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VOLTAGE RESONANT INVERTER AS A POWER SOURCE

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Abstract: The operation mode of a voltage resonant inverter as a power source with variable load is analyzed. In order to reduce load power variations, an approach to development of the inverter's load power response based on providing similar positive and negative power deviations from its nominal value has been proposed. The design procedure for resonant inverter with open loop structure as a power source has been elaborated. For a high pressure sodium lamp as a load, the power deviation of about 4% from its nominal power in case of two fold increase of the lamp resistance thriughout its lifetime has been achieved.

Key words: voltage resonant inverter, power source, high frequency operation, variable load resistance.

1. Inroduction

High-frequency voltage resonant inverters (VRI) have numerous applications as an output stage of various electrical systems including induction heating, welding, melting, plasma generation, resonant dc-dc converters and radio-frequency power amplifiers, etc. They have the advantages over other converters due to their soft-switching characteristics, low electromagnetic interference, small volume and weight. Besides main function of voltage inverting, VRIs also realize the functions of load voltage or current control and stabilization. The theory on operation and design strategy of resonant inverters is well known [1].

However, there is a wide application area which needs to stabilize the output (load) power rather then voltage or current. Firstly, it concerns electrical systems and apparatus whose loads are discharge light sources – fluorescent lamps, high pressure sodium lamps, metal-halide lamps, etc. High pressure sodium lamps (HPS) are the main light sources in exterior lighting due to high efficiency and good color visualization. For this purpose the topology of serial- parallel resonant LC_pC_s inverters is suitable and is widely used to provide the high startup voltage and to regulate the current through the lamp [2]. If the inverter operation frequency is close to its resonant frequency, such VRI behaves as a current source. It is known that equivalent HPS lamp resistance rises up to 200% of its nominal value during the lamp life [3]. Therefore, the power delivered to lamp increases and may exceed the upper limit defined by the standard. In order to maintain the power supplied to loads within the operation area, an inverter circuit must be designed accordingly.

Hence the problem of maintenance of resonant inverters output power in established limits for variable loads appears. Such resonant inverter operation mode corresponds to operation mode of power source [4].

2. Statement of the problem

Some closed loop structures of systems with power control of resonant inverters have been proposed [1-3]. The main advantage of such structures is their high accuracy of power control, but its practical realization is complicated due to additional efforts on feedback power design, while solving stability problems of systems with such nonlinear loads as HPS lamps. For instance, the power control intermediate stage buck-boost [2] or buck converters [3] are additionally included in system structures to control VRI output power with accuracy about 0.6%. But additional power processing stage reduces system efficiency and increases its cost.

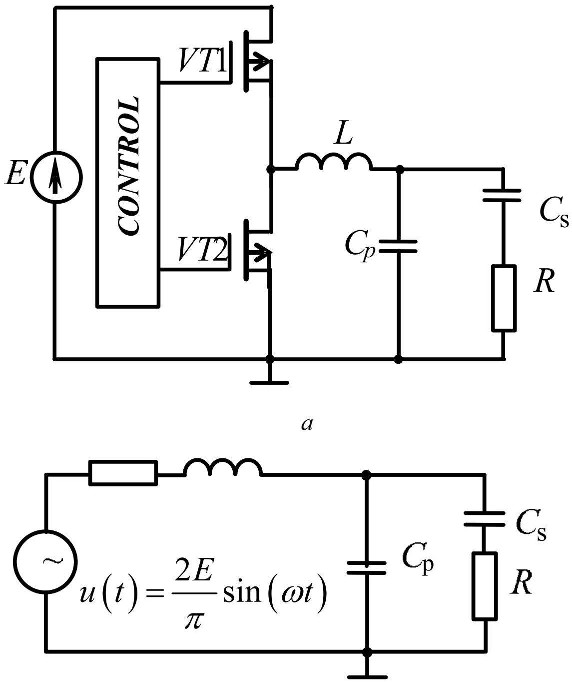
A power variations limits of some loads such as HPS lamps are relatively wide (±20% and even more) [5]. Therefore, the simpler, cost-effective and high efficiency solutions for power maintenance in such loads are the actual problem. One of the solutions is the use of the open loop of system structure on the basis of resonant inverter output power dependence on a load resistance value [6, 7]. The design sequence [6] limits the expected load power increase to +15% of nominal power regarding load resistance increasing to 100% of the nominal value. The results of analysis [7] demonstrate the possibility of reaching the load power deviation to +6.5% at the same conditions.

