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PROBLEM WITH HOMOGENEOUS INTEGRAL CONDITION FOR NONHOMOGENEOUS EVOLUTION EQUATION

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We propose a method of solving the problem with homogeneous integral condition for non-homogeneous evolution equation with abstract operator in linear space H. For the right-hand side of the equation, which for fixed t belongs to special subspace $N \subseteq H$ and is represented as a Stieltjes integral over a certain measure, the solution of the problem is also represented as a Stieltjes integral over the same measure.

Key words: differential-symbol method, evolution equation, problems with integral conditions.

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Introduction

The semigroup theory is very important tool for the research on problems for evolution equations in Banach spaces (see, e.g., [1–4]).

Nowadays, problems with integral conditions become more and more popular since they arise when modeling a lot of physical phenomena, e.g. diffusion of particles in a turbulent medium, processes of heat conduction, moisture transfer in capillary-porous media, problems of describing the dynamics of population and problems of demography as well. Similar problems have been studied in a lot of works (see, e. g., works [5–10]).

I. Statement of the problem

Let H be a linear space and A be a given linear operator acting in it ($A\colon H \to H$).

Assume that for $\lambda \in \mathbb{C}$ exists in H the solution of the equation

$$Ay = \lambda y,$$

which obviously is the eigenvector $y(\lambda)$ of the operator A, which corresponds to its eigenvalue $\lambda \in \mathbb{C}$.

Consider entire function

$$a(\lambda) = \sum_{k=0}^{\infty} a_k \lambda^k,$$

which is not a constant and is the symbol of linear operator $a(A)$.

Since $A^j y(\lambda) = \lambda^j y(\lambda)$, $j = 2, 3, \ldots$, we conclude

$$a(A)y(\lambda) = a(\lambda)y(\lambda), \quad \lambda \in \mathbb{C}. \tag{1}$$

Note that, for the operator $a\left(\frac{d}{dx}\right)$ and the eigenvector $y(\lambda) = e^{\lambda x}$ of the operator $A = \frac{d}{dx}$ in the space $H = C^\infty(\mathbb{R})$, equality (1) gets the form

$$a\left(\frac{d}{dx}\right) e^{\lambda x} = a(\lambda)e^{\lambda x}. \tag{2}$$

Definition 1. We shall say that for arbitrary $t \in (0, T)$, $T > 0$, vector $f(t)$ from H belongs to N_F, if on $\Lambda \subseteq \mathbb{C}$ there exist a measure $\mu(\lambda)$ and analytical in t linear operator $F_f(t, \lambda)\colon H \to H$ such that $f(t)$ can be represented in the form of Stieltjes integral

$$f(t) = \int_\Lambda F_f(t, \lambda)y(\lambda)d\mu(\lambda). \tag{3}$$

We consider for abstract operator $A : H \to H$ the following problem:

$$\left[\frac{d}{dt} - a(A)\right] U(t) = f(t), \quad t \in (0, T), \tag{4}$$

$$\int_0^T U(t)dt = 0, \tag{5}$$

where $U : (0, T) \to H$ is an unknown vector-function, $f\colon (0, T) \to H$ is a given vector-function from N_F, i.e. can be represented in the form (3).

In this paper, we will show an approach to solving the abstract problem (4), (5). Note that this work is the continuation of previous investigations (see [11–13]).

II. Main results

Consider the entire function

$$\eta(\lambda) \equiv \int_0^T e^{a(\lambda)t}dt = \frac{e^{a(\lambda)T} - 1}{a(\lambda)}. \tag{6}$$

Also we consider the entire function with respect to t, ν and λ

$$F(t,\nu,\lambda)\equiv\int\limits_0^t e^{\nu\tau+a(\lambda)(t-\tau)}d\tau=\frac{e^{\nu t}-e^{a(\lambda)t}}{\nu-a(\lambda)}. \quad (7)$$

