On the set of variants of bar structures for topology optimization problem

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Abstract* – *The article suggests a method of specifying a set of variants of bar structures with variable number of nodes and bars for the formulation and solution of topology optimization problems. The suggested approach is implemented in the program OptCAD.

Keywords – bar structures, steel structures, topology optimization, discrete design variables.

I. Statement of the Problem

Bar steel structures are used in many types of buildings and structures. It is known that their technical and economic parameters are to a large extent determined by the accepted topology (position and number of nodes and bars) of structures [4].

To select the optimal topology of complex of several structures it is needed to compare possible variants of their topology. They are usually formed from combinations of variants of each topology structure. For example, topology variants of covering for industrial buildings consist of possible variants of truss and eaves truss. In this case, topology of separate structures may depend both on the parameters of the whole system and on the parameters of the other structures. For example, the number of panels of eaves truss depends on column spacing and the number of eaves truss which are between them. In such a case to determine the optimal parameters and topology of each structure it is needed to formulate and solve the structure optimization problem. To formulate the optimization problem, we need a method of forming a set of variants of topology structure.

II. The analysis of researches and publications

Optimization problem of geometry and cross sections of elements of bar structures is formulated as a problem of finding such values of design variables $\vec{X}$ under which the value of objective function is the smallest:

$$f\left(\vec{X}\right) \to \min . \qquad (1)$$

At the same time, constraints that describe the normative, technical and other requirements for the structure should be performed:

$$\psi_k\left(\vec{X}\right) = 0,\ k = \overline{1, N_\psi}, \qquad (2)$$

$$\varphi_p\left(\vec{X}\right) \le 0,\ p = \overline{N_\psi + 1, N_\varphi}, \qquad (3)$$

where $f\left(\vec{X}\right)$ – objective function; $\vec{X} = \left\{X_1, X_2, \ldots, X_{N_x}\right\}^T$ – vector of design variables; $\psi_k\left(\vec{X}\right)$ – equality constraint functions; $\varphi_p\left(\vec{X}\right)$ – inequality constraint functions; N_x – total number of design variables; N_ψ – number of equality constraints; N_φ – total number of constraints

Bar structure optimization problem consist in determining the optimal position and number of nodes and bars of structure [2]. Synthesis of variants of bar structure in the process of finding the optimal topology can occur by random or aimed variation of the original topology [1].

A common method of synthesis of new topological solutions for bar structures is ground structure method, first proposed in [5]. The essence of method is to remove "excess" bars from the initial ground structure, which includes determined by the designer grid of nodes and all possible combination of these nodes by bars. On the basis of "ground structure" method modified methods of synthesis of new topological solutions can be developed [6, 7, 8].

During the synthesis of new topological solutions by this method it is difficult to avoid the appearance of "bad" structures, which are not stable or instantaneously variable system, not technological or obviously unacceptable.

An alternative is the way in which for the synthesis of a new topology of structure position of bars in bar structure is changed [9]. The position of the bar in the bar structure is determined by two nodes, to which the beginning and end of the bar are joined. To change the position of the bar is enough to change the numbers of nodes to which the bar is joined. Possible positions of the bar are described by two sequences (tuples) of numbers of nodes. The exact position of the bar can be set by the ordinal number that identifies two nodes, one of the tuple.

Optimization algorithm modifies this ordinal number and, therefore, the position of the bar in the structure. Design variables in this way is the ordinal number of nodes in the tuples that define variants for positions of the bar.

It is possible to combine several tuples of nodes that define possible variants of position of several bars with the help of the value of one design variable. This allows the designer to set all the variants of structure on the begin stage of the data input of optimization problem and respectively control the emergence of "bad" structure.

This method was used in [9] to set the variants of construction topology with the same number of nodes and bars.

III. Aims and objectives of the research

The aim of research is to develop a method of specifying a set of variants of topology structure with different number of nodes and bars for optimization problem. While developing the method, the monitoring of the emergence of "bad" variants of topology must be ensured.

IV. The main material

It is consider the bar structure for which it is needed to determine the optimal topology. Vector of design variables in the optimization problem (1)-(3) of bar structure should contain variables, the current value of which select the variant of bar structure from some set of variants of topology. To specify set of variants of bar structure with different number of nodes and bars will use the discrete design variables, which in [9] are used to describe the position of separate bars.

It is divide the bar structure into two subsystems – basic and additional. Any required variant of bar structure we will form in the basic subsystem. From the elements that are not included in the basic subsystem in some variants of topology, we form an additional subsystem. Additional and basic subsystems must be stable, and not to influence strained state of each other. While solving the optimization problem in calculating the value of objective function and constraints we will take into consideration only the basic subsystem.