But designing procedures [6, 7] take into account only positive power variations. However, equal distribution of these variations between positive and negative deviations of load power regard to its nominal value may provide further decreasing relative power deviations in such converters.

Thus, the purpose of this paper is to study the way of decreasing the relative power deviation in resonant inverter with variable load and to elaborate a new design procedure for the LC_pC_s inverter on the basis of the proper choice of its parameters.

3. Theoretical part

Fig. 1(a) shows the electrical diagram of a serial-parallel resonant inverter. The LC_pC_s resonant tank loaded by resistor R serves as a bandpass filter for the input voltage square pulses [1]. These pulses are produced by commutation of transistor switches *VT1-VT2* with constant high frequency (20-200 kHz). As the fundamental approximation is considered for analysis, the inverter is replaced by an equivalent diagram, according to Fig.1 (a), where *u(t)* is the first harmonic of square pulses with its amplitude equal to $2E/\pi$, E is inverter power supply DC voltage, r is loss equivalent resistance.



a

b

Fig.1. (a) Half-bridge series-parallel inverter;
(b) Simplified circuit for analysis purposes.

The parameters of the series-parallel resonant inverter and relationship between them are presented in Table 1.

Table 1

Parameters of series-parallel resonant inverter

Parallel Resonant Frequency	Characteristic Impedance	Quality Factor
$\omega_0 = \frac{1}{\sqrt{LC_p}}$	$Z_0 = \sqrt{\frac{L}{C_p}}$	$Q = \frac{R}{Z_0}$
Relative Operation Frequency	Capacitor Relationship	Relative Loss Resistance
$\Omega = \frac{\omega}{\omega_0}$	$c = \frac{C_p}{C_s}$	$\alpha = \frac{r}{R}$

The expressions for VRI output power P and resonant frequency Ω_r obtained in [2, 7] are taken as the basis of future analysis:

$$P = \frac{2E^2}{\pi^2 Z_0} \cdot \frac{Q}{\left[\Omega + c\left(\Omega - \frac{1}{\Omega}\right) + \alpha\Omega Q\right]^2 + Q^2\left[1 - \Omega^2 + \alpha\left(1 + c\right)\right]^2} \quad (1)$$

$$\Omega_r = \frac{\omega_r}{\omega_0} = \sqrt{\frac{1}{2}\left\{1 - \frac{1}{Q^2}\left(1 + c\right)^2 + \sqrt{\left[1 - \frac{1}{Q^2}\left(1 + c\right)^2\right]^2 + \frac{4}{Q^2}c\left(1 + c\right)}\right\}}. \quad (2)$$

Due to negligible inverter component losses (r>>R), the expression for output power may be simplified:

$$P = \frac{2E^2}{\pi^2 Z_0} \frac{Q}{\left[\Omega + c\left(\Omega - \frac{1}{\Omega}\right)\right]^2 + Q^2\left(1 - \Omega^2\right)^2}. \quad (3)$$

The load resistance is supposed to increase from initial (minimum) value R_0 to final (maximum) value R_F and this increase modifies quality factor from Q_0 to Q_F, accordingly. The curve of output power as a function of quality factor in accordance with proposed approach is presented in Fig. 2. It starts at a point O of minimum power P_0, reaches point N of nominal power P_N, point M of maximum power P_M and, at last, point F of finishing power P_F. Let final power be equal to nominal power (P_F=P_N) and the maximum positive power deviation equal to maximum negative power deviation from nominal power (i. e. P_M-P_N=P_N-P_0=ΔP). The power trajectory *ONMF* (Fig. 2) may be very useful for such specific loads as HPS lamps. The light flux of a new lamp is the highest, and at the end of life it essentially falls down. Therefore, due to a frugal use of lamp resource at initial part of its life, this approach will give the possibility for prolonging the lamp life or decreasing its light flux falling at the end of life.

Let us obtain the analytical relations between quality factors Q_O, Q_N, Q_M and Q_F, and VRI parameters for determinative points of curve *ONMF* (Fig. 2) which correspond to powers P_0, P_N, P_M, P_F and load resistances R_0, R_N, R_M, R_F.

