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RESEARCH OF DYNAMICS OF THE VIBRATIONAL SYSTEM BY DESIGN MODELING TOOLS

Для вибору конструкційних та кінематичних параметрів вібраційної системи оброблення виробів з метою підвищення її продуктивності запропоновано методику дослідження її динаміки на основі нелінійної комплексної математичної моделі руху системи "вібраційна система – оброблювальне середовище". Як приклад наведено модель системи з двома вібрознудниками, які мають незалежний привод, та робочого середовища її контейнера.

Ключові слова: вібраційна система, нелінійна комплексна математична модель.

In order to calculate designed parameters for vibrational machine of volumetric vibrational treatment with the purpose to increase its productivity, the technique for investigating its dynamics based on non-linear complex mathematical model of the "vibromachine – treating medium" – system is suggested in this paper. As an example models of the machine with two independently driven vibrational exciters and the treating medium of its container are described.

Key words: vibrational machine, non-linear complex mathematical model.

Raising of problem. Widespread application of volumetric treatment calls for a new approach to calculating and designing vibromachines (unbalanced drives, elastic elements, container cases, and other dynamically loaded parts), investigation of the processes which take place in the loaded container of the vibromachine while it moves as well as the influence of its parameters upon the process of treating, creation of the methods of computer designing of machines in general, and therefore thorough mathematical description of dynamic processes which take place in vibromachines during their work. Dynamic processes which take place in such complicated systems were described mainly in linear formulation that usually unadequately describes real physical phenomena of the process. The necessities of practice call for prognostication of treatment results depending on the form of the parts treated, their structures, physico-mechanical properties kind of friable treating medium as well as vibromachine characteristics taking into account the non-linear forces which take place in the case of their interaction. Solution of such complex problems which take into account the influence of the drive upon dynamics of different kinds of the medium and the body treated enables us to prognosticate the intensivity of interaction between them (process of removing metal layer or superficial deformation of the parts treated), to properly choose geometrical and kinematic parameters of the vibromachine, and finally to lower material – and energy imputs as well as to increase the intensity of the treatment of articles.

It is suggested to solve the problems of this type through the elaboration and investigation of complex non-linear mathematic models of the "vibromachine – treating medium" – system by means of creating interconnected models of the motion of working tool of machine and friable medium; and on the basis of this dynamic phenomena of the system should be investigated in order to determine the influence of machine parameters and the medium upon factors of intensivity of article treatment. The models are derived with taking into account unification and adequacy. The former (unification) enables us to apply them for designing and operating many types of machines provided they are parameterized; and this leads to time and material imputs reduction. The latter, because of the complication of the dynamics of "vibromachine – treating medium" – system, is possible only when the models obtained are non-linear.

Raising of task. By an example of vibromachine with two independently driven unbalance vibrational exciters, the complex model of the vibromachine and its medium is shown below.

Vibromachine with two independently driven unbalance vibrational exciters are of several peculiarities which advantage them over other vibromachines. These advantages are the following: a) generation of oscillations of various forms (because of the two unbalance vibrational exciters), b) the possibility of the separation of the articles from the treating medium by means of redistribution of the oscillation amplitude of the working container on its plane during the treatment of articles or at the completion of treatment process due to the two independently driven vibrational exciters, c) relative simplicity of their designs and relatively high reliability of their parts in operation, d) their universality for modelling: the model includes, for example, the values of the masses of both unbalanced masses arbitrarily placed in the vibromachine; taking zero – value for one of the unbalanced masses enables us to employ this model for single – balance vibromachines with the unbalanced mass arbitrary placed with respect to its working container. In its turn, by means of the model derived, this enables us to investigate dynamic processes in vibromachines of other types; e) the possibility of automation of the process of volumetric vibrotreatment using the software elaborated on the basis of adequate mathematical models obtained as well as using results of investigating dynamic processes in vibromachines.

Exposition of basic material. For the creation of the complex model of "vibromachine – treating medium" – system, separate models describing the motion of vibromachine container and the medium were drived, and finally they are united into the complex model of the system.

To derive the vibromachine container motion model, its diagram is drown (Fig. 1) where the vibromachine is depicted as plane mechanical system having several degrees of freedom, and the main geometric and kinematic parameters are given symbolically (parametrically).