Note that function (7) is a solution of the Cauchy problem

$$\left[\frac{d}{dt}-a(\lambda)\right]F=e^{\nu t},$$

$$F(0,\nu,\lambda)=0.$$

Denote by P the set of zeros of function (6):

$$P=\left\{\lambda\in\mathbb{C}\colon\ a(\lambda)T=2\pi ki,\ i^2=-1,\ k\in\mathbb{Z}\backslash\{0\}\right\}. \quad (8)$$

Theorem 1. *Function of the form*

$$G\equiv G(t,\nu,\lambda)=\frac{\nu\eta(\lambda)e^{\nu t}-e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)} \quad (9)$$

has the following properties:

1) $G(t,\cdot,\lambda)$ is entire function for $\lambda\in\mathbb{C}\setminus P$, where P is set (8)*;*

2) $G(t,\nu,\cdot)$ is analytical on $\mathbb{C}\setminus P$ function for $\nu\in\mathbb{C}$;

3) $G(t,\nu,\lambda)$ satisfies the equation

$$\left[\frac{d}{dt}-a(\lambda)\right]G=e^{\nu t}; \quad (10)$$

4) $G(t,\nu,\lambda)$ satisfies the integral condition

$$\int\limits_0^T Gdt=0; \quad (11)$$

□ *Proof.* For proving properties 1) and 2) we shall show that function (9) can be represented in the form

$$G(t,\nu,\lambda)=F(t,\nu,\lambda)-\frac{F(T,\nu,\lambda)-F(T,0,\lambda)}{\nu}\cdot\frac{e^{a(\lambda)t}}{\eta(\lambda)}, \quad (12)$$

where F is function (7), $\eta(\nu)$ is function (6).

In fact,

$$\frac{e^{\nu t}-e^{a(\lambda)t}}{\nu-a(\lambda)}-\frac{\left[\frac{e^{\nu T}-e^{a(\lambda)T}}{\nu-a(\lambda)}-\frac{1-e^{a(\lambda)T}}{-a(\lambda)}\right]}{\nu}\cdot\frac{e^{a(\lambda)t}}{\eta(\lambda)}=$$

$$=\frac{e^{\nu t}-e^{a(\lambda)t}}{\nu-a(\lambda)}-$$

$$-\frac{a(\lambda)\left(e^{\nu T}-e^{a(\lambda)T}\right)+(\nu-a(\lambda))\left(1-e^{a(\lambda)T}\right)}{\nu a(\lambda)(\nu-a(\lambda))}\cdot\frac{e^{a(\lambda)t}}{\eta(\lambda)}=$$

$$=\frac{e^{\nu t}-e^{a(\lambda)t}}{\nu-a(\lambda)}-\frac{a(\lambda)e^{\nu T}+\nu-a(\lambda)-\nu e^{a(\lambda)T}}{\nu a(\lambda)(\nu-a(\lambda))}\cdot\frac{e^{a(\lambda)t}}{\eta(\lambda)}=$$

$$=\frac{e^{\nu t}-e^{a(\lambda)t}}{\nu-a(\lambda)}-\frac{a(\lambda)\left[e^{\nu T}-1\right]-\nu\left[e^{a(\lambda)T}-1\right]}{\nu a(\lambda)(\nu-a(\lambda))}\cdot\frac{e^{a(\lambda)t}}{\eta(\lambda)}=$$

$$=\frac{e^{\nu t}}{\nu-a(\lambda)}-\frac{\left[e^{\nu T}-1\right]e^{a(\lambda)t}}{\nu(\nu-a(\lambda))\eta(\lambda)}=G(t,\nu,\lambda).$$

From the representation (12) of function G, and the fact that the function $F(t,\nu,\lambda)$ as well as the function $\frac{F(T,\nu,\lambda)-F(T,0,\lambda)}{\nu}$ are entire with respect to ν and λ, we conclude that properties 1) and 2) hold.

