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EVALUATION OF VARIATION RANGES OF ELECTRICAL PARAMETERS IN THREE-PHASE UNEARTHED NETWORKS

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Abstract. Operating parameters of low (LV) and medium (MV) voltage electrical networks such as phase voltages, leakage and ground fault currents are decisive for safe operation of these systems. Knowledge of maximum possible values of the network operating parameters is necessary for the correct assessment of numerous hazards. So far no simple methods for the determination of these parameters highest levels have been available. In the paper, there is presented a new approach to evaluation of variation ranges of operating parameters in three-phase unearthed networks with the help of Wolfram Mathematica 9.

Key words: three-phase networks, unearthed neutral, insulation parameters, maximum value, phase voltage, leakage current, active power loss.

1. Introduction

An important task of exploitation of three-phase LV and MV networks with unearthed neutral (isolated neutral point - IT) (see Fig.1) is maintaining its insulation-to-ground at a required level. The insulation resistance and capacitance exert influence on such operating parameters as phase voltages, leakage and ground fault currents. These parameters values are decisive for safe operation of these networks. An excessive phase voltage rise (overvoltage) increases risk of insulation breakdown, whereas the high leakage and ground fault currents may cause a human shock, fire or explosion.

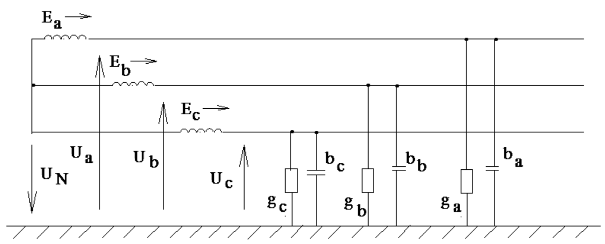

Fig. 1. Equivalent circuit of 3-phase IT network.

Knowledge of maximum possible values of the network operating parameters is necessary for the correct assessment of the mentioned hazards. However, no simple methods for the determination of these parameters highest levels have been published so far.

2. Operating levels of three-phase network voltages

With symmetrical insulation leakage conductances and capacitances of single phases (i.e. $g_a = g_b = g_c$, $b_a = b_b = b_c$), wire-to-ground (phase) voltages in a network with symmetrical supply source voltages are equal. In case of insulation parameters deterioration or asymmetry, this balance may be disturbed. With a dead ground fault of any phase, the voltages of the remaining two phases increase by 73 %, and the neutral point displacement voltage U_N grows to E. However, this is not the greatest possible rise of the network voltages as compared to symmetrical condition. Fig. 2 depicts an example of still higher overvoltages.

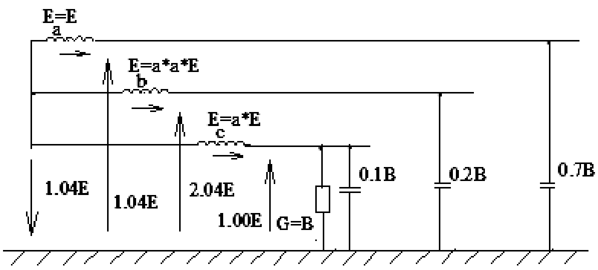

Fig. 2. Three-phase network voltages with asymmetrical insulation parameters – an example.

This example rises a question about a possible variation range of phase voltages in 3-phase IT networks. In order to determine the highest possible phase and displacement overvoltages, it is necessary to find maximum values of the functions given by the following formulas [1]

$$U_N = E\cdot\left|\frac{(g_a + jb_a) + e^{j\frac{4\pi}{3}}\cdot(g_b + jb_b) + a^{j\frac{2\pi}{3}}\cdot(g_c + jb_c)}{(g_a + g_b + g_c) + j\cdot(b_a + b_b + b_c)}\right| \quad (1)$$

$$U_a = \left|E - \underline{U_N}\right|,\ U_b = \left|E\cdot e^{j\frac{4\pi}{3}} - \underline{U_N}\right|,\ U_c = \left|E\cdot e^{j\frac{2\pi}{3}} - \underline{U_N}\right| \quad (2)$$

It is convenient for this analysis to assume fixed values of the network's insulation-to-ground equivalent (i.e. total) conductance G and susceptance B:

$$g_a + g_b + g_c = G,\ b_a + b_b + b_c = B \quad (3)$$

This approach is based on the fact that in practice insulation parameters for single phases are not known, but merely their equivalent values are measured. According to formulas (3) any set of three independent phase conductances and separately of three phase susceptances is represented by single, unique insulation condition indicators G and B. The insulation equivalent resistance $R=1/G$ is indicated by an insulation monitor whereas the total network-to-ground capacitance $C=B/2\pi f$ should be known for maximum possible configuration of the network. In practice these equivalent values for LV networks may vary from few µS to ∞ (for G) and from few nF to few µF (for B). From equations (3), insulation parameters for the phase c can be expressed as $g_c=G-g_a-g_b$, $b_c=B-b_a-b_b$. Therefore, the problem of U_N, U_a, U_b, U_c maximum values determination is the task with four independent variables $0\le g_a\le G$, $0\le g_b\le G$, $0\le b_a\le B$, $0\le b_b\le B$ which fulfill conditions $0\le g_a+g_b\le G$, $0\le b_a+b_b\le B$.