Effective load voltage V_N corresponding to nominal power P_N is calculated from (3) by equating nominal power P_N to the expression U_N^2 / R_N:

$$U_N = \frac{\sqrt{2}E}{\pi} \frac{Q_N}{\sqrt{\left[\Omega + c\left(\Omega - \frac{1}{\Omega}\right)\right]^2 + Q_N^2\left(1 - \Omega^2\right)^2}}. \quad (4)$$

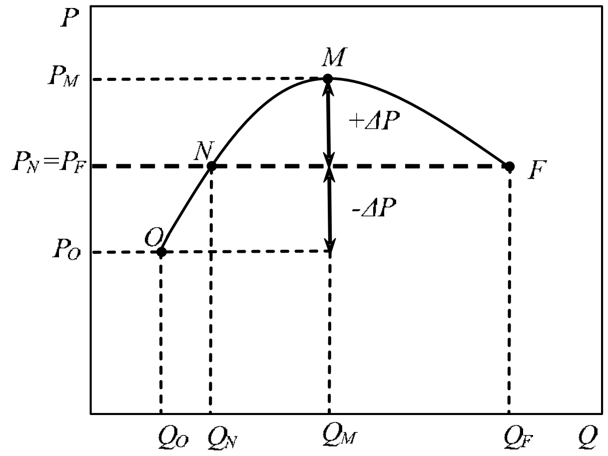


Fig.2. Output power as a function of quality factor.

From the above expression the quality factor Q_N is calculated:

$$Q_N=\frac{\Omega+c\left(\Omega-\frac{1}{\Omega}\right)}{\sqrt{\left(\frac{\sqrt{2}E}{\pi U_N}\right)^2-\left(1-\Omega^2\right)^2}}. \tag{5}$$

The power delivered to the load reaches its maximum value, when the quality factor is equal to Q_M. By applying the maximum power condition $dP/dQ=0$ to expression (3), the quality factor Q_M is determined:

$$Q_M=\frac{\Omega+c\left(\Omega-\frac{1}{\Omega}\right)}{1-\Omega^2}. \tag{6}$$

To obtain the expression for Q_F it must be taken into consideration that quality factor reaches its maximum value at a point F (Fig. 4). Therefore resonant frequency (2) also reaches its maximum value. To ensure zero-voltage switching of inverter transistors, the resonant frequency must be higher than its resonant frequency [1]. For limit case solving $\Omega=\Omega_r$ (2) relative to $Q=Q_F$, the value of quality factor Q_F is obtained:

$$Q_F=\sqrt{\frac{\left(1+c\right)\left(c-\Omega^2\right)}{\Omega^2\left(1-\Omega^2\right)}}. \tag{7}$$

Initial quality factor Q_0 equals:

$$Q_0=\frac{Q_K}{\beta}, \tag{8}$$

where $\beta= R_F/R_0$ is load resistor relative variation.

By substituting (6) in (3), the maximum power value P_M is obtained:

$$P_M=\frac{E^2}{\pi^2 Z_0\left(1-\Omega^2\right)\left[\Omega+c\left(\Omega-\frac{1}{\Omega}\right)\right]}. \tag{9}$$

The maximum relative deviation δ_P of power from nominal value P_N is equal to:

$$\delta_P=\frac{\Delta P}{P_N}=$$

$$=\frac{P_M-P_N}{P_N}=\frac{\left[\Omega+c\left(\Omega-\frac{1}{\Omega}\right)\right]^2+Q_N^2\left(1-\Omega^2\right)^2}{2Q_N\left(1-\Omega^2\right)\left[\Omega+c\left(\Omega-\frac{1}{\Omega}\right)\right]}-1 ,\tag{10}$$

where nominal power P_N was calculated by substituting (5) in (3).

By dealing with (5), (6), (9) and (10) the next expression is obtained:

$$1+\delta_P=$$

$$=\frac{1-\Omega^2}{2\sqrt{\left(\frac{\sqrt{2}E}{\pi U_N}\right)^2-\left(1-\Omega^2\right)^2}}+\frac{\sqrt{\left(\frac{\sqrt{2}E}{\pi U_N}\right)^2-\left(1-\Omega^2\right)^2}}{2\left(1-\Omega^2\right)}, \tag{11}$$

which is reduced to biquadratic equation. And solving the last equation gives the expression for relative switching frequency to ensure the power deviation being no more than ΔP (Fig. 2):

$$\Omega=\sqrt{1-\frac{\sqrt{2}E}{\pi U_0}\sqrt{\frac{1}{2}-\frac{\sqrt{\delta_P^2+2\delta_P}}{2(\delta_P+1)}}}. \tag{12}$$

Thus, the operation frequency is the function of relative power deviation δ_P, power supply voltage E and nominal load voltage U_N.