Using Lagrange's equations of II-nd type, the motion of the vibromachine container is described by means of the system of non-linear differential equations:

$$\begin{cases} \ddot{x}_c + \omega^2 x_c = \varepsilon f_x(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega_1 t + \alpha_0, \omega_2 t + \psi_0); \\ \ddot{y}_c + \omega^2 y_c = \varepsilon f_y(\varphi, \dot{\varphi}, \ddot{\varphi}, \omega_1 t + \alpha_0, \omega_2 t + \psi_0); \\ \ddot{\varphi} + \omega_\varphi(t)\varphi = \varepsilon f_\varphi(\varphi, \dot{\varphi}, \ddot{x}_c, \ddot{y}_c), \end{cases} \tag{1}$$

where x_c, y_c are coordinates of geometric centre of the container at arbitrary time instant t; φ – angle of its turn; ω – frequency of natural oscillations of the container; ε – small parameter; f_x, f_y, f_φ – functions which take into account physical and geometric non-linearity of mechanical system; $\omega_\varphi(t)$ – frequency of circular oscillations of container, unbalances taken into account; $\omega_i\,(i = 1,2)$ – angular velocities of vibrational exciters motion, $\omega_1 t + \alpha_0$ and $\omega_2 t + \psi_0$ are their phases.

The first approximation of the solutions of system (1) is obtained by means of asymptotic methods of non-linear mechanics:

$$\begin{cases} x_{o3} = x_0\, sin(\omega t + \alpha_x) + \dfrac{\varepsilon}{\omega} \displaystyle\int_0^t f_x\left(\varphi^*, \dot{\varphi}^*, \ddot{\varphi}^*, \omega_1 t + \alpha_0, \omega_2 t + \psi_0\right) sin\left(\omega(t-u)\right) du; \\ y_{o3} = y_0\, sin(\omega t + \alpha_x) + \dfrac{\varepsilon}{\omega} \displaystyle\int_0^t f_y\left(\varphi^*, \dot{\varphi}^*, \ddot{\varphi}^*, \omega_1 t + \alpha_0, \omega_2 t + \psi_0\right) sin\left(\omega(t-u)\right) du; \\ \varphi^* = \varphi_0\, cos\left(\omega_0 t + \theta(t)\right), \end{cases} \tag{2}$$

where $\theta(t)$ is known function; ω_0, x_0, y_0, α_x and α_y are constants determined by parameters of the system and its initial condition.