It should be noted that the minimum displacement voltage U_N is of course zero for symmetrical insulation parameters. A minimum phase voltage is zero too, and it appears when the entire insulation conductance and susceptance are concentrated at this one phase only.

3. Determination of displacement voltage and phase voltages maximum levels

For the limitations given above, there should be found maximum values of the voltages described by formulas (1) and (2) where

$$(4)\quad \underline{U}_N=\frac{E}{G+jB}\cdot\left\{\begin{array}{l}(g_a+jb_a)+e^{j\frac{4\pi}{3}}\cdot(g_b+jb_b)+\\ \left[(G-g_a-g_b)+j(B-b_a-b_b)\right]\cdot e^{j\frac{2\pi}{3}}\end{array}\right\}$$

This problem can be solved in the following way. Note those mappings $g_a\to|\underline{U}_N(g_a;\ b_a;\ g_b;\ b_b)|^2$, $b_a\to|\underline{U}_N(g_a;\ b_a;\ g_b;\ b_b)|^2$, $g_b\to|\underline{U}_N(g_a;\ b_a;\ g_b;\ b_b)|^2$, $b_b\to|\underline{U}_N(g_a;\ b_a;\ g_b;\ b_b)|^2$, $g_a\to|\underline{U}_N(g_a;\ b_a;\ G-g_a;\ b_b)|^2$, $b_a\to|\underline{U}_N(g_a;\ b_a;\ g_b;\ B-b_a)|^2$ (with other arguments fixed arbitrarily) are all convex or affine, so each of them attains its maximum on the boundary (see [2], [3]).

Taking above remarks into account we consider that:

$$\max|\underline{U}_N(g_a;b_a;g_b;b_b)|=$$
$$=\max\{\max|\underline{U}_N(g_a;b_a;0;b_b)|,\ \max|\underline{U}_N(g_a;b_a;G\text{-}g_a;b_b)|\}=$$
$$\max\{\max|\underline{U}_N(0;b_a;0;b_b)|,\ \max|\underline{U}_N(G;b_a;0;b_b)|,$$
$$\max|\underline{U}_N(0;b_a;G;b_b)|\}=$$
$$\max\{\max|\underline{U}_N(0;b_a;0;0)|,\ \max|\underline{U}_N(0;b_a;0;B\text{-}b_a)|,$$
$$\max|\underline{U}_N(G;b_a;0;0)|,\ \max|\underline{U}_N(G;b_a;0;B\text{-}b_a)|,$$
$$\max|\underline{U}_N(0;b_a;G;0)|,\ \max|\underline{U}_N(0;b_a;G;B\text{-}b_a)|\}=$$
$$=\max\{\max|\underline{U}_N(0;0;0;0)|,\ \max|\underline{U}_N(0;B;0;0)|,$$
$$\max|\underline{U}_N(0;0;0;B)|,\ \max|\underline{U}_N(G;B;0;0)|,$$
$$\max|\underline{U}_N(G;0;0;B)|,\ \max|\underline{U}_N(G;0;0;0)|,$$
$$\max|\underline{U}_N(0;0;G;0)|,\ \max|\underline{U}_N(0;0;G;B)|,$$
$$\max|\underline{U}_N(0;B;G;0)\}$$

So we conclude that the point $(g_a;\ b_a;\ g_b;\ b_b)$ at which $|\underline{U}_N|$ attains its maximum, must satisfy $\{g_a, g_b\}\subset\{0, G\}$, $\{b_a, b_b\}\subset\{0, B\}$. One can check, that

$$\max|\underline{U}_N(g_a;b_a;g_b;b_b)|=E\sqrt{\frac{B^2+G^2+\sqrt{3}BG}{B^2+G^2}}\qquad(5)$$

and is attained for the points $(g_a;\ b_a;\ g_b;\ b_b)\in\{(0,\ 0,\ G,\ 0),\ (0,\ B,\ 0,\ 0),\ (G,\ 0,\ 0,\ B)\}$. In the same way, we find the maximum phase voltages:

