11}$$

In the sources concerning the pressure sensors with piezoresistors, various piezoresistors arrangements on the membrane surface are presented in figures; however, in support of those arrangements rather laconic explanations are given. The following analyses are an attempt to discuss that question in more detailed way.

3. Arc-shaped piezoresistor

When no strain ε is applied to the membrane, the resistance R_{0U} of the arc-shaped piezoresistor supplied by a current I in radial direction (see Fig.2a), can be calculated as (12):

$$R_{0U} = \rho_0 \cdot \frac{\ln \frac{r_2}{r_1}}{g \cdot (\vartheta_2 - \vartheta_1)} \tag{12}$$

where g is the thickness of the piezoresistor.

As the strain has been applied, the resistance R_U of the same piezoresistor supplied by a current I in radial direction can be calculated as (13), provided that the resistivity $\rho(r,\theta)$ is a function of r only:

$$R_U = \frac{\int_{r_1}^{r_2} \frac{\rho(r)}{r} dr}{g \cdot (\vartheta_2 - \vartheta_1)} \tag{13}$$

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Because in the discussed case of thick film piezoresistor the function $\rho(r)$ is given by equation (10), R_U is:

$$R_U = \frac{\int_{r_1}^{r_2} \frac{2\mathrm{C}\,r^2 + \mathrm{C}_1}{r}\,\mathrm{d}r}{g\cdot(\vartheta_2-\vartheta_1)} = \frac{\mathrm{C}(r_2^2-r_1^2)+\mathrm{C}_1\cdot\ln\frac{r_2}{r_1}}{g\cdot(\vartheta_2-\vartheta_1)} \tag{14}$$

After using formulae (9), the equation (15) can be obtained in form similar to (12):

$$R_U = \rho_0\cdot\frac{\mathrm{C}'(r_2^2-r_1^2)+(1-\mathrm{C}'\cdot a^2)\cdot ln\frac{r_2}{r_1}}{g\cdot(\vartheta_2-\vartheta_1)} \tag{15}$$

The relative change of resistance caused by strain on the top surface of the membrane χ can be written as:

$$\chi_U = \frac{R_U}{R_{0U}} = \frac{\rho_0\cdot\dfrac{\mathrm{C}'(r_2^2-r_1^2)+(1-C'\cdot a^2)\cdot\ln\frac{r_2}{r_1}}{g\cdot(\vartheta_2-\vartheta_1)}}{\rho_0\cdot\dfrac{\ln\frac{r_2}{r_1}}{g\cdot(\vartheta_2-\vartheta_1)}} = 1+\frac{\mathrm{C}'(r_2^2-r_1^2)}{\ln\frac{r_2}{r_1}} - C'\cdot a^2 = 1+\Delta\chi_U \tag{16}$$

If the substitution λ=C'a^2 is used, the relative increase of resistance $\Delta\chi_U$ can be written in the short form:

$$\Delta\chi_U = \lambda\left[\frac{\left(\frac{r_2}{a}\right)^2-\left(\frac{r_1}{a}\right)^2}{\ln\frac{r_2}{r_1}}-1\right] \tag{17}$$

The expression in square brackets can be denoted as η_1 , and then: $\Delta\chi_U = \lambda\cdot\eta_1$.

For a given value: $r_2 - r_1 = t$ (in this case t is the length of the piezoresistor), an analysis of the relationship between r_2 and $\Delta\chi_U$ can be made using a as the relative unit of length. The results are presented in Table 1 and shown in Fig.2b.

Table 1

r_2		a	$0.9a$	$0.8a$	$0.7a$	$0.6a$	$0.5a$	$0.4a$	$0.3a$	$0.2a$
$\frac{\Delta\chi_U}{C'a^2}$	$t = 0.1a$	0.803	0.443	0.123	-0.157	-0.397	-0.597	-0.757	-0.877	-0.957
	$t = 0.2a$	0.613	0.273	-0.027	-0.287	-0.507	-0.687	-0.827	-0.927	—
	$t = 0.3a$	0.430	0.110	-0.170	-0.410	-0.610	-0.771	-0892	—	—

For the arc-shaped piezoresistor supplied by a current I in radial direction, the following statements can be made:

– the relative increase of resistance $\Delta\chi_U$ (proportional to the sensitivity) is a function of two variables: r_2 – the radius of the outer side of the piezoresistor, and $t = r_2 - r_1$: the length of the piezoresistor.

– when the value of t is given, the maximum of positive values of $\Delta\chi_U$ is obtain for $r_2 = a$, i.e. at the edge of the membrane. The maximum of negative values of $\Delta\chi_U$ is obtain for $r_2 \approx t$, i.e. at the centre of the membrane. Theoretically, $\Delta\chi_U$ is included within the interval : $-1 < \Delta\chi_U < 1$.

