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Integral photoelasticity relations for inhomogeneously strained dielectrics

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A model for interaction of polarized light with inhomogeneously strained non-magnetizable dielectric solid is considered in the paper. The model establishes ray photoelasticity integrals connecting distributions of strain tensor components on any direction on the body volume with measurable optical parameters of polarized light beam propagating in this direction. The model can be used for developing mathematical methods for polarized-optical computational tomography of stress-strained states of dielectric solids.

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1. Introduction

Photoelasticity is widely used for non-destructive determination of 2-d strain-stressed state of isotropic dielectric solids [1]. Application of the method is based on the simple relation, knowing as stress-optic law:

$$(\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}}) = \frac{1}{Ch}\delta. \tag{1}$$

It connects the difference of the first σ_{I} and the second σ_{II} principal stresses with the phase retardation δ of polarized light propagating in the object in the direction normal to the both principal axes I and II. Here C is the stress-optic coefficient, h is the dimension of the object in the direction of light propagation.

The formula (1) was obtained in assumption of homogeneous strain-stressed state. So, the method based on it is applicable only for cases when the stress components are non-changeable in the directions of light propagation. That takes place for objects being in plane stress state.

There is a case of inhomogeneous strain-stressed state, when we can obtain a posteriori information about it, sounding the object by polarized light. It is the case when the principal axes of stress tensor are non-changeable in the direction of light propagation. Then we have integral stress-optic law:

$$\int_0^h (\sigma_1 - \sigma_2)\, dz = \frac{1}{C}\delta. \tag{2}$$

where σ_1 and σ_2 are the projections of the principal axes on the plane normal to direction of light propagation.

It is a very spatial case. It is realized when the object, being in a plane strain state, is sounded in the directions lying in the plane of translation symmetry. So, in this case, as in the previous one, the stress-optic law can be used for determination of 2-d stress-strained state.

The relations (1) and (2) represent mathematical models for polarized light interaction with strained solids for the two special cases. The methods for non-destructive determination of 2-d stress fields can be implemented on this basis with the use of polarized-optical measurement. This technique is very precise can be automated [2–4].

That encourages researchers to search for possibilities to apply the photoelasticity phenomenon for determination parameters of arbitrary 3-d strain-stressed state [5–9]. In aggregate with a tomographic approach it would to enable creation of a powerful method for nondestructive testing of stress-strained state of dielectric solids.

The paper deals with mathematical model for interaction of polarized light with strain fields in solids. An isotropic and homogeneous non-magnetizable lossless dielectric body $\mathcal{B}$ is considered. The body occupies a 1-connected area $\mathcal{V} \subset \mathbb{R}^3$ bounded by sufficiently smooth surface $\partial\mathcal{V}$. The body is in inhomogeneous elastic stress-strained state and possesses features of optic anisotropy and inhomogeneity induced by strain. The body is subjected to sounding in different directions $\mathbf{K} \subset \mathcal{V}$ by narrow parallel beams of polarized light in order to gather a posteriori information about its stress-strained state. The polarization of incident light beams are known; polarization of emerging light beams can be measured by polarized-optical methods. The objective is to establish integral relations connecting distributions of strain tensor's e components $e_{ij}(\mathbf{r})$, $r \in \mathbf{K}$, $i, j = 1, 2, 3$ along the directions $\mathbf{K}$ of light's propagation to measurable parameters of polarization of emerging light beams.

2. Optical anisotropy and inhomogeneity of strained body

Light propagation in the body is defined by the two pairs of vector parameters [10] — electric field intensity $\mathbf{E} = \{E_i\}$ and displacement $\mathbf{D} = \{D_i\}$, magnetic field intensity $\mathbf{H} = \{H_i\}$ and magnetic induction $\mathbf{B} = \{B_i\}$, which, in the general case, are functions of spatial coordinate $r \in \mathcal{V}$ and time $t \in \mathbb{R}^+$:

$$\mathbf{E} = \mathbf{E}(\mathbf{r}, t), \quad \mathbf{D} = \mathbf{D}(\mathbf{r}, t), \quad \mathbf{H} = \mathbf{H}(\mathbf{r}, t), \quad \mathbf{B} = \mathbf{B}(\mathbf{r}, t). \tag{3}$$

Here E_i, D_i, B_i and H_i stand for components of the vectors $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$ and $\mathbf{H}$ in the orthonormal basis of a global Cartesian coordinate system (x_1, x_2, x_3).