$$\max|\underline{U}_a(g_a;b_a;g_b;b_b)|=\max|E-\underline{U}_N(g_a;b_a;g_b;b_b)|=$$
$$|E-\underline{U}_N(0,0,G,0)|=E\sqrt{3\frac{B^2+G^2+\sqrt{3}BG}{B^2+G^2}}$$
$$\max|\underline{U}_b(g_a;b_a;g_b;b_b)|=\max|Ee^{j4\pi/3}-\underline{U}_N(g_a;b_a;g_b;b_b)|=\qquad(6)$$
$$|Ee^{j4\pi/3}-\underline{U}_N(0,B,0,0)|=E\sqrt{3\frac{B^2+G^2+\sqrt{3}BG}{B^2+G^2}}$$
$$\max|\underline{U}_c(g_a;b_a;g_b;b_b)|=\max|Ee^{j2\pi/3}-\underline{U}_N(g_a;b_a;g_b;b_b)|=$$
$$|Ee^{j2\pi/3}-\underline{U}_N(G,0,0,B)|=E\sqrt{3\frac{B^2+G^2+\sqrt{3}BG}{B^2+G^2}}$$

The maximum values of both the displacement voltage and phase voltages with any possible insulation parameters $0<g_a<\infty$, $0<g_b<\infty$, $0<g_c<\infty$, $0<b_a<\infty$, $0<b_b<\infty$, $0<b_c<\infty$ are obviously obtained when $B=G$. These values are equal respectively

$$U_{N\max}=E\sqrt{1+\frac{\sqrt{3}}{2}}=1.366\cdot E,\ U_{phase\max}=E\sqrt{3+\frac{3\sqrt{3}}{2}}=2.366\cdot E$$

This extreme case for phase a voltage ($g_b=G=$ $=b_c=B$) is presented as a vector diagram in Fig.3. Such unequal distribution of insulation conductance and capacitance is possible when conductors of phases a, b are much shorter than that of phase c and there is a ground leakage in the phase b only.

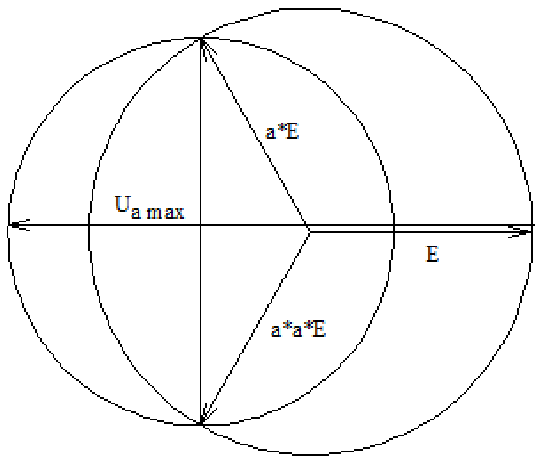

Fig. 3. Vector diagram of phase "a" voltage for $g_b=G=b_c=B$ (the remaining phase conductances and susceptances are zero), $U_{a\ max}=2.366$ E.

Based on formulas (5) and (6), the dependence of $U_{N\ max}$ and $U_{phase\ max}$ on B/G ratio can be plotted in relative units U/E (see Fig.4). These formulas, as well as the curves in Fig.4 enable to evaluate the sought maximum overvoltages for any possible values of insulation parameters. These maximum overvoltages are attained for the points $(g_a;\ b_a;\ g_b;\ b_b)$ $\{(0, 0, G, 0), (0, B, 0, 0), (G, 0, 0, B)\}$.

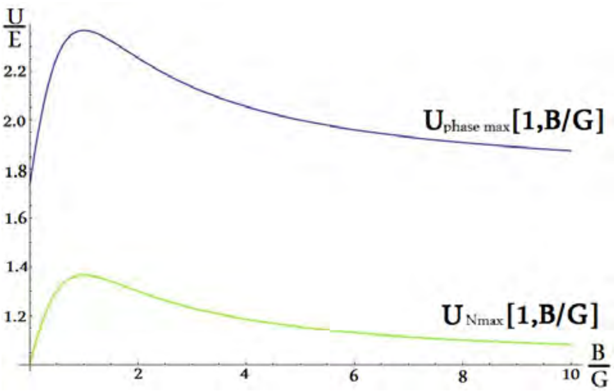

Fig. 4. Dependence of maximum relative values of displacement U_{Nmax} and phase $U_{phase\ max}$ voltages on B/G ratio.