Let's show that function (9) satisfies equation (10):

$$\left[\frac{d}{dt}-a(\lambda)\right]\frac{\nu\eta(\lambda)e^{\nu t}-e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)}=$$

$$=\frac{d}{dt}\frac{\nu\eta(\lambda)e^{\nu t}-e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)}-$$

$$-a(\lambda)\frac{\nu\eta(\lambda)e^{\nu t}-e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)}=$$

$$=\frac{\nu^2\eta(\lambda)e^{\nu t}-a(\lambda)e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)}-$$

$$-a(\lambda)\frac{\nu\eta(\lambda)e^{\nu t}-e^{a(\lambda)t}\left(e^{\nu T}-1\right)}{\nu(\nu-a(\lambda))\eta(\lambda)}=$$

$$=\frac{\nu^2\eta(\lambda)e^{\nu t}-a(\lambda)\nu\eta(\lambda)e^{\nu t}}{\nu(\nu-a(\lambda))\eta(\lambda)}=e^{\nu t}.$$

Next we shall prove the realization of integral condition (11):

$$\int\limits_0^T Gdt=\frac{1}{\nu-a(\lambda)}\left(\int\limits_0^T e^{\nu t}dt-\frac{e^{\nu T}-1}{\nu\eta(\lambda)}\int\limits_0^T e^{a(\lambda)t}dt\right)=$$

$$=\frac{1}{\nu-a(\lambda)}\left(\frac{e^{\nu T}-1}{\nu}-\frac{e^{\nu T}-1}{\nu\eta(\lambda)}\frac{e^{a(\lambda)T}-1}{a(\lambda)}\right)=$$

$$=\frac{e^{\nu T}-1}{\nu(\nu-a(\lambda))}\left(1-\frac{1}{\eta(\nu)}\eta(\nu)\right)=0.$$

This completes our proof. ■

Lemma 1. *On the set $(0,T)\times\mathbb{C}\times(\mathbb{C}\setminus P)$ there holds the following identity*

$$\left[\frac{d}{dt}-a(A)\right]\left\{G(t,\nu,\lambda)y(\lambda)\right\}\equiv e^{\nu t}y(\lambda).$$

□ *Proof.* From equality (1) and property 3) of function G, for arbitrary $(t,\nu,\lambda)\in(0,T)\times\mathbb{C}\times(\mathbb{C}\setminus P)$ we have

$$\left[\frac{d}{dt}-a(A)\right]\left\{G(t,\nu,\lambda)y(\lambda)\right\}=$$

$$=\frac{d}{dt}\left\{G(t,\nu,\lambda)y(\lambda)\right\}-a(A)\left\{G(t,\nu,\lambda)y(\lambda)\right\}=$$

$$=\frac{d}{dt}\left\{G(t,\nu,\lambda)y(\lambda)\right\}-a(\lambda)\left\{G(t,\nu,\lambda)y(\lambda)\right\}=$$

$$=\left[\frac{d}{dt}-a(\lambda)\right]\left\{G(t,\nu,\lambda)y(\lambda)\right\}=$$

$$=\left\{\left[\frac{d}{dt}-a(\lambda)\right]G(t,\nu,\lambda)\right\}y(\lambda)=e^{\nu t}y(\lambda).$$

This proves our lemma. ■

Theorem 2. *Let the vector-function $f(t)$ in equation (4) for arbitrary $t \in (0,T)$ belongs to N_F, i.e. could be represented in the form (3), and $G(t,\nu,\lambda)$ be function (9), $\Lambda = \mathbb{C} \setminus P$, where P is the set (8). Then the formula*

$$U(t) = \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{G(t,\nu,\lambda)y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) \quad (13)$$

defines a formal solution of problem (4), (5).