The field vectors (3) satisfy in $\mathcal{V}$ four Maxwell macroscopic equations [10]. In the absence of volumetric charges and currents they look like

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{D} = 0, \\ \nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0. \end{aligned} \tag{4}$$

For electromagnetic fields of low intensity, when external electric field is small enough in comparison with molecular fields of medium, linear material equations, expressing the electric displacement vector $\mathbf{D}$ in term of electric field $\mathbf{E}$ and magnetic induction vector $\mathbf{B}$ in terms of magnetic field $\mathbf{H}$, can be used

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E}, \quad \mathbf{B} = \mu_0 \mathbf{H}. \tag{5}$$

Here $\boldsymbol{\varepsilon} = \{\varepsilon_{ij}\}$ is a dielectric permittivity tensor in current configuration of body, μ_0 is the vacuum magnetic permeability; in SI units $\mu_0 = 4\pi \cdot 10^{-7} N/m$.

We introduce a relative permittivity tensor $\widehat{\boldsymbol{\varepsilon}}$ and its disturbance $\tilde{\boldsymbol{\varepsilon}}$ caused by strain, in terms of which the permittivity tensor $\boldsymbol{\varepsilon}$ can be expressed as

$$\boldsymbol{\varepsilon} = \varepsilon\, \widehat{\boldsymbol{\varepsilon}} = \varepsilon \left(\mathbf{I} + \tilde{\boldsymbol{\varepsilon}}\right), \tag{6}$$

where ε is the scalar dielectric permittivity of body in its initial (unstrained) state; $\mathbf{I}$ is the unit tensor of rank two.

As the optical anisotropy and inhomogeneity induced by elastic strain are small, in linear approximation we can take

$$\tilde{\boldsymbol{\varepsilon}} = \mathbf{P} \cdot\cdot \mathbf{e}. \tag{7}$$

Here $\mathbf{P} = \{P_{ijkl}\}$ stands for photoelasticity tensor of rank four, $\mathbf{e} = \{e_{ij}\}$ stands for linear strain tensor.

As in its initial unstrained state the medium is isotropic, the components P_{ijkl} of photoelasticity tensor can be expressed in two independent dimensionless material constants P_e and $\hat{P}_e$:

$$P_{ijkl} = P_e\delta_{ij}\delta_{kl} + \hat{P}_e\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \tag{8}$$

where δ_{ij} is Kronecker delta.

In view of (8), the relation (7) can be rewritten as

$$\tilde{\boldsymbol{\varepsilon}} = P_e \mathrm{Tr}\left(\mathbf{e}\right)\mathbf{I} + \hat{P}_e\mathbf{e}. \tag{9}$$

If to decompose tensor $\tilde{\boldsymbol{\varepsilon}}$ into the isotropic $\tilde{\varepsilon}$ and deviatoric $\hat{\boldsymbol{\varepsilon}} = \{\hat{\varepsilon}_{ij}\}$ parts:

$$\tilde{\boldsymbol{\varepsilon}} = \tilde{\varepsilon}\,\mathbf{I} + \hat{\boldsymbol{\varepsilon}}, \tag{10}$$

where $\tilde{\varepsilon} \equiv 1/3\mathrm{Trace}\left(\tilde{\boldsymbol{\varepsilon}}\right)$, $\hat{\boldsymbol{\varepsilon}} \equiv \tilde{\boldsymbol{\varepsilon}} - \tilde{\varepsilon}\,\mathbf{I}$, we can use instead of (9) the relations

$$\tilde{\varepsilon} = P_e e, \quad \hat{\boldsymbol{\varepsilon}} = 2\hat{P}_e\hat{\mathbf{e}}. \tag{11}$$

Here we used denotation $P_e = 1/3(3P_1 + 2\hat{P}_e)$, $e = \mathrm{Tr}(\mathbf{e})$, $\hat{\mathbf{e}} = \mathbf{e} - 1/3e\,\mathbf{I}$, where e is volumetric strain (dilatation), $\hat{\mathbf{e}}$ is strain deviator.

When the strain is elastic, strain tensor $\mathbf{e}$ can be explicitly termed via the Cauchy stress tensor $\boldsymbol{\sigma} = \{\sigma_{ij}\}$ and we can rewrite relationships (11) in the form

$$\tilde{\varepsilon} = P_\sigma\sigma, \quad \hat{\boldsymbol{\varepsilon}} = 2\hat{P}_\sigma\hat{\boldsymbol{\sigma}}. \tag{12}$$

Here we used denotation $P_\sigma = P_e/K = (3P_1 + 2\hat{P}_e)/(3K)$, $\hat{P}_\sigma = \hat{P}_e/G$, $\sigma = -1/3\mathrm{Tr}(\boldsymbol{\sigma})$, $\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} + \sigma\mathbf{I}$, where K and G are dilatation and shear modules of medium [11]; σ and $\hat{\boldsymbol{\sigma}} = \{\hat{\sigma}_{ij}\}$ are isotropic and deviatoric parts of stress tensor $\boldsymbol{\sigma}$.