- for $r_2 \approx t$, the current density on the inner conductive termination increases, so the minimal value of r_1 is limited by the allowable current density J_{max} .

- when the value of r_2 is fixed, the increase of the value of t involves the decrease of $\Delta\chi_U$, although this effect becomes less noticeable for lower values of r_2 .

- for each of values of t, there is a certain value $r_2 = r_{20}$ such that $\Delta\chi_U = 0$ and no change of resistance in the strained piezoresistor occurs. However, the implicit equation $\Delta\chi_U = f(r_2,t) = 0$ can be resolved approximately only. The value of "neutral" radius $r_2 = r_{20}$ depends on t, and is approximately equal to (18) :

$$r_{20} = \frac{3t+\sqrt{t^2+8a^2}}{4} \tag{18}$$

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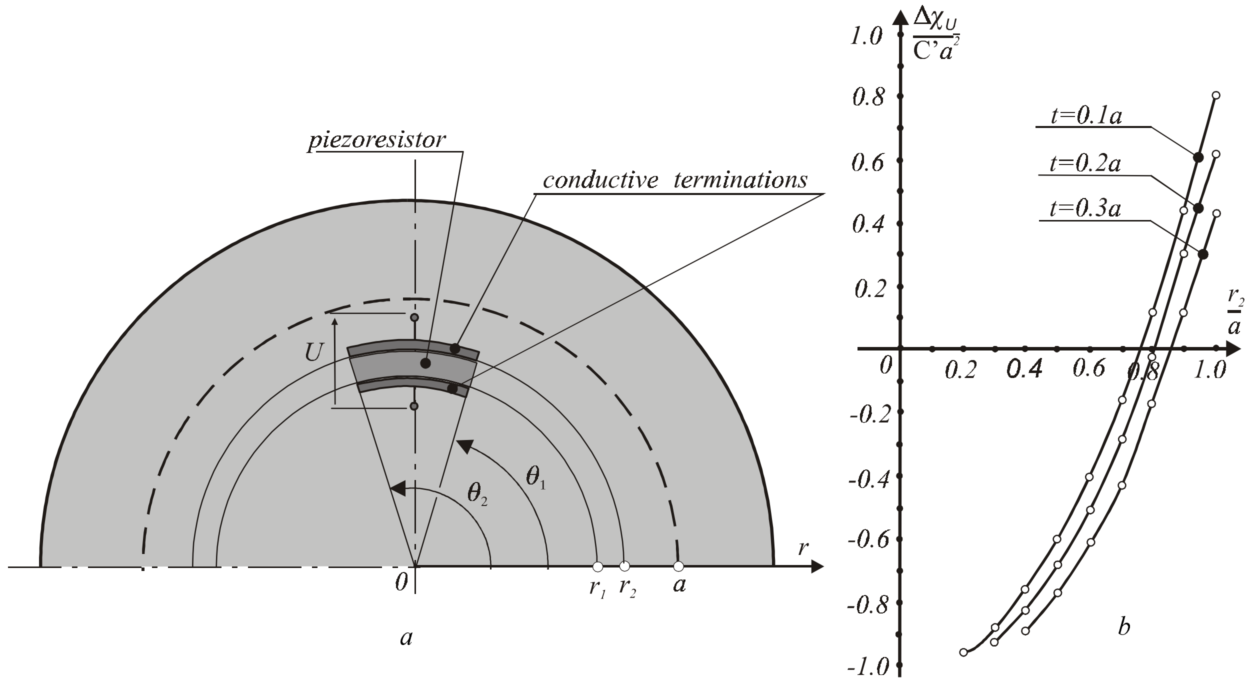

Fig. 2. a)Schematic arrangement of the arc-shaped piezoresistor;
b) The graph of $\Delta\chi_U$ *at various values of* t

4. Rectangle-shape piezoresistor

In general, in the case of rectangular strip the calculations are more complicated than for the circular-ring segment. When non-strained rectangle-shaped piezoresistor is supplied by a current I in radial direction, in accordance with Fig.3a its resistance R_{0LU} can be calculated as (19):

$$R_{0LU} = \rho_0 \cdot \frac{r_2 - r_1}{g \cdot 2b} \tag{19}$$

As the strain has been applied, the resistance R_{LU} of the same piezoresistor supplied by a current I in radial direction can be calculated as follows, provided that along vertical axis of symmetry (x=0) and along both vertical sides of the rectangular strip (x=-b, x=b) the conditions (20) are fulfilled:

$$\int_{r_1}^{r_2} E(0,y)\mathrm{d}y = U, \qquad \int_{r_1}^{r_2} E(b,y)\mathrm{d}y = U = \int_{r_1}^{r_2} E(-b,y)\mathrm{d}y \tag{20}$$