We denote the set of nodes of bar structure – N, and set of bars of bar structure – B. Number of nodes and bars in sets B and N for all variants of bar structure is the same. Therefore, in the sets B and N should be enough nodes and bars to form any variant of topology. The necessary number of nodes and bars in these sets is determined by comparing possible variants of topology. For most problems the number of nodes is determined from the variant in which there is the maximum number of nodes and the number of bars – from the variant which is the maximum number of bars.

Suppose sets N_1 and N_2 are subsets of the set N, and the set B_1 and B_2 are subsets the set B.

$$\begin{aligned} &N_1 \subseteq N,\\ &N_2 \subseteq N,\\ &N_1 \cup N_2 = N,\\ &N_1 \cap N_2 = \varnothing, \end{aligned} \tag{4}$$

$$\begin{aligned} &B_1 \subseteq B,\\ &B_2 \subseteq B,\\ &B_1 \cup B_2 = B,\\ &B_1 \cap B_2 = \varnothing. \end{aligned} \tag{5}$$

Subsets N_1 and B_1 contain nodes and bars of the basic subsystem and subsets N_2 and B_2 – additional. Subsets N_2 and B_2, except for nodes and bars that are used to form certain variants in the basic subsystem, can contain special nodes and bars that never belong to the sets N_1 and B_1. They are used to provide stability of an additional subsystem. In the set N_2 there are always two special nodes. They are fixed from displacement in all directions and serve as supports of an additional subsystem. In the set B_2 special bars are added only when in some variants all bars are used for formation the basic subsystem ($B_2 = \varnothing$), meanwhile not all the nodes from the set N are used (not counting special nodes). In this case, the unused nodes should be connected to the additional system where they need to be connected by special bars. For each such node we must have two special bars.

We form each variant of bar structure topology dividing nodes from set N and bars from set B between subsets (subsystems). To list the bar to the subset B_1 (B_2) it is needed to set its position using the nodes from subset N_1 (N_2). To list a node to the subset N_1 (N_2) it is needed to attach it to the bars from the subset B_1 (B_2).

It is showed the specify of design variables for setting the bar structure topology variants in OptCAD program [11] on the example of eaves truss. Eaves truss is simple supported with a span of 18 m. The load of 10 tons applied to the average node of the top part of the truss. The truss bars can have three types of rectangular tube cross sections, separately for the top, bottom parts and angle braces. We have to determine the type of grating on the truss, number of panels, size of cross sections of bars and truss height, under which the mass of eaves truss is minimal. Height of the truss can range from 1.5 to 3.5 meters. Possible variants (types of grating and the number of panels) of the truss are depicted in Figure 1.

To set the topology of truss described above method with using discrete design variables was used. The design variable X_t was formed so that each its value corresponded to one variant of the truss topology. It allows avoiding the emergence of "bad" structures in the solution the optimization problem, as all the variants of topology were defined on the begin stage of the data input. To create a variable X_t, tuples of nodes K_{ib}, K_{ie} were formed to which can be linked the beginning and the end of the bar i in each variant of the truss topology. Tuples are formed considering the distribution of bars between the subsystems in each variant of topology. Number of tuples is equal to doubled number of bars that change its position in at least one variant of topology.

In the figure 1 for each variant of topology the basic (left) and additional (right) subsystems are represented. Nodes and bars, which are used to form a basic subsystem, have been numbered (bar numbers are depicted in the frame). All the bars and nodes have been used to form the topology of basic subsystem in the variant I with the greatest amount of nodes and bars. The additional subsystem of the variant I has only two special nodes n_{d1} and n_{d2}. The additional subsystem is depicted conventionally in the Figure 1 and in other variants includes nodes and bars which didn't appear in basic subsystem.

Table 1 gives the instance of the specifying of tuples K_{ib}, K_{ie} with the nodes, to which it is possible to add the following bars 1-6, 13, 17, 18, 27, 30, 38, 40, and the meaning of design variables $X_{t.}$ as well.
The nodes of additional subsystem in the Table 1 have been printed in bold.

To solve the task of topology optimization of eaves truss, the harmony search method have been applied [3, 10]. The optimization results are pictured in Table 2.