If the components e_{ij} of strain tensor are given as functions specified in area $\mathcal{V}$:

$$e_{ij} = e_{ij}(\mathbf{r}), \quad \mathbf{r} \in \mathcal{V} \tag{13}$$

we can determine, with the using the relations (6), (9), the distributions of components ε_{ij} of permittivity tensor in the area $\mathcal{V}$

$$\varepsilon_{ij} = \varepsilon_{ij}(\mathbf{r}), \quad \mathbf{r} \in \mathcal{V} \tag{14}$$

To introduce a quantitative measure for optical inhomogeneity of medium, induced by the strain, we consider a unit vector $\mathbf{k} = \{k_1, k_2, k_3\}$ and a point $M(\xi_1, \xi_2, \xi_3) \in \mathcal{V}$. The pair $\mathbf{K} = \{\mathbf{k}, M\}$ determines a unique line passing through the body. We can consider a directional permittivity component distributions: $\widehat{\varepsilon}_{ij}\,(z) = \widehat{\varepsilon}_{ij}\,(k_1z + \xi_1,\ k_2z + \xi_2,\ k_3z + \xi_3)$, where z is a coordinate on K.

Let $\|\partial\,\widehat{\varepsilon}_{ij}\,/\partial z\|$ be a scalar norm of tensor-valued function $\partial\,\widehat{\varepsilon}_{ij}\,/\partial z$. Then the length $l_{\mathbf{K}} = \|\partial\widehat{\varepsilon}_{ij}/\partial z\|^{-1}$ is a characteristic parameter of medium's optical inhomogeneity in the direction $\mathbf{K}$. We say that optical inhomogeneity of the medium in direction $\mathbf{K}$ is week if the wavelength λ of sounding light is small as compared to the length $l_{\mathbf{K}}$.

Introducing the dimensionless parameter $\mu_{\mathbf{K}} = \lambda/l_{\mathbf{K}}$, we can express this definition in the form: inhomogeneity in the direction $\mathbf{K}$ is week if $\mu_{\mathbf{K}} \ll 1$. Similarly, we say that the body $\mathcal{B}$ is slightly inhomogeneous if $\mu \ll 1$, where $\mu = \max\limits_{\mathbf{K}\in\mathcal{K}}(\mu_{\mathbf{K}})$, $\mathcal{K}$ stands for the set of straights crossing the domain $\mathcal{V}$.

3. Model of interaction of polarized light with strained solid

Let the body $\mathcal{B}$ is sounded in the direction $\mathbf{K}$ by a parallel light beam which falls on the boundary $\partial\mathcal{V}$ from external homogeneous isotropic lossless medium with permittivity ε_{out}. Electromagnetic field of a light beam refracted into the body's volume $\mathcal{V}$ satisfies the system (4), (5).

Since the body is non-magnetizable, it is convenient to reduce the system (4), using relations (5), to one vector wave equation in electric field $\mathbf{E}$ or to equivalent linear system of three coupled scalar wave equations in Cartesian components E_i, $i = 1, 2, 3$ of electric field

$$\frac{1}{c^2}\,\widehat{\varepsilon}_{ij}\,\frac{\partial^2 E_j}{\partial t^2} = \Delta E_i - \frac{\partial}{\partial x_i}\frac{\partial E_j}{\partial x_j}. \tag{15}$$

Here $c = 1/\sqrt{\varepsilon\mu_0}$ is velocity of light in unstrained medium.

Monochromatic light is harmonic in time electromagnetic wave:

$$\mathbf{E}(\mathbf{r}, t) = \operatorname{Re}\left(\dot{\mathbf{E}}(\mathbf{r})\exp(-i\omega t)\right), \tag{16}$$

where $\dot{\mathbf{E}}(\mathbf{r})$ is complex wave amplitude (phasor); i stands for imaginary unit; ω is a real constant (circular frequency).

Substituting (16) into equations (15), we come to time-independent wave equations for the phasors $\dot{E}_j$:

$$-\frac{\omega^2}{c^2}\,\widehat{\varepsilon}_{ij}\,\dot{E}_j = \Delta\dot{E}_i - \frac{\partial}{\partial x_i}\frac{\partial\dot{E}_j}{\partial x_j}, \quad i, j = 1, 2, 3. \tag{17}$$

The functions $\dot{E}_j$ satisfies on the boundary $\partial\mathcal{V}$ the conditions

$$\begin{gathered}\left(\dot{E}_i + \dot{E}_i^{out} - \dot{E}_i^{I}\right)\Big|_{\mathcal{S}_K}(\delta_{ij} - n_i n_j) = 0, \quad \left(\varepsilon\,\widehat{\varepsilon}_{ij}\,\dot{E}_j + \varepsilon_{out}\dot{E}_i^{out} - \varepsilon_{out}\dot{E}_i^{I}\right)\Big|_{\mathcal{S}_K} n_i = 0,\\ \left(\dot{E}_j + \dot{E}_j^{out}\right)\Big|_{\partial\mathcal{V}/\mathcal{S}_K}\cdot(\delta_{ij} - n_i n_j) = 0, \qquad \left(\varepsilon\,\widehat{\varepsilon}_{ij}\,\dot{E}_j + \varepsilon_{out}E_i^{out}\right)\Big|_{\partial\mathcal{V}/\mathcal{S}_K} n_i = 0.\end{gathered} \tag{18}$$