As $E(0,y)= J(0,y)\cdot\rho(0,y)=J_0\cdot\rho(0,y)$, and $E(-b,y)= E(b,y)= J(b,y)\cdot\rho(b,y)=J_1\cdot\rho(b,y)$, the specific (given) values of current density J_0, J_1 are:

$$J(0,y) = J_0 = \frac{U}{\int_{r_1}^{r_2} \rho(0,y)\mathrm{d}y}; \qquad J(b,y) = J_1 = \frac{U}{\int_{r_1}^{r_2} \rho(b,y)\mathrm{d}y} \tag{21}$$

For $\rho(x,y)=2Cx^2+2Cy^2+C_1$ is $\rho(0,y)=2Cy^2+C_1$, $\rho(b,y)=2Cy^2+2Cb^2+C_1$, and then J_0, J_1 can be calculated and expressed as (22) and (23):

$$J_0 = \frac{U}{(r_2 - r_1)\left[\frac{2C}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right) + C_1\right]} = \frac{U}{M_0} \tag{22}$$

$$J_1 = \frac{U}{(r_2 - r_1)\left[\frac{2C}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right) + 2Cb^2 + C_1\right]} = \frac{U}{M_1} \tag{23}$$

The functional relationship: $J(x,r_1)$ is complicated, but the linear approximation (24) can be applicable:

$$J(x, r_1) = \frac{(J_1 - J_0)}{b} \cdot x + J_0 \tag{24}$$

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Then the current I is equal to (25):

$$I = 2g\int_0^b\left(\frac{J_1 - J_0}{b}\cdot x + J_0\right)dx = g\cdot b\cdot\frac{M_0 + M_1}{M_0\cdot M_1}\cdot U \tag{25}$$

The resistance $R_{LU}=U/I$ is expressed by the formula (26):

$$R_{LU} = \frac{M_0\cdot M_1}{b\cdot g\cdot(M_0 + M_1)} \tag{26}$$

If the substitutions (9) and $A'=⅔\cdot C'[(r_1^2+ r_1\cdot r_2+ r_2^2)+C_1]$ are applied to (26), then:

$$R_{LU} = \rho_0\cdot\frac{(r_2 - r_1)\cdot A'\cdot\left(A' + 2C'\cdot b^2\right)}{2\cdot b\cdot g\cdot\left(A' + C'\cdot b^2\right)} \tag{27}$$

The relative change of resistance caused by strains on the top surface of the membrane χ_{LU} can be written as:

$$\chi_{LU} = \frac{R_{LU}}{R_{0LU}} = \frac{A' + 2C'b^2}{1 + \dfrac{C'b^2}{A'}} \tag{28}$$

If the substitution $\lambda=C'a^2$ is used, the formula for A' can be written shorter:

$$A' = 1 + \lambda\left\{\frac{2}{3}\cdot\left[\left(\frac{r_2}{a}\right)^2 + \left(\frac{r_1}{a}\right)\left(\frac{r_2}{a}\right) + \left(\frac{r_1}{a}\right)^2\right] - 1\right\} = 1 + \lambda\cdot k_1 \tag{29}$$

The expression in braces $k_1=\{\}$ is a function of r_2 and r_1 (or: r_2 and t). $C'b^2$ can be denoted as μ; then (30):

$$\chi_{LU} = \frac{1 + \lambda\cdot k_1 + 2\mu}{1 + \dfrac{\mu}{1 + \lambda\cdot k_1}} \approx 1 + \lambda\cdot k_1 + \mu;\quad \Delta\chi_{LU} = 1 - \chi_{LU} = \lambda\cdot k_1 + \mu \tag{30}$$

provided that $\lambda<< 1$, and $\mu<< 1$.

For a given value: $r_2 - r_1 = t$ (t is the length of the piezoresistor) and b=0.44a, an analysis of the relationship between r_2 and $\Delta\chi_{LU}$ can be made using a as the relative unit of length (because of the rectangular shape with b=0.44a, r_2 cannot exceed 0.9a). The results are presented in Table 2 and shown in Fig.3b.

Table 2

r_2		0.9a	0.8a	0.7a	0.6a	0.5a	0.4a	0.3a	0.2a
$\dfrac{\Delta\chi_{LU}}{C'a^2}$	t = 0.1a	0.637	0.317	0.037	-0.203	-0.403	-0.563	-0.683	-0.763
	t = 0.2a	0.447	0.177	-0.083	-0.303	-0.483	-0.623	-0.723	—
	t = 0.3a	0.330	0.050	-0.190	-0.390	-0.550	-0.670	—	—

For the rectangle-shaped piezoresistor supplied by a current I in radial direction, the following statements can be made:

– the relative increase of resistance $\Delta\chi_{LU}$ is the sum of two components: $\lambda\cdot k_1$, and μ.

– the component $\mu = C'b^2$ depends on b only; its value is always positive, and for varying r_2 and t the value of μ remains unchanged.