For the compactness of the next expressions, the parameters of power variation and voltage are introduced:

$$F=\sqrt{\frac{1}{2}-\frac{\sqrt{\delta_P^2+2\delta_P}}{2(\delta_P+1)}}, \tag{13}$$

$$A=\frac{\sqrt{2}E}{\pi U_N}. \tag{14}$$

Using the expressions (12), (13), equations (5), (6) and (7) for quality factors Q_N, Q_M and Q_F may be written as:

$$Q_N=\frac{1-A\cdot F\left(1+c\right)}{A\sqrt{\left(1-F^2\right)\left(1-A\cdot F\right)}}; \tag{15}$$

$$Q_M=\frac{1-AF\left(1+c\right)}{AF\sqrt{1-AF}}; \tag{16}$$

$$Q_F=\sqrt{\frac{\left(1+c\right)\left(1-AF-c\right)}{AF\left(1-AF\right)}}. \tag{17}$$

As it is obtained in [7] for the case of equality between final and nominal powers, the ratio ξ of corresponding quality factors Q_F and Q_N is equal to:

$$\xi = \frac{Q_K}{Q_N} = \frac{1-F^2}{F^2}. \tag{18}$$

Expression (18) is the basis for calculating the values of load resistance R_N and load voltage U_N at nominal power (Fig. 4) through equation (18):

$$R_N = \frac{\beta R_0}{\xi}, \tag{19}$$

$$U_N = \sqrt{U_N P_N}. \tag{20}$$

By working with (15), (16), (17) and (18) the simple connection between quality factors is obtained:

$$Q_M^2 = Q_N Q_F. \tag{21}$$

This expression allows formulating the next assertion: in resonant inverter the square of the quality factor which corresponds to the maximum power level is equal to the product of two quality factors which correspond to some less power level.

By using (15), (16), (17) and (22), the condition under which the inverter behavior approaches that of power source is obtained:

$$F^2 \sqrt{\frac{A(1+c)(1-AF-c)}{F\left(1-F^2\right)}} = 1 - AF(1+c). \tag{22}$$

The expression (22) is accepted as the first equation with two unknown variables A and c. For introducing the design sequence, the second equation with these variables may be introduced when considering output power variation due to component tolerances in resonant inverter. As it is shown in [8], the inductance L is the most critical element whose variation produces the highest power variations due to its highest tolerance and power sensitivity. The sensitivity of output power regarding the inductance variations is obtained in [8]:

$$S_L^P = -\frac{2\left[\Omega^2 + c\left(\Omega^2 - 1\right)\right] - 2Q\Omega^2\left(1-\Omega^2\right)}{\left[\Omega + c\left(\Omega - \frac{1}{\Omega}\right)\right]^2 + Q^2\left(1-\Omega^2\right)^2}. \tag{23}$$

After applying (13), (14) and (15) to (23), the next equation with variables A and c is obtained:

$$S_L^P = 2(1-AF)\left[\frac{F}{A} - \frac{1-F^2}{1-AF(1+c)}\right]. \tag{24}$$

The equations (22), (24) form the equation set with two unknown variables A and c. This set is the basis for the calculation of resonant inverter parameters but it has not an analytical solution, therefore the numerical method must be used for its solving. The possible solutions are limited by design requirements.

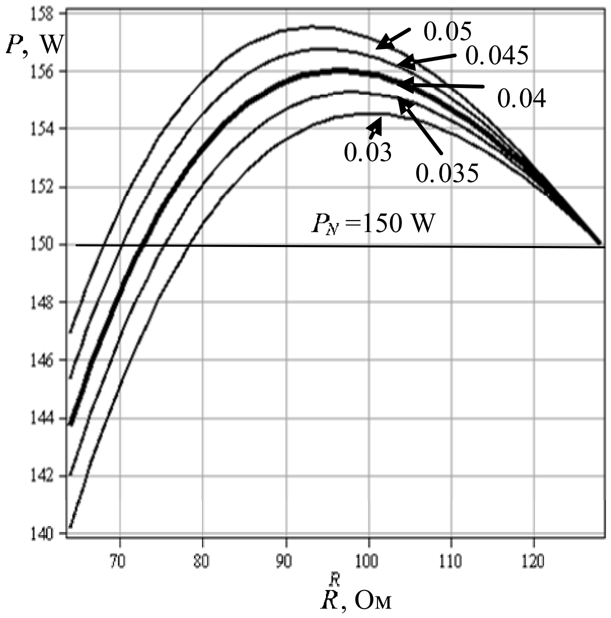


Fig.3. Load characteristics of resonant inverter calculated for relative power variation from 0.03 to 0.05.