Here $\dot{\mathbf{E}}^I = \left\{\dot{E}_i^I\right\}$ stands for phasor of incident light beam, $\mathcal{S}_K$ is the domain of intersection of the incident beam and the boundary $\partial\mathcal{V}$, $\dot{\mathbf{E}}^{out} = \left\{\dot{E}_i^{out}\right\}$ stand for the electric phasor of the field arising outside the boundary $\partial\mathcal{V}$. It satisfy the equation

$$\Delta\dot{E}_i^{out} + \frac{\omega^2}{c_{out}^2}\dot{E}_i^{out} = 0, \quad c_{out} = 1/\sqrt{\varepsilon_{out}\mu_0}. \tag{19}$$

We can consider $\dot{\mathbf{E}}^I$ in conditions (16) as a given vector-function specified in the domain $\mathcal{S}_{\mathbf{K}}$:

$$\dot{\mathbf{E}}^I = \dot{\mathbf{E}}^I(\mathbf{r}), \quad \mathbf{r} \in \mathcal{S}_K \tag{20}$$

The equations (17), (19) with boundary conditions (18) and given functions (14), (20) specify the correct direct problem for determination of the parameters $\dot{\mathbf{E}}(\mathbf{r})$ and $\dot{\mathbf{E}}^{out}(\mathbf{r})$ of electromagnetic field the body's volume $\mathcal{V}$ and outside its boundary $\partial\mathcal{V}$. We will consider this problem as a mathematical model for sounding of the body with polarized light.

4. Simplification of the model

The model is too complicated to use it for developing the methods of tensor field tomography. But the model can be simplified if to take into account the typical conditions of implementation of polarized-optical measurements and characteristic features of the body $\mathcal{B}$. Let consider them.

1. To prevent reflection and refraction of light on the boundary $\partial\mathcal{V}$, the object is immersed in transparent optically isotropic and homogeneous liquid, permittivity ε_{out} of which is very close to permittivity of object in its actual state. Due to that the intensity of reflected by the boundary $\partial\mathcal{V}$ in the area $\mathcal{S}_{\mathbf{K}}$wave is very small and we can neglect by reflection and refraction of the light waves on the boundary $\partial\mathcal{V}$.

2. Sounding of body is carried out by parallel beams of monochromatic light. The beam's diameter is much larger than the wavelength λ and smaller than the characteristic length $l = \lambda/\mu$. So, in some approximation we can consider the sounding beam as a plane harmonic wave falling on the body surface.

3. Optical inhomogeneity of the body induced by its strain-stressed state is very small. This means that on distances about several wavelengths the medium permittivity is practically invariable. Due to this and taking into account the conditions 1 and 2, we can:

- neglect by light refraction in the body's volume $\mathcal{V}$ and its surface $\partial\mathcal{V}$ and consider the light rays as straight line,
- neglect by light reflection in the body volume $\mathcal{V}$ and its surface $\partial\mathcal{V}$ and consider only direct wave initiated by incident wave,
- neglect by gradients of permittivity in directions normal to the light ray on its polarization state.

Under these assumptions, the system (17) is reduced to system of two second-order ordinary equations with regard to the transversal components $\dot{E}_1$, $\dot{E}_2$ of complex amplitude $\dot{\mathbf{E}}$ of direct refracted wave:

$$\frac{d^2E_k}{dz^2} + \frac{\omega^2}{c^2}\,\widehat{\varepsilon}^{K}_{kl}\,E_l = 0, \quad k,l = 1,2 \tag{21}$$

and the conditions, given on the part of boundary, on which the incident falls:

$$E_k|_{z=0} = E_k^0, \tag{22}$$

Here E_1^0 and E_2^0 are given complex constants — the components of complex amplitude $\dot{\mathbf{E}}_0$ of the incident wave, $\widehat{\varepsilon}^{\mathbf{K}}_{kl}$ $(k,l = 1,2)$ stands for components of relative permittivity $\widehat{\boldsymbol{\varepsilon}}$. In Cartesian coordinate system $\{x,y,z\}$ associated with the direction $\mathbf{K}$ (z axis is directed along $\mathbf{K}$) we have:

$$\widehat{\varepsilon}^{\mathbf{K}}_{kl} = A^{\mathbf{K}}_{kn}A^{\mathbf{K}}_{lm}\,\widehat{\varepsilon}_{nm}, \quad k,l = 1,2, \quad m,n = 1,2,3, \tag{23}$$

where $\widehat{\varepsilon}_{kl}$ stands for the components of tensor $\widehat{\boldsymbol{\varepsilon}}$ in the global system $\{x_1,x_2,x_3\}$; $A^{\mathbf{K}}_{ik}$ is the matrix for coordinates transformation from the global system to $\{x,y,z\}$.