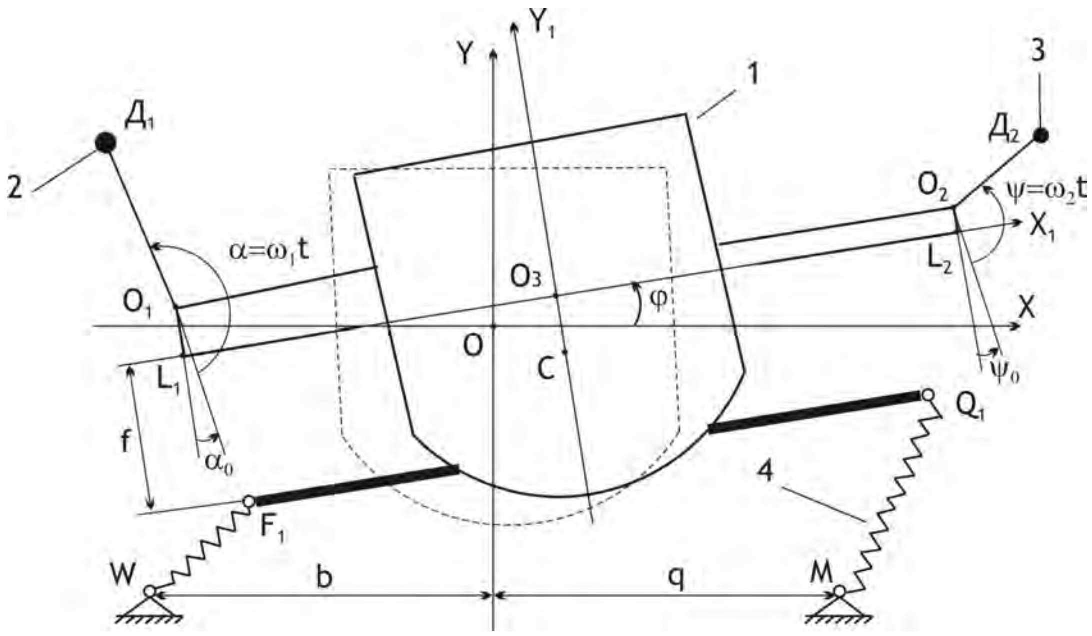

Fig. 1. Diagram of generalized two-unbalance vibromachine: 1 – working container; 2 – left unbalance ($Д_1$); 3 – right unbalance ($Д_2$); XOY – fixed coordinate system; $X_1O_3Y_1$ – moving coordinate system connected with geometric centre and symmetry axes of container; C – mass centre of container; f – distance between geometric centre and bottom of container; O_3L_1 and O_3L_2 – displacement of centers of rotation of unbalances with respect to vertical symmetre axis of container; WF_1 and MQ_1 – lengthes of suspension springs at arbitrary instant of time during the motion of container

An important stage in deriving models of such kind is the analysis of the stability of the solutions obtained in the form (2), i.e. the determination of the influence of parameters of this equation upon the character of its solution. Such analysis is necessary for ensuring perfect adequacy of the mathematical model to the real physical process both during designing and in the course of operating. It is necessary that the equation deseribing the model to have solution at intervals of time of any duration (the duration of treating articles in vibromachine may be considered unlimited). These laws of motion must meet the operational requirements of the system during the time interval of treating. As the result of investigation, it is found that for really existing parameters of the vibromachine the derived system of analytical expressions describes stable motion of the container.

The next stage of the work is the creation of the treating medium motion model of the vibromachine container [4]. For this purpose, the following hypotheses are assumed:

1. The material of the medium is continuous and homogeneous represented as stratification (multy-layer) of planar elastic-plastic beams, the thickness of which is considerably less than their length, and which in certain way contact with the container walls; in mathematical model this enables us to take into account both different kinds of interaction of working medium with the container and the motion of the container itself.

2. The medium moves layerwise and is in compound motion (in the plane of the motion of the container); the transportation motion of a layer is the motion together with the vibromachine container, relative motion of a layer of the medium is the longitudinal oscillations along layer ξ. The schematic diagram of the compound motion of the cross-section of layer i of the medium (point M of a volume element of the medium layer) is drawn in Fig. 2. Such assumption enables us to mathematically describe the circulation of the medium in the container (volumwise) at different velocities.

3. The correlation between stress and deformation in the medium material is described by non-linear Föcht's law; the following two kinds of non-linearity are considered here:

a) non linearity of viscous stresses $\sigma = E\zeta + k_0 \left(\frac{d\zeta}{dt} \right)^{\nu+1}$; (3)

b) non linearity caused by elastic properties of the medium $\sigma = E(\zeta)^{\nu+1} + k_0 \frac{d\zeta}{dt}$, (4)

where σ is normal stress in a layer of the medium; E, k_0, ν are constants which characterize elastic and viscouse properties of the medium; $\zeta = \frac{\partial u}{\partial \xi}$ is its relative deformation (here $u = u(\xi, t)$ is displacement of arbitrary crose – section of medium layer along axis ξ during a small interval of time t).