The load characteristics of resonant inverter, loaded by HPS lamp SON-T 150W PHILIPS, were calculated for the case of relative power variation δ_P from 0.03 to 0.05 with step equal to 0.005. These characteristics are shown at Fig. 3. In its graphs we can observe that for this load the maximum positive power deviation is equal to the maximum negative power deviation if the relative power variation δ_P is near to 0.04.

4. Design procedure

The design objective of the proposed approach is to find the values of the inverter components L, C_p, C_s. The initial data for design procedure are nominal power P_N, minimum and maximum load resistances R_0 and R_F, sensitivity of output power regarding the inductance S_L^P.

The proposed design procedure consists of following next steps.

1. For minimum and maximum values of load resistance the parameter F of power variation is calculated from the expression (18).
2. From characteristics (Fig. 3) the value of relative power variation δ_P is so chosen that the maximum positive power deviation is almost equal to the maximum negative deviation from the nominal power.
3. The solution to the set of equations (22), (24) gives the parameter A of voltage and capacitor relationship c.
4. By using (12), (14), δ_P and A the relative operation frequency Ω is determined.
5. By substituting the calculated values F, c, and A into (15), (16) and (17), the quality factors Q_N, Q_M, Q_F

are calculated. The quality factor Q_0 is determined by using (8).

6. By obtaining Q_0 from the Table 1, the characteristic impedance Z_0 is calculated.

7. From expressions (19) and (20) the resistance R_N and the voltage U_N corresponding to nominal load power are determined.

8. The voltage E of the inverter power supply is calculated from (14).

9. The switching frequency $\omega=\Omega\omega_0$ is chosen on the assumption of component frequency possibilities and taking into account the load frequency properties.

10. Finally, the resonant inverter parameters L, C_p, C_s are calculated on the basis of relationships given in Table 1.

As a design example, the resonant inverter for operating the HPS lamp SON-T 150W PHILIPS was calculated with initial data: P_N=150W; R_0=64Ω; R_F=128 Ω; S_L^P=-1. The calculation results are as follows: E=242V; δP=0.04; Ω=0.61; Q_0=0.62; Q_N=0.70; Q_M=0.93; Q_K=1.23; Z_0=103.7Ω; f=120кHz; L=84μH; C_p=7.8 nF; C_s=317 nF. Calculated powers corresponding to points *O, N, M* and *F* (Fig. 2) are equal to P_0=143.7 W, P_N=150W, P_M=156W and P_F=150W.

The load characteristic as a calculation result is shown in Fig. 4. Observing Fig. 4 reveals that the maximum positive and negative power deviations are about 4% for double changing of load resistance. Thus, the relative power deviation is reduced by 30% as compare to known result [7] (δ_P=4% vs. δ_P=6,5%). It is more than enough for loads similar to a discharge light sources.

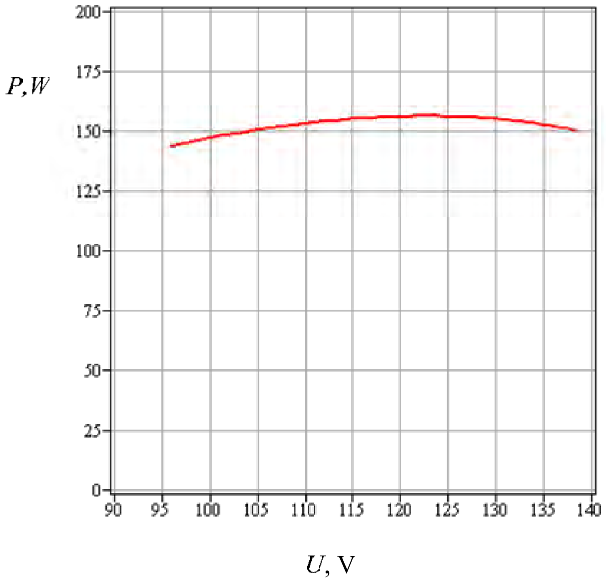


Fig. 4. Power vs. voltage delivered to load in case of double load changing.

5. Simulation results

To verify the results of analytical study the electrical circuit (Fig. 5) of resonant inverter with above calculated parameters was simulated using MicroCap-9.0.

Fig. 6 shows the simulated waveforms of inverter load voltages and average powers for three values of load resistance corresponding to point *O*, *M* and *F* (Fig. 2). Simulation results demonstrate good agreements with theoretical ones.