System with variable coefficients (21) describes evolution of polarization state of monochromatic light beam on the way of its propagation in the direction $\mathbf{K}$ in the body volume.

Introducing Jones vector — 2×1-matrix $\mathbf{E} = (E_1,E_2)^{\mathrm{T}}$ — and 2×2-matrix composed of relative permittivity components:

$$\widehat{\mathcal{E}}_{\mathbf{K}} = \begin{pmatrix} \widehat{\varepsilon}^{\mathbf{K}}_{11} & \widehat{\varepsilon}^{\mathbf{K}}_{12} \\ \widehat{\varepsilon}^{\mathbf{K}}_{21} & \widehat{\varepsilon}^{\mathbf{K}}_{22} \end{pmatrix}, \tag{24}$$

we can rewrite system (21) in the matrix form

$$\frac{d^2\mathbf{E}}{dz^2} + \frac{\omega^2}{c^2}\,\widehat{\mathcal{E}}_{\mathbf{K}}\cdot\mathbf{E} = 0. \tag{25}$$

Taking into account relation (6), we can shape (21) into

$$\frac{d^2E_k}{dz^2}+\frac{\omega^2}{c^2}E_k+\frac{\omega^2}{c^2}\tilde{\varepsilon}^{\mathbf{K}}_{kl}E_l=0 \tag{26}$$

where $\tilde{\varepsilon}^{\mathbf{K}}_{ij}$, $i,j=1,2$ stands for Cartesian components of relative permittivity tensor disturbance $\tilde{\boldsymbol{\varepsilon}}$ in Cartesian coordinate system associated with $\mathbf{K}$.

In the case of unstrained medium $\tilde{\varepsilon}_{ij}=0$ and (26) turn into

$$\frac{d^2E_k}{dz^2}+\frac{\omega^2}{c^2}E_k=0 \tag{27}$$

This system describes plane waves propagation in homogeneous isotropic medium with permittivity ε. It has a solution in the form of linearly polarized plane wave:

$$E^0_k(z)=A_k\exp\left(-iCz\right),\quad k=1,2, \tag{28}$$

where A_k are constants, $C\equiv\omega/c$.

Let $E_kt(z)$ be a solutions of system (21), then functions $E_k(z)/E^0_k(z)$, $k=1,2$ define the deviation of the wave, propagating in the strained medium, with respect to the plane wave (28), propagating in this medium when strain vanishes. Introducing the new variables

$$\tilde{E}_k(z)=E_k(z)\exp(-iCz),\quad k=1,2 \tag{29}$$

we reduce the system (26) to the next form

$$\frac{d^2\tilde{E}_k}{dz^2}-2i\frac{\omega}{c}\frac{d\tilde{E}_k}{dz}+\frac{\omega^2}{c^2}\tilde{\varepsilon}^{\mathbf{K}}_{kl}\tilde{E}_l=0 \tag{30}$$

If to use the dimensionless coordinate $\zeta=z/l_{\mathbf{K}}$, we can rewrite (30) in the form

$$\mu^2_{\mathbf{K}}\frac{d^2\tilde{E}_k}{d\zeta^2}-4i\mu_{\mathbf{K}}\pi\frac{d\tilde{E}_k}{d\zeta}+4\pi^2\tilde{\varepsilon}^{\mathbf{K}}_{kl}\tilde{E}_l=0. \tag{31}$$

As $\mu^2_{\mathbf{K}}\ll 1$, system (31) is a singularly disturbed one, because it contains the small parameter $\mu^2_{\mathbf{K}}$ as coefficients at the second (higher) derivatives. Solutions of such systems, as it is known [12], contain two components —fast and slowly changing ones. The fast component is localized near the boundary and quickly decays with the distance from it. It is a so called boundary layer. This component describes the wave reflected in the volume and it is not interesting for further consideration.