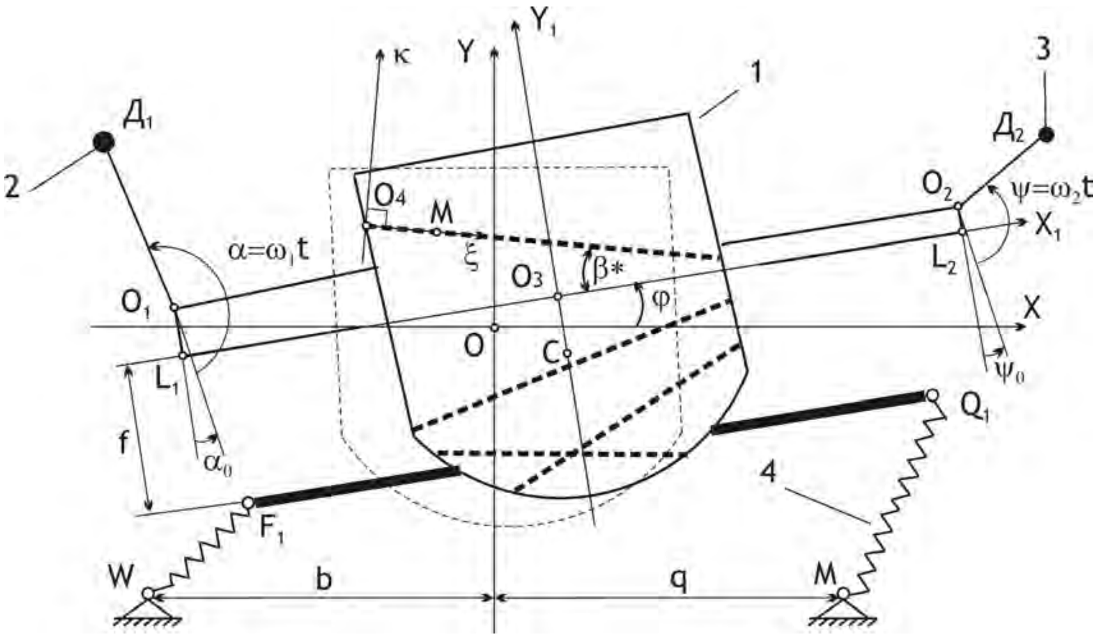

Fig. 2. Graphic representation of vibromachine medium motion model: 1–working container; 2–i^th layer of the medium that represent elastic-plastic beam which elastically contacts the walls, of container; $\xi O_4 \kappa$ – coordinate system bound with i^th layer of the medium; $u(\xi, t)$ (point M) – arbitrary cross-section of i^th layer of the medium, in the relative motion this layer oscillates along axis ξ; β^* – angle between coordinate system $\xi O_4 \kappa$ and moving coordinate system $Y_1O_3X_1$ which is bound with the geometric centre of the container (angle of inclination of i^th layer of the medium to the bottom of the container)

4. In a layer of the medium as well as between leyers, internal friction force is determined by means of Bolotin's law:

$$R = \frac{\partial u}{\partial t}\left(B + B_0 u^2 \right), \tag{5}$$

where B, B_0 are constants determined by the kind of medium material.

Under the assumptions mentioned, the equation of longitudinal oscillations (equation of relative motion of cross-section) of the medium layer takes the following forms:

a) form (3) in the case on non-linear Föcht's law

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial^2 u}{\partial \xi^2} - \beta \left(\frac{\partial^3 u}{\partial \xi^2 \partial t} \right)^{\nu+1} = f(t) + \frac{\partial u}{\partial t}\left(\vartheta + \delta u^2 \right); \tag{6}$$

b) form (4) in the case of nonlinear-elastic properties of friable medium

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial}{\partial\xi}\left(\frac{\partial u}{\partial\xi}\right)^{\nu+1}-\beta\frac{\partial^3 u}{\partial\xi^2\partial t}=f(t)+\frac{\partial u}{\partial t}\left(\vartheta+\delta u^2\right), \qquad (7)$$

where α, β, ϑ, δ are coefficients determined by the kind of medium material (metal balls, abrasive, etc.); $f(t)$ is external disturbance caused by the motion of drive unbalanced masses.

It is assumed that in equations (6) and (7) the forces of viscouse friction are small in comparison with non-linear-elastic (restoring) force in this layer, i.e. $\beta,\vartheta,\delta \ll \alpha^2$.

The model of medium layer motion with non-linear viscouse component of stress is of the form:

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial^2 u}{\partial\xi^2}=\varepsilon\left[\left(\frac{\partial^3 u}{\partial\xi^2\partial t}\right)^{\nu+1}+\left(\vartheta_1+\delta_1 u^2\right)\frac{\partial u}{\partial t}+b_1\sin\mu t\right], \qquad (8)$$

Where εb_1 and μ are parameters expressing the influence of external disturbance on the medium motion.

Unifrequent dynamic processes in the medium the motion of which can be described by means of (8), provided the interaction between the medium and the vibromachine container corresponds to hinge model of the contact between the medium and the working container ($u(0,t)=u(l,t)=0$), is described by the formula:

$$u_k(\xi,t)=a(t)\tilde{\Xi}_k(\xi)\cos(\omega_k^* t+\theta(t)), \qquad (9)$$

where $\omega_k^*=\alpha\frac{k\pi}{l}$ (here l is geometric parameter of container); $k=1,2,...$.

The model of medium layer motion with non-linear stress component is of the form

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial}{\partial\xi}\left(\frac{\partial u}{\partial\xi}\right)^{\nu+1}=\varepsilon\left[b_1\sin\mu t-(\vartheta_1+\delta_1)\frac{\partial u}{\partial t}u^2+\beta\frac{\partial^3 u}{\partial\xi^2\partial t}\right]. \qquad (10)$$

The solutions of differential equations are obtained by means of special Ateb-functions.