☐ *Proof.* We shall show that vector-function (13) satisfies equation (4). In fact, using the lemma 1 and equality (2), we obtain:

$$\left[\frac{d}{dt} - a(A)\right]U(t) =$$

$$= \left[\frac{d}{dt} - a(A)\right]\int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{G(t,\nu,\lambda)y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{\left[\frac{d}{dt} - a(A)\right]G(t,\nu,\lambda)y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{\left[\frac{d}{dt} - a(\lambda)\right]G(t,\nu,\lambda)y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{e^{\nu t}y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f(t,\lambda)\left\{e^{\nu t}y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\,(t,\lambda)\,y(\lambda)d\mu(\lambda) = f(t).$$

Besides, from the property 4) of function G and the linearity of operator F_f, we obtain:

$$\int_0^T U(t)dt =$$

$$= \int_0^T\left\{\int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\{G(t,\nu,\lambda)y(\lambda)\}\Big|_{\nu=0} d\mu(\lambda)\right\}dt =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{\int_0^T G(t,\nu,\lambda)dt\right\}y(\lambda)\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\{0\}\,y(\lambda)\Big|_{\nu=0} d\mu(\lambda) = 0.$$

This proves our theorem. ■

Remark 1. The Stieltjes integral in formula (13) is taken only on $\Lambda = \mathbb{C} \setminus P$ since the function $G(t,\nu,\lambda)$ by the property 2) is analytical with respect to λ in this domain.

Remark 2. Vector-function (13) defines a formal solution of the problem (4), (5), since we assume the following equalities to hold:

$$\left[\frac{d}{dt} - a(A)\right]\int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{G(t,\nu,\lambda,)y(\lambda)\right\}\Big|_{\nu=0} d\mu(\lambda) =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{\left[\frac{d}{dt} - a(A)\right]G(t,\nu,\lambda)y(\lambda)\right\}\Big|_{\nu=0} d\mu_f(\lambda),$$

$$\int_0^T\left\{\int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\{G(t,\nu,\lambda)y(\lambda)\}\Big|_{\nu=0} d\mu(\lambda)\right\}dt =$$

$$= \int_\Lambda F_f\left(\frac{d}{d\nu},\lambda\right)\left\{\int_0^T G(t,\nu,\lambda)dt\right\}y(\lambda)\Big|_{\nu=0} d\mu(\lambda).$$

as well as the integrals in those formulas to exist.

Remark 3. If $a(\lambda) \equiv a = \text{const}$ and $aT \neq 2\pi ki, k \in \mathbb{Z}\setminus\{0\}$ then $P = \varnothing$, so in Theorem 2 $\Lambda = \mathbb{C}$. Whereas if for $k_0 \in \mathbb{Z}\setminus\{0\}$ the condition $aT = 2\pi k_0 i$ holds, then $P = \mathbb{C}$. In this case, a solution of problem (4), (5) does not exist for $f(t) \not\equiv 0$, and the problem has nonzero solutions when $f(t) \equiv 0$.

III. Applying the abstract approach to solving a problem with integral condition for PDE

Consider the problem

$$\left[\frac{\partial}{\partial t} - a\left(\frac{\partial}{\partial x}\right)\right]U(t,x) = f(t,x),\ (t,x) \in (0,T)\times\mathbb{R}, \quad (14)$$

$$\int_0^T U(t,x)dt = 0, \quad x \in \mathbb{R}, \quad (15)$$

where $a\left(\frac{\partial}{\partial x}\right)$ is a differential expression with entire symbol $a(\lambda) \neq \text{const}$. Problem (14), (15) has been studied in the work [14] by means of the differential-symbol method [15, 16].

We can treat problem (14), (15) as a particular case of problem (4), (5) with operator $A = \frac{\partial}{\partial x}$ and its eigenvector $e^{\lambda x}$.