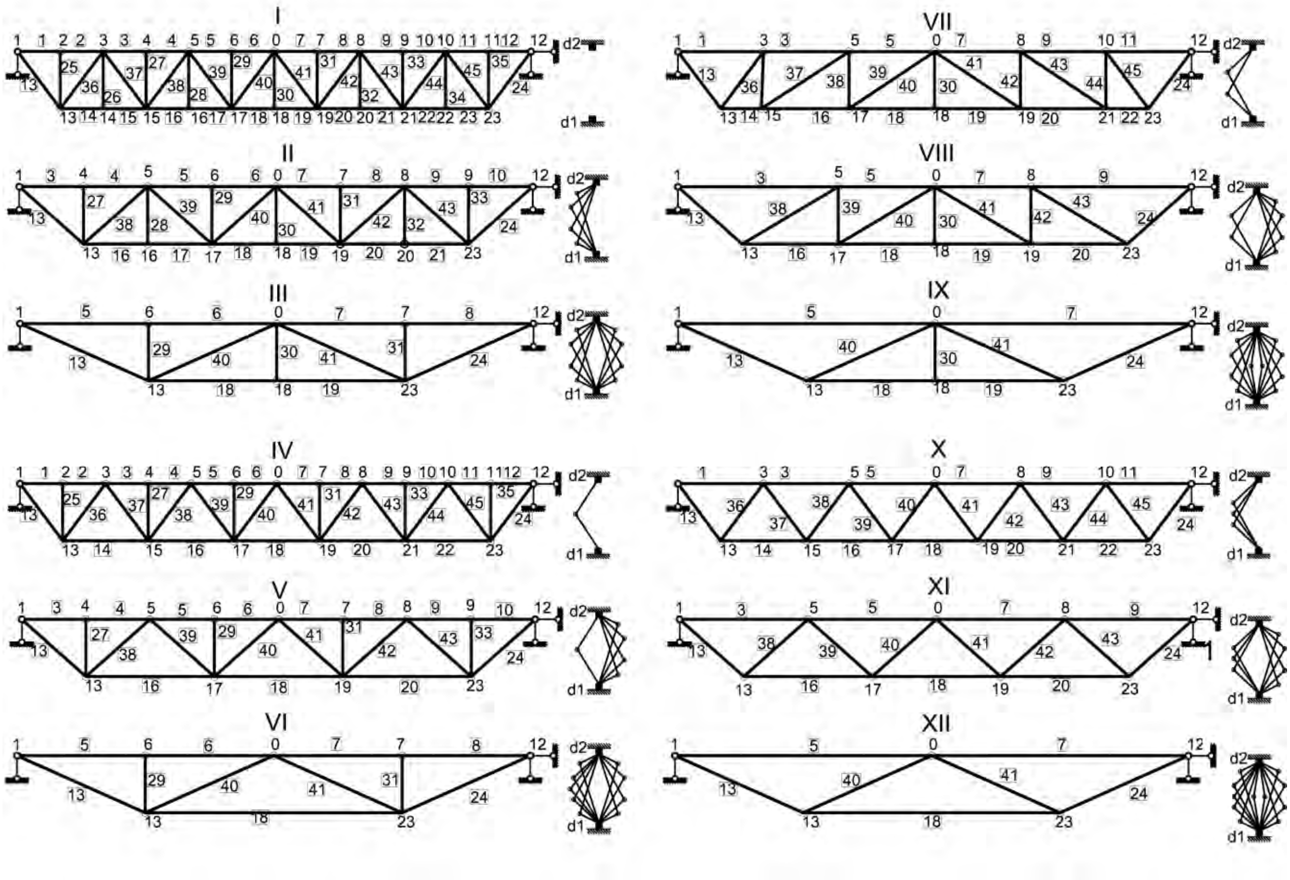

Fig. 1. Topology variants of eaves truss

TABLE 1

THE TUPLES OF NODES THAT DEFINE POSITIONS OF THE BAR IN EACH VARIANT OF TOPOLOGY OF TRUSS

Bar №		Tuple	Topology variants											
			I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1	begin	K_{1b}	n_1	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	n_1	n_1	n_1	n_1	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	n_1	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$
	end	K_{1e}	n_2	n_2	n_2	n_2	n_2	n_2	n_3	n_3	n_3	n_3	n_3	n_3
5	begin	K_{5b}	n_5	n_5	n_1	n_5	n_5	n_1	n_5	n_5	n_1	n_5	n_5	n_1
	end	K_{5e}	n_6	n_6	n_6	n_6	n_6	n_6	n_0	n_0	n_0	n_0	n_0	n_0
6	begin	K_{6b}	n_6	n_6	n_6	n_6	n_6	n_6	n_6	n_6	n_6	n_6	n_6	n_6
	end	K_{6e}	n_0	n_0	n_0	n_0	n_0	n_0	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$
13	begin	K_{13b}	n_1	n_1	n_1	n_1	n_1	n_1	n_1	n_1	n_1	n_1	n_1	n_1
	end	K_{13e}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}	n_{13}
17	begin	K_{17b}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}	n_{16}
	end	K_{17e}	n_{17}	n_{17}	n_{17}	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$	$\mathbf{n_{d2}}$
18	begin	K_{18b}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}
	end	K_{18e}	n_{18}	n_{18}	n_{18}	n_{19}	n_{19}	n_{23}	n_{18}	n_{18}	n_{18}	n_{19}	n_{19}	n_{23}
27	begin	K_{27b}	n_4	n_4	n_4	n_4	n_4	n_4	n_4	n_4	n_4	n_4	n_4	n_4
	end	K_{27e}	n_{15}	n_{13}	n_{d2}	n_{15}	n_{13}	n_{d2}	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$
30	begin	K_{30b}	n_0	n_0	n_0	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	n_0	n_0	n_0	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$	$\mathbf{n_{d1}}$
	end	K_{30e}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}	n_{18}
38	begin	K_{38b}	n_{15}	n_{13}	$\mathbf{n_{d2}}$	n_{15}	n_{13}	$\mathbf{n_{d2}}$	n_{15}	n_{13}	$\mathbf{n_{d2}}$	n_{15}	n_{13}	$\mathbf{n_{d2}}$
	end	K_{38e}	n_5	n_5	n_5	n_5	n_5	n_5	n_5	n_5	n_5	n_5	n_5	n_5
40	begin	K_{40b}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}	n_{17}	n_{17}	n_{13}
	end	K_{40e}	n_0	n_0	n_0	n_0	n_0	n_0	n_0	n_0	n_0	n_0	n_0	n_0
The value of X_t			1	2	3	4	5	6	7	8	9	10	11	12

TABLE 2

THE RESULTS OF SOLVING THE TOPOLOGY OPTIMIZATION PROBLEM OF EAVES TRUSS