– the component $\lambda\cdot k_1$ depends on r_2 and t; for varying b the value of k_1 remains unchanged.

– because of the positive value of μ, for rectangle-shaped piezoresistor the surface area of positive value of $\Delta\chi$ is larger than for arc-shaped piezoresistor.

– the value of "neutral" radius $r_2 = r_{20}$ depends on t and b, and is approximately equal to (31) :

$$r_{20} = \frac{t + \sqrt{t^2 + 2\left(a^2 - b^2\right)}}{2} \tag{31}$$

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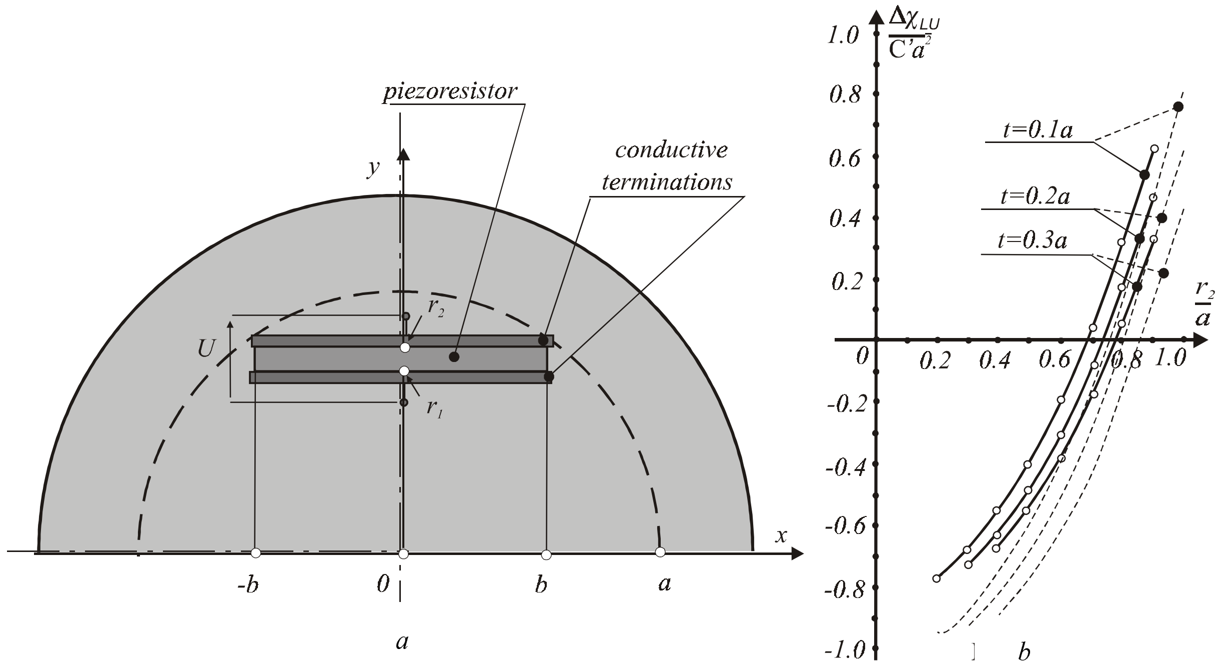

Fig. 3. a) Schematic arrangement of the rectangle-shaped piezoresistor; b) The graph of $\Delta\chi_{LU}$ at various values of t (the thin dashed curves correspond to arc-shaped piezoresistors).

6. Conclusions

The shape of the thick film piezoresistor can be of importance – it seems in some cases at least – when the arrangement on the membrane is considered. Then the following observations may be useful:

– at the edge of the membrane ($r_2 = a$), the arc-shaped piezoresistor exhibits $\Delta\chi_{LU}$=0.803 λ (at t=0.1a): when the greatest possible positive value of $\Delta\chi$ is required, the piezoresistor should be arc-shaped and placed at the very edge of the membrane. However, if (even small) distance from the edge should be kept, the rectangle-shaped piezoresistor exhibits greater values of $\Delta\chi$ than the arc-shaped one; the increase of $\Delta\chi$ is proportional to squared breadth b^2.

– when the maximum of negative value of $\Delta\chi$ is required, the arc-shaped piezoresistor is adequate. Moreover, if a given $\Delta\chi$ must be obtained, the appropriate arc-shaped piezoresistor needs to be placed less closely to the membrane centre than the rectangle-shaped piezoresistor.

– because the “neutral” radius r_{20} for rectangle-shaped piezoresistors is shifted nearer the membrane centre than for arc-shaped piezoresistors, the position of corrective piezoresistors should be shifted, correspondingly.

– the influence of spread of given values r_2 and t is less significant for piezoresistors placed nearer the membrane centre.

– The choice of the shape of the thick film piezoresistor can be a helpful supplementary resource for designers and manufacturers of pressure sensors.

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