Let H be a class of entire functions $f(t,x)$ and N_F be a subclass of H, namely:

N_F is a class of entire functions $f(t,x)$ which for any $t \in (0,T)$ are quasipolynomials from $K_{\mathbb{C}\setminus P}$ [14], i.e.

$$f(t,x) = \sum_{j=1}^m Q_j(t,x)e^{\alpha_j x},$$

where $\alpha_1, \ldots, \alpha_m \in \mathbb{C} \setminus P$, $Q_1(t,x), \ldots, Q_m(t,x)$ are entire functions which for $t \in (0,T)$ are polynomials, $m \in \mathbb{N}$. For $f \in N_F$, as a measure $\mu(\lambda)$ on $\Lambda = \mathbb{C} \setminus P$, we take the Dirac delta-measure on Λ, for which the integral $\int_\Lambda g(\lambda)d\mu(\lambda)$ equals $g(0)$ when $0 \in \Lambda$ and equals 0 when $0 \notin \Lambda$. As an analytical linear operator F_f, we take the operator whose action is defined as follows:

$$F_f(t,\lambda)e^{\lambda x} \equiv f\left(t, \frac{\partial}{\partial \xi}\right) e^{\xi x}\Big|_{\xi=\lambda}.$$

We shall show that for $f \in N_F$ there holds the equality (analogue of equality (3)):

$$f(t,x) = \int_\Lambda F_f(t,\lambda)e^{\lambda x} d\mu(\lambda).$$

In fact,

$$\int_\Lambda F_f(t,\lambda)e^{\lambda x} d\mu(\lambda) = \int_\Lambda f\left(t, \frac{\partial}{\partial \lambda}\right) e^{\lambda x}\delta(\lambda)d\lambda =$$

$$= \int_\Lambda f(t,x)e^{\lambda x}\delta(\lambda)d\lambda = f(t,x)e^{\lambda x}\big|_{\lambda=0} = f(t,x).$$

From formula (13) of the solution of problem (4), (5), for problem (14), (15) we have

$$U(t,x) = \int_\Lambda f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right)\left\{G(t,\nu,\lambda)e^{\lambda x}\right\}\Bigg|_{\nu=0} d\mu(\lambda) =$$

$$= f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right)\left\{G(t,\nu,\lambda)e^{\lambda x}\right\}\Bigg|_{\lambda=\nu=0}.$$

We obtain the following result that has been obtained in the work [14, theorem 1] by means of the differential-symbol method.

Theorem 3. *Let $f(t,x)$ in equation (14) be entire function and, for arbitrary $t \in (0,T)$ belongs to $K_{\mathbb{C}\setminus P}$, where P is set (8). Then, in the class of entire functions $U(t,x)$ which for fixed $t \in (0,T)$ belong to $K_{\mathbb{C}\setminus P}$, there exists a unique solution of problem (14), (15), which can be represented in the form*

$$U(t,x) = f\left(\frac{\partial}{\partial \nu}, \frac{\partial}{\partial \lambda}\right)\left\{G(t,\nu,\lambda)e^{\lambda x}\right\}\Bigg|_{\lambda=\nu=0},$$

where $G(t,\nu,\lambda)$ is the function (9).

Conclusions

We propose an approach to solving a problem with homogeneous integral condition for nonhomogeneous evolution equation with abstract operator in a linear space. The solution of the problem is represented in the form of Stieltjes integral over a certain measure. We give an example of application of the proposed abstract approach to solving the problem with homogeneous integral time condition for nonhomogeneous PDE of first order in time and, in general, infinite order in spatial variable.

In future research, it would be interesting to develop a similar method for solving higher order differential-operator equations.

References

[1] *Hille E., Phillips R. S.* Functional analysis and semigroups. – Amer. Math. Soc., 1982. – **31.** – 820 p.

[2] *Krein S.* Linear differential equation in Banach space. – Amer. Math. Soc., 1971. – **29.** – 395 p.

[3] *Pazy A.* Semigroups of linear operators and applications to partial differential equations. – Springer-Verlag, New York, 1983. – 287 p.

[4] *Yosida K.* Functional analysis. – Springer-Verlag, New York, 1980. – 513 p.

[5] *Ионкин Н. И.* Решение одной краевой задачи в теории теплопроводности с нелокальными краевыми условиями // Дифференц. уравнения. – 1977. – **13**, №2. – С. 294–304.

[6] *Фардигола Л. В.* Критерий корректности в слое краевой задачи с интегральными условиями // Укр. мат. журн. – 1990. – **42**, № 11. – С. 1546–1551.