For stabilized mode of motion the equation of medium layer motion under the condition of rigid contact between the medium and container is of the form

$$u(\xi,t)=a(\nu)sa\left(1,\frac{1}{\nu+1},\Pi_\xi\frac{\xi}{l}\right)ca(\nu+1,1,\psi), \qquad (11)$$

where

$$a(\nu)=\sqrt{\left(\frac{8\pi\Gamma^2\left(1+\frac{\nu}{2}\right)\Gamma^2\left(\frac{1}{\nu+2}\right)}{(\nu+2)^2 l\Gamma^2\left(\frac{3}{2}+\frac{1}{\nu+2}\right)}-\frac{4\vartheta_1\sqrt{\pi}\,pl\Gamma\left(\frac{1}{\nu+2}\right)}{(\nu+2)\Gamma\left(\frac{3}{2}+\frac{1}{\nu+2}\right)}\right)\left(\frac{3\delta_1\pi\Gamma\left(\frac{3}{\nu+2}\right)\Gamma\left(\frac{\nu+1}{\nu+2}\right)}{8\Gamma\left(\frac{3}{2}+\frac{3}{\nu+2}\right)\Gamma\left(\frac{5}{2}+\frac{\nu+1}{\nu+2}\right)}\cdot\frac{l}{\Pi_\xi}\right)^{-1}};$$

$\Pi_\xi=\sqrt{\pi}\Gamma\left(\frac{\nu+1}{\nu+2}\right)\Gamma^{-1}\left(\frac{1}{2}+\frac{\nu+1}{\nu+2}\right)$; $\Gamma(...)$ is γ – function of corresponding argument.

Finally, the complex model of “vibromachine-treating medium”-system takes the form[5]:

$$\begin{cases} x_M(t) = x_{03}(t) + S\sin\varphi(t) - w\cos\varphi(t) - q\sin\varphi(t) + (\xi + u(\xi,t))\cos\left(\varphi(t) + \beta^*\right), \\ y_M(t) = y_{03}(t) - S\cos\varphi(t) - w\sin\varphi(t) + q\cos\varphi(t) + (\xi + u(\xi,t))\sin\left(\varphi(t) + \beta^*\right), \end{cases} \quad (12)$$

where $S = O_3C$ is the distance between geometric centre of the container and the mass centre of the medium, $\xi = O_4M$, is a position of an element of medium layer at initial instant of time, $u(\xi,t)$ is the law of longitudinal oscillations of the layer (law of relative motion), which is described by corresponding equations for a resonance case of medium motion or non-resonance one, $w = LO_3$, $q = LO_4$.

Conclusion. By means of the complex model obtained it is possible to investigate different modes of the motion of the vibromachine and the medium, to plot trajectories of motion for arbitrary points of the container and the medium, to investigate the influence of the parameters of the vibromachine and the medium on the factors of the intensivity of articles treatment according to corresponding criterion of intensivity.

1. Митропольский Ю. А., Мосеенков Б. И. Асимптотические решения уравнений в частичных производных. – К.: Вища школа, 1976. – 592 с. 2. Субач А. П. Динамика процессов и машин объемной обработки. – Рига: Зинатне, 1991. – 240 с. 3. Опірський Б. Я., Денисов П. Д. Новые вибрационные станки, конструирование и расчет. – Львів: Світ, 1991. – 158 с. 4. Стоцько З. А., Топільницький В. Г., Кусий Я. М. “Математична модель опису динаміки вібраційного сепаратора з дебалансним приводом”, Автоматизація виробничих процесів в машинобудуванні та приладобудуванні. Міжгалузевий збірник наукових праць. – Львів: Видавництво Львівської політехніки. – 2013. – Вип. 47. – С. 28–36. 5. Zinovij Stotsko, Bogdan Sokil, Volodymyr Topilnytskyj, Darija Rebot. The influence of the loose medium parameters on the process of vibratory separation. Journal of Manufacturing and Industrial Engineering. Vol 12, No 1–2 (2013), s.17–19. 6. Стоцько З. А., Топільницький В. Г., Кусий Я. М. “Математична модель опису динаміки вібраційного сепаратора з дебалансним приводом”, Автоматизація виробничих процесів в машинобудуванні та приладобудуванні. Міжгалузевий збірник наукових праць. – Львів: Вид-во Львівської політехніки. – 2013. – Вип. 47. – С. 28–36.