№ var.	Weight, t	Heigh m, м	Bars of rectangular cross-section tube A x B x t, mm		
			bottom chord	lower chord	diagonal
Topology optimization problem					
XII	1,88	1,75	200x200x8	160x200x8	160x200x8
Simple optimization problems for each topology variant					
I	2,41	1,602	180x320x8	200x200x8	100x140x4
II	2,31	1,602	250x250x8	200x200x8	120x120x5
III	2,04	1,751	200x200x8	160x200x8	160x200x6
IV	2,29	1,747	250x250x8	160x200x8	120x80x5
V	2,22	1,602	180x320x8	200x200x8	120x120x5
VI	1,99	1,751	250x250x8	160x200x8	120x80x5
VII	2,45	1,720	200x300x8	200x200x8	120x200x4
VIII	2,29	1,751	300x200x8	200x200x8	140x140x5
IX	1,93	1,751	200x200x8	160x200x8	160x200x6
X	2,18	1,646	200x200x8	160x200x8	160x200x6
XI	2,07	1,760	200x300x8	200x200x8	80x120x5
XII	1,88	1,751	200x200x8	160x200x8	160x200x6

Additionally, it is solve the problem of optimization of cross sections and height of truss for each variant of topology separately. The initial data for all the tasks remains the same and has been described above. The results of solution of these tasks with harmony search method are and pictured in Table 2 and Figure 2.

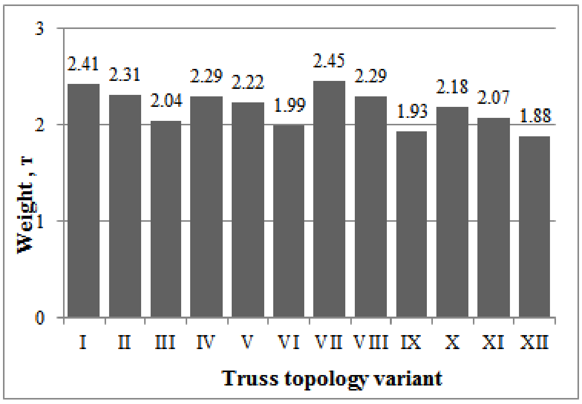

Fig. 2 The results of task solution of eaves truss optimization

The least mass of truss was received for the variant XII of the truss topology. The same results were received in the task of optimization with the alternating topology (table 2). This proves the credibility of the obtained results of eaves truss topology optimization and the possibility of implementing this tendered way for the optimization tasks of eaves truss topology.

Conclusion

The way of assigning the variants of structure topology, which can have different amount of nodes and bars to define the task of construction topology optimization, has been elaborated. This approach the discrete variables of designing have been applied which in [9] have been used to set the variants of the location of bars of the constructions.

The efficiency of application of this approach has been confirmed by the example of eaves truss topology optimization in the program OptCAD.

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