[7] *Ільків В. С., Магеровська Т. В.* Задача з інтегральними умовами для рівняння з частинними похідними другого порядку // Вісн. Нац. ун-ту "Львівська політехніка". Серія "Фіз.-мат. науки". – 2008. – № 625. – С. 12–19.

[8] *Cannon J. R.* The solution of the heat equation. Subject to the specification of energy // Quart. Appl. Math. – 1963. – **21**. – P. 155–160.

[9] *Molaei H.* Optimal control and problem with integral boundary conditions // Int. J. Contemp. Math. Sci. – 2011. – **6,** No. 48. – P. 2385–2390.

[10] *Bouziani A.*Initial boundary-value problems for a class of pseudoparabolic equations with integral boundary conditions // J. Math. Anal. Appl. – 2004. – **291**. – P. 371–386.

[11] *Kalenyuk P. I., Nytrebych Z. M., Drygaś P.* Method of solving the Cauchy problem for evolutionary equation in Banach space // Мат. методи та фіз.-мех. поля – 2004. – **47**, № 4. – С. 46–50.

[12] *Kalenyuk P. I., Kohut I. V., Nytrebych Z. M., Yarka U. B.* Problem with inhomogeneous integral time condition for a partial differential equation of the first order with respect to time and of infinite order

with respect to the space variables // J. Math. Sci. – 2014. – **198**, No. 1. – P. 1–12.

[13] *Каленюк П. І., Нитребич З. М., Когут І. В.* Однозначна розв'язність задачі з інтегральними умовами для рівняння із частинними похідними другого порядку за часом // Вісн. Нац. ун-ту "Львівська політехніка". Серія "Фіз.-мат. науки". – 2013. – № 768. – С. 5–11.

[14] *Kalenyuk P. I., Kohut I. V., Nytrebych Z. M.* Problem with integral condition for a partial differential equation of the first order with respect to time // J. Math. Sci. – 2012. – **181**, No. 3. – P. 293–304.

[15] *Каленюк П. І., Нитребич З. М.* Узагальнена схема відокремлення змінних. Диференціально-символьний метод. – Львів: Вид-во Нац. ун-ту "Львівська політехніка", 2002. – 292 с.

[16] *Каленюк П. И., Баранецкий Я. Е., Нитребич З. Н.* Обобщенный метод разделения переменных. – К.: Наук. думка, 1993. – 232 с.

ЗАДАЧА С ОДНОРОДНЫМ ИНТЕГРАЛЬНЫМ УСЛОВИЕМ ДЛЯ НЕОДНОРОДНОГО ЭВОЛЮЦИОННОГО УРАВНЕНИЯ

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Предложен метод решения задачи с однородным интегральным условием для неоднородного эволюционного уравнения с абстрактным оператором в линейном пространстве H. Для правой части уравнения, принадлежащей для фиксированного t специальному подпространству $N \subseteq H$ и представленной интегралом Стилтьеса по некоторой мере, решение задачи представлено тоже в виде интеграла Стилтьеса по этой же мере.

Ключевые слова: дифференциально-символьный метод, эволюционное уравнение, задачи с интегральными условиями.

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ЗАДАЧА З ОДНОРІДНОЮ ІНТЕГРАЛЬНОЮ УМОВОЮ ДЛЯ НЕОДНОРІДНОГО ЕВОЛЮЦІЙНОГО РІВНЯННЯ

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Запропоновано метод розв'язання задачі з однорідною інтегральною умовою для неоднорідного еволюційного рівняння з абстрактним оператором у лінійному просторі H. Для правої частини рівняння, що для фіксованого t належить до спеціального підпростору $N \subseteq H$ і зображається інтегралом Стілтьєса за деякою мірою, розв'язок задачі зображено також у вигляді інтеграла Стілтьєса за цією ж мірою.

Ключові слова: диференціально-символьний метод, еволюційне рівняння, задачі з інтегральними умовами.

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