4. Evaluation of leakage current variation range

A high leakage current i.e. an active component of the current flowing through insulation-to-ground conductances poses a threat of excessive temperature rise in places with a lowered insulation level. This may cause fire or explosion in the presence of flammable or explosive substances.

The leakage current (RMS value) in the phase e.g. *a* is given by formula (7) where $\underline{U_N}$ is described by (4)

$$I_a = U_a \cdot g_a = \left| E - \underline{U_N} \right| \cdot g_a \qquad (7)$$

Again, note these mappings $b_a \rightarrow I_a(g_a;\ b_a;\ g_b;\ b_b)^2$, $g_b \rightarrow I_a(g_a;\ b_a;\ g_b;\ b_b)^2$, $b_b \rightarrow I_a(g_a;\ b_a;\ g_b;\ b_b)^2$, $b_a \rightarrow I_a(g_a;\ b_a;\ g_b;\ B-b_a)^2$ (with other arguments fixed arbitrarily) are all convex or affine. Hence, repetition of the procedure from point 3., reduces the problem of finding the maximum value I_{amax} for the fixed B and G, to the set $\{ (g_a, b_a, g_b, b_b) \mid (g_b{=}0 \text{ or } g_b{=}G{-}g_a) \text{ and } ((b_a, b_b) = (0, 0) \text{ or } (b_a, b_b) = (0, B) \text{ or } (b_a, b_b) = (B, 0)) \}$. With the help of Wolfram Mathematica 9 [4], we find formulas for the maximum value of leakage current:

$$I_{a\max}(B,G) = E \cdot Max\ \left[\ \left\{\frac{\sqrt{3}BG}{\sqrt{B^2+G^2}}\ for\ B,G>0\right\},\right.$$

$$\left\{\frac{\sqrt{3}\sqrt{61B^4+104\sqrt{3}B^3G+152B^2G^2+32\sqrt{3}BG^3+16G^4+\sqrt{\Delta_1}}}{16\sqrt{2}\sqrt{B^2+G^2}}\right.$$

$$\left. for\ \frac{\sqrt{3}}{2}G \geq B > 0\right\}, \qquad \left\{\frac{\sqrt{3}G^2}{2\sqrt{B^2+G^2}}\ for\ B,G>0\right\},$$

$$\left\{\frac{\sqrt{3}\sqrt{61B^4-104\sqrt{3}B^3G+152B^2G^2-32\sqrt{3}BG^3+16G^4+\sqrt{\Delta_2}}}{16\sqrt{2}\sqrt{B^2+G^2}}\right.$$

$$for\ G>0\ and\ 0<B\leq Second\ root\ of\ polynomial$$

$$\left. P(x):=-4G^4+12\sqrt{3}G^3x+8G^2x^2-4\sqrt{3}Gx^3+x^4\right\},$$

$$\left\{\frac{\sqrt{3}\sqrt{-8B^4+20B^2G^2+G^4+G\sqrt{(G^2-8B^2)^3}}}{4\sqrt{2}\sqrt{B^2+G^2}}\ for\ G>0\ and\right. \qquad (8)$$

$$\left.0<B\leq Second\ root\ of\ polynomial\ P(x):=-G^4+11G^2x^2+x^4\right\}\ \Big]$$

$$where\ \Delta_1:=-375B^8+400\sqrt{3}B^7G+1840B^6G^2-1344\sqrt{3}B^5G^3-$$

$$-4128B^4G^4+768\sqrt{3}B^3G^5+3840B^2G^6+1024\sqrt{3}BG^7+256G^8,$$

$$\Delta_2:=-375B^8-400\sqrt{3}B^7G+1840B^6G^2+1344\sqrt{3}B^5G^3-$$

$$-4128B^4G^4-768\sqrt{3}B^3G^5+3840B^2G^6-1024\sqrt{3}BG^7+256G^8$$

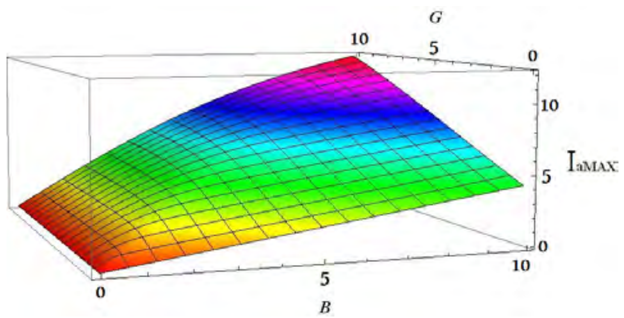

Fig. 5. Dependence of Iamax [mA] on B [mS] and G [mS] for E=1 [V].