To solve the system (31) we can apply an asymptotic expansion method [13]. In zero order expansion we come to the next system for the slow component

$$\frac{d\tilde{E}_k}{dz}+i\frac{1}{2}C\tilde{\varepsilon}^{\mathbf{K}}_{kl}\tilde{E}_2=0. \tag{32}$$

System (32) represents the mathematical model for polarized light propagation in sligtly inhomogeneous anisotropic medium in terms of complex amplitudes $\tilde{E}_1$, $\tilde{E}_2$. Introducing Jones vector [14] $\tilde{\mathbf{E}}=(\tilde{E}_1,\tilde{E}_2)^{\mathrm{T}}$, we can rewrite it in the matrix form:

$$\frac{d\tilde{\mathbf{E}}}{dz}+iC\tilde{\mathcal{E}}_{\mathbf{K}}\cdot\tilde{\mathbf{E}}=0. \tag{33}$$

where $\tilde{\mathcal{E}}_{\mathbf{K}}$ is 2×2-matrix:

$$\tilde{\mathcal{E}}_{\mathbf{K}}=\begin{pmatrix}\tilde{\varepsilon}^{\mathbf{K}}_{11} & \tilde{\varepsilon}^{\mathbf{K}}_{12}\\ \tilde{\varepsilon}^{\mathbf{K}}_{21} & \tilde{\varepsilon}^{\mathbf{K}}_{22}\end{pmatrix} \tag{34}$$

If to decompose tensor $\tilde{\boldsymbol{\varepsilon}}_{\mathbf{K}}$ into the isotropic $\tilde{\varepsilon}_{\mathbf{K}}$ and deviatoric $\hat{\boldsymbol{\varepsilon}}_{\mathbf{K}} = \{\hat{\varepsilon}_{kl}^{\mathbf{K}}\}$ parts, we can rewrite system (32) in the form

$$\frac{d\tilde{E}_k}{dz} + iC\tilde{\varepsilon}_{\mathbf{K}}\tilde{E}_k + iC\hat{\varepsilon}_{kl}^{\mathbf{K}}\tilde{E}_l = 0, \tag{35}$$

where $\hat{\varepsilon}_{kl}^{\mathbf{K}}$, $k,l = 1,2$ stands for components of deviatoric part $\hat{\boldsymbol{\varepsilon}}_{\mathbf{K}}$ of permittivity tensor disturbance $\tilde{\boldsymbol{\varepsilon}}$ in the coordinate system $\{x_1, x_2, z\}$ associated with $\mathbf{K}$.

If strain is isotropic $\hat{\boldsymbol{\varepsilon}}_{\mathbf{K}} = 0$, and system (35) is braked up into two independent equations:

$$\frac{d\tilde{E}_k}{dz} + iC\tilde{\varepsilon}_{\mathbf{K}}\tilde{E}_k = 0. \tag{36}$$

Their solutions describe the wave:

$$\tilde{E}_j^{0} = B\exp\left(-iC\int_0^z \tilde{\varepsilon}_{\mathbf{K}}dz\right), \tag{37}$$

which phase is changing along coordinate due to directional dependence of isotropic part $\tilde{\varepsilon}(z)$ of tensor $\tilde{\boldsymbol{\varepsilon}}$.

As we can see, the isotropic part $\tilde{\varepsilon}$ of tensor $\tilde{\boldsymbol{\varepsilon}}$ does not affect on the phase retardation of waves propagating in the body. So, we can exclude it from the equations (35). For this we represent the solution of the system (35) as

$$\tilde{E}_j(z) = \hat{E}_j(z)\exp\left(-iC\int_0^z \tilde{\varepsilon}dz\right), \tag{38}$$

where $\hat{E}_j(z)$, $j = 1,2$ are new unknown complex-valued functions.

Substituting (38) into (35), we reduce the model to the form

$$\frac{d\hat{E}_k}{dz} + i\frac{1}{2}C\hat{\varepsilon}_{kl}^{\mathbf{K}}\hat{E}_l = 0. \tag{39}$$

To satisfy the conditions (22) we should subordinate the function $\hat{E}_k(z)$, $k = 1,2$ to the conditions:

$$\hat{E}_j(z)\Big|_{z=0} = E_j^0, \quad j = 1,2, \tag{40}$$

Introducing Jones vector $\hat{\mathbf{E}} = (\hat{E}_1, \hat{E}_2)^{\mathrm{T}}$ and 2×2-matrix

$$\hat{\mathcal{E}}_{\mathbf{K}} = \begin{pmatrix} \hat{\varepsilon}_{11}^{\mathbf{K}} & \hat{\varepsilon}_{12}^{\mathbf{K}} \\ \hat{\varepsilon}_{21}^{\mathbf{K}} & \hat{\varepsilon}_{22}^{\mathbf{K}} \end{pmatrix}, \tag{41}$$

we can rewrite system (39) in the matrix form

$$\frac{d\hat{\mathbf{E}}}{dz} + iC\hat{\mathcal{E}}_{\mathbf{K}}\cdot\hat{\mathbf{E}} = 0. \tag{42}$$

So, mathematical model for interaction of polarized light with slightly inhomogeneous anisotropic medium (21), (22) was reduced to Cauchy problem for the system of the ordinary first-order differential equations (39) with variable coefficients and initial conditions (40)