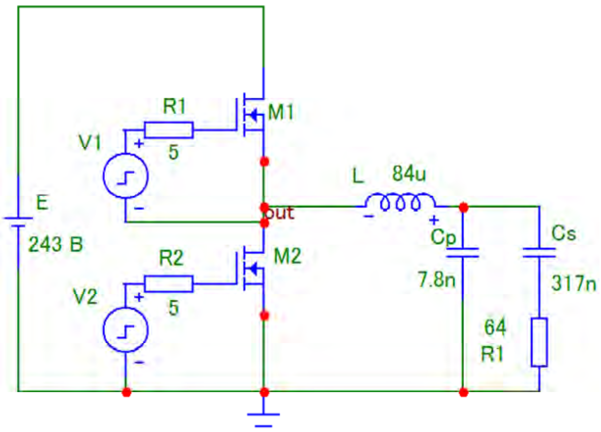
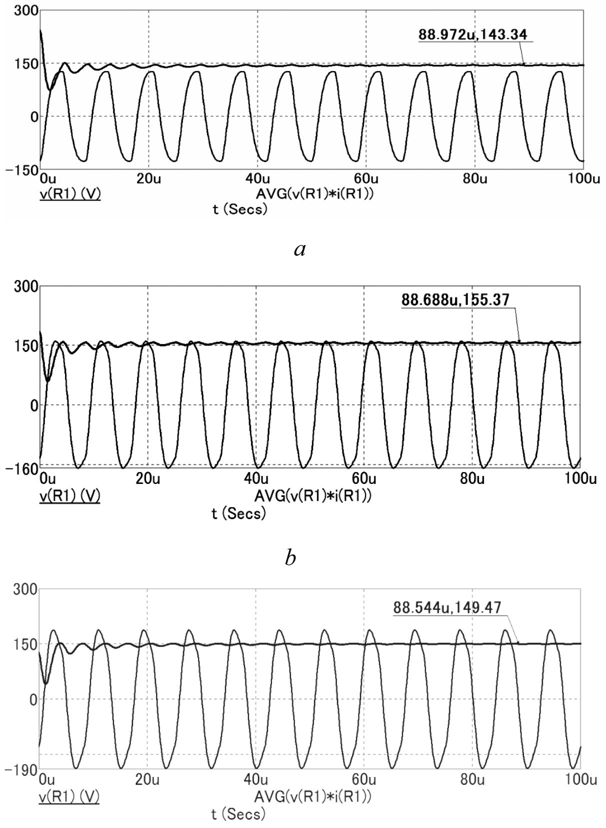


Fig. 5. Electrical circuit for resonant inverter simulation.



a

b

c

Fig. 6. Simulated waveforms of inverter load voltages and average powers: a) for minimum load resistance; b) for maximum load power; c) for maximum load resistanceю.

6. Conclusion

Resonant properties of the VRI may be successfully used for the maintenance of its load power at a nearly constant level in the case of variable loads without using close loop control. It is shown that resonant inverter designed as open loop circuit is able to achieve the power deviations about 4% of its nominal value for double change in load resistance.

For the serial-parallel resonant LC_pC_s inverter the next assertion is established: the square of the quality factor which corresponds to the maximum load power level is equal to the product of two quality factors which correspond to some less load power level.

Proposed approach presents a better solution then known ones related to designing resonant inverter as a power source for variable loads operation. The design procedure for the open loop resonant inverter is elaborated. Simulation results are in good agreement with theoretical ones. The results of this study may be used in designing electrotechnical systems for the operation of discharge light sources and other similar loads.

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РЕЗОНАНСНИЙ ІНВЕРТОР НАПРУГИ ЯК ДЖЕРЕЛО ПОТУЖНОСТІ

Анатолій Лупенко, Петро Стахів

Проаналізовано режим роботи резонансного інвертора напруги на змінне навантаження, близький до режиму джерела потужності. Для зменшення варіації потужності в навантаженні запропоновано метод формування потужнісної навантажувальної характеристики резонансного інвертора шляхом забезпечення однакових додатних та від’ємних відхилень його потужності від номінального значення. Розроблено методику розрахунку резонансного інвертора з розімкненою структурою як джерела потужності. На прикладі натрієвої лампи високого тиску як навантаження отримано відхилення потужності 4 % від номінальної за двократної зміни опору лампи впродовж всього строку її служби.

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Research interests: electrotechnical systems of high-frequency operations and control for nonlinear loads, power electronics, parametric identification of dynamic objects.