The surface plotted in Fig.5 according to formulas (8) presents the dependence of the maximum leakage current $I_{a\ max}$ on the insulation equivalent parameters B and G. This diagram can be used for an approximate quick evaluation of the sought maximum level.

5. Evaluation of maximum power loss in a phase-to-ground insulation

The active power loss produced by the leakage currents may cause an impermissible temperature rise leading to fire or explosion hazards. Therefore, it is the second to I_{amax} parameter describing the above mentioned risk. The power loss produced in insulation-to-ground of a single phase e.g. *a* is given by the following formula

$$P_a = U_a^2 \cdot g_a = \left| E - \underline{U_N} \right|^2 \cdot g_a \qquad (9)$$

The formula for $P_{a\ max}$ can be obtained in the same way as $I_{a\ max}$:

$$P_{a\max}(B,G) = \frac{E^2}{9(B^2+G^2)} \cdot Max\ \left[\ \{27B^2G\ for\ B,G>0\},\right.$$

$$\left\{18B^2G+2G^3+2\sqrt{(-3B^2+G^2)^3}\ \ for\ \frac{G}{2}\geq B>0\right\}, \qquad (10)$$

$$\{4G^3\ for\ B,G>0\},\ \{3\sqrt{3}B^3+3\sqrt{3}BG^2+9B^2G+2G^3+$$

$$+2G\sqrt{3\sqrt{3}GB^3+9B^2G^2+3\sqrt{3}BG^3+G^4}$$

$$for\ \ G>0\ and\ \ 0<B\leq\sqrt{3}G\ \},$$

$$\{-3\sqrt{3}B(B^2+G^2)+9B^2G+2G^3+$$

$$+2G\sqrt{-3\sqrt{3}GB^3+9B^2G^2-3\sqrt{3}BG^3+G^4})$$

$$\left. for\ \ G>0\ \ and\ \ 0<B\leq\frac{G}{(2+\sqrt{3})}\}\ \ \right]$$

Some results of the calculation according to the above formulas are enclosed in the following table. Each row (row number: *k=1,2,3,4,5* - then B=0.5k in [mS]) contains P_{amax} values in [mW] for five values of *G* from 0.5 to 2.5 [mS] (then G=0.5l (l-column number) in [mS])). The calculations were done for E=1 [V]. For the other *E* values, each result should be multiplied by E^2.

Table 1

Values of P_{amax}[k,l] for {k, 0.5, 2.5, 0.5}, {l, 0.5, 2.5, 0.5}]

{{0.845108, 1.11962, 1.33299, 1.54219, 1.75358},
{1.2, 1.69022, 1.99955, 2.23923, 2.45594},
{1.35, 2.09338, 2.53532, 2.86202, 3.12585},
{1.41176, 2.4, 2.95715, 3.38043, 3.71728},
{1.44231, 2.58621, 3.31067, 3.81295, 4.22554}}

Based on the calculation results contained in the above table, it is possible to plot a special diagram presenting dependence of P_{amax} on the insulation equivalent parameters *B* and *G*. This however has been omitted here because of the similar appearance to Fig.5.

6. Conlusion

1. Knowledge of maximum possible operating parameters such as voltages, ground fault and leakage currents of unearthed networks is necessary for the correct assessment of hazards to electrical systems safety. As shown in the paper, these parameters may in certain conditions exceed the levels commonly assumed to be the highest ones.

2. In case of extremely asymmetrical distribution of insulation conductances and susceptances, the highest phase-to-ground overvoltage to be expected in three-phase unearthed networks is 2.366 x nominal phase voltage.

3. With the help of Wolfram Mathematica it is possible to obtain exact formulas and numerical values for maximum levels of the analyzed operating parameters of unearthed networks. Application range of the presented analytical methods is theoretically unlimited and covers all values of insulation parameters met in real LV and MV networks.

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ОЦІНКА ДІАПАЗОНІВ ЗМІНИ ЕЛЕКТРИЧНИХ ПАРАМЕТРІВ У ТРИФАЗНИХ НЕУЗЕМЛЕНИХ МЕРЕЖАХ

Цезари Ольшовець, Пьотр Ольшовець

Робочі параметри електричних мереж, такі як фазові напруги, струми витоку, струми короткого замикання на землю, є вирішальними для безпечної работи цих систем. Володіння знаннями про максимально можливі значення робочих параметрів мереж є необхідними для правильної оцінки численних ризиків. На цей час все ще не існує жодних простих методів, які б дали змогу визначати найвищий рівень цих параметрів. У цій статті представлено новий підхід до оцінки діапазонів змінності робочих параметрів у трифазних неуземлених мережах з допомогою Wolfram Mathematica 9.

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