If optical inhomogeneity and anisotropy of the medium are induced by its strain-stressed state, we can rewrite the systems, with use of relationships (15) or (17), in terms of strain or stress field parameters. For instance for system (39) we will have

$$\frac{d\hat{E}_k}{dz} + i\hat{C}_e \hat{e}^{\mathbf{K}}_{kl} \hat{E}_l = 0, \tag{43}$$

$$\frac{d\hat{E}_k}{dz} + i\hat{C}_\sigma \hat{\sigma}^{\mathbf{K}}_{kl} \hat{E}_l = 0, \tag{44}$$

where $\hat{C}_e = 2C\hat{P}_e = \hat{P}_e\omega/c$, $\hat{C}_\sigma = 2C\hat{P}_\sigma = \hat{P}_\sigma\omega/c$.

In the matrix form these systems look like

$$\frac{d\hat{\mathbf{E}}}{dz} + i\hat{C}_e \hat{\mathcal{E}}_{\mathbf{K}} \cdot \hat{\mathbf{E}} = 0, \tag{45}$$

$$\frac{d\hat{\mathbf{E}}}{dz} + i\hat{C}_\sigma \hat{\mathcal{S}}_{\mathbf{K}} \cdot \hat{\mathbf{E}} = 0, \tag{46}$$

where

$$\hat{\mathcal{E}}_{\mathbf{K}} = \begin{pmatrix} \hat{e}^{\mathbf{K}}_{11} & \hat{e}^{\mathbf{K}}_{12} \\ \hat{e}^{\mathbf{K}}_{21} & \hat{e}^{\mathbf{K}}_{22} \end{pmatrix}, \quad \hat{\mathcal{S}}_{\mathbf{K}} = \begin{pmatrix} \hat{\sigma}^{\mathbf{K}}_{11} & \hat{\sigma}^{\mathbf{K}}_{12} \\ \hat{\sigma}^{\mathbf{K}}_{21} & \hat{\sigma}^{\mathbf{K}}_{22} \end{pmatrix}.$$

System similar to (44) were suggested in [5] as a model of integrated photoelasticity. The author starts his consideration directly from the (21), then, using presentation (29) and some qualitative reasoning about smallness of the medium inhomogeneity, postulates system (32). We have shown here that system (30) is singularly disturbed and system (32) follows from it as zero order term in asymptotic expansion of solution.

5. Measurable polarized-optical parameters

Let $\mathbf{E}_0 = (\dot{E}^0_1, \dot{E}^0_2)^{\mathrm{T}}$ and $\mathbf{E}_h = (\dot{E}^h_1, \dot{E}^h_2)^{\mathrm{T}}$ be Jones vectors of sounding light beam, propagating in the direction $\mathbf{K}$, at two points: $z = 0$ (input) and $z = h_{\mathbf{k}}$ (output). Here $h_{\mathbf{k}}$ stands for dimension of the body in the direction $\mathbf{K}$. As medium is linear, we can write down a general relation connecting two complex 2×1-matrix $\mathbf{E}_h$ and $\mathbf{E}_0$ in the form [14]:

$$\mathbf{E}_h = \mathbf{J}_K \cdot \mathbf{E}_0, \tag{47}$$

where $\mathbf{J}_{\mathbf{K}}$ is a complex 2×2-matrix (Jones matrix).

The Jones matrix $\mathbf{J}_{\mathbf{K}}$ identically defines a change of polarization of light ray passed through the object in the direction $\mathbf{K}$: if $\mathbf{J}_{\mathbf{K}}$ is known we can calculate by the formula (47) the output light state $\mathbf{E}_h = (\dot{E}^h_1, \dot{E}^h_2)^{\mathrm{T}}$ for any given input state $\mathbf{E}_0 = (\dot{E}^0_1, \dot{E}^0_2)^{\mathrm{T}}$.

On the other hand, if to measure the light's polarization at the points $z = 0$ and $z = h_k$ and to use relationship (47), we can calculate matrix $\mathbf{J}_{\mathbf{K}}$ [14]. In this sense we can treat $\mathbf{J}_{\mathbf{K}}$ as a measurable characteristic polarized-optical parameter of object in the direction $\mathbf{K}$.

Consider the matrix

$$\hat{\mathbf{J}}_{\mathbf{K}} = \mathbf{J}_{\mathbf{K}} \exp\left(-i\theta_{\mathbf{K}}\right), \tag{48}$$

where $\theta_{\mathbf{K}} = 1/2(\varphi^{\mathbf{K}}_1 + \varphi^{\mathbf{K}}_2)$, $\varphi^{\mathbf{K}}_1 = \mathrm{Arg}\,(\lambda^{\mathbf{K}}_1)$, $\varphi^{\mathbf{K}}_2 = \mathrm{Arg}\,(\lambda^{\mathbf{K}}_2)$; $\lambda^{\mathbf{K}}_1$, $\lambda^{\mathbf{K}}_2$ stand for the eigenvalues of the matrix $\mathbf{J}_{\mathbf{K}}$.

It's easy to see, that eigenvalues of matrix $\hat{\mathbf{J}}_{\mathbf{K}}$ are

$$\hat{\lambda}^{\mathbf{K}}_1 = \exp\left(i\varphi_{\mathbf{K}}\right), \quad \hat{\lambda}^{\mathbf{K}}_2 = \exp\left(-i\varphi_{\mathbf{K}}\right), \quad \varphi_{\mathbf{K}} = 1/2\left(\varphi^{\mathbf{K}}_1 - \varphi^{\mathbf{K}}_2\right). \tag{49}$$

It is known [15], that logarithm of a 2×2 matrix $\mathbf{X}$, with the eigenvalues ξ_1 and ξ_2 such that $\xi_1 \neq 0$, $\xi_2 \neq 0$, $\xi_2 \neq \xi_1$, is determined as

$$\ln(\mathbf{X}) = \frac{\mathbf{X} - \xi_2 \mathbf{I}}{\xi_1 - \xi_2} \ln \xi_1 + \frac{\mathbf{X} - \xi_1 \mathbf{I}}{\xi_2 - \xi_1} \ln \xi_2 \tag{50}$$

Let us determine the matrix $\hat{\mathbf{L}}_{\mathbf{K}} = -i \ln(\hat{\mathbf{J}}_{\mathbf{K}})$ such that $\exp(-i\hat{\mathbf{L}}_{\mathbf{K}}) = \hat{\mathbf{J}}_{\mathbf{K}}$. Substituting into (50) $\mathbf{X} = \hat{\mathbf{J}}_{\mathbf{K}}$, $\xi_1 = \hat{\lambda}_1^{\mathbf{K}} = \exp(i\varphi_{\mathbf{K}})$ and $\xi_2 = \hat{\lambda}_2^{\mathbf{K}} = \exp(-i\varphi_{\mathbf{K}})$, after simple algebraic manipulations we obtain

$$\tilde{\mathbf{L}}_{\mathbf{K}} = -i \frac{\tilde{\mathbf{J}}_{\mathbf{K}} - \cos(\varphi_{\mathbf{K}})\mathbf{I}}{\sin(\varphi_{\mathbf{K}})} (\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}), \quad n_{\mathbf{K}} = 0, \pm 1, \pm 2, \ldots. \tag{51}$$

With accounting (49), we can rewrite (51) in the form

$$\hat{\mathbf{L}}_{\mathbf{K}} = \frac{\operatorname{Im}(\tilde{\mathbf{J}}_{\mathbf{K}})}{\sin(\varphi_{\mathbf{K}})} (\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}) = \begin{pmatrix} \cos(2\alpha_{\mathbf{K}}) & \sin(2\alpha_{\mathbf{K}}) \\ \sin(2\alpha_{\mathbf{K}}) & -\cos(2\alpha_{\mathbf{K}}) \end{pmatrix} (\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}), \tag{52}$$

where $\alpha_{\mathbf{K}}$ is the angle between the eigenvectors of the matrix $\hat{\mathbf{L}}_{\mathbf{K}}$ and coordinate axes x, y.

$\hat{\mathbf{L}}_{\mathbf{K}}$ is a real symmetric matrix defined by two independent real constants $\alpha_{\mathbf{K}}$ and $\varphi_{\mathbf{K}} \equiv \varphi_{\mathbf{K}} + n_{\mathbf{K}}$. It posses the feature $\operatorname{Trace}(\hat{\mathbf{L}}) = 0$.

Jones matrix $\mathbf{J}_{\mathbf{K}}$ can be represented in terms of matrixes $\hat{\mathbf{J}}_{\mathbf{K}}$ and $\hat{\mathbf{L}}_{\mathbf{K}}$ as

$$\mathbf{J}_{\mathbf{K}} = \exp(i\theta_{\mathbf{K}}\mathbf{I}) \cdot \hat{\mathbf{J}}_{\mathbf{K}} = \exp(i\theta_{\mathbf{K}}\mathbf{I}) \cdot \exp(i\hat{\mathbf{L}}_{\mathbf{K}}) = \exp\Big(i(\theta_{\mathbf{K}}\mathbf{I} + \hat{\mathbf{L}}_{\mathbf{K}})\Big). \tag{53}$$

This enables to rewrite relation (47) in the form

$$\mathbf{E} = \exp\Big(i(\theta_{\mathbf{K}}\mathbf{I} + \hat{\mathbf{L}}_{\mathbf{K}})\Big) \cdot \mathbf{E}_0. \tag{54}$$

As it follows from (52), (53), if the parameters $\theta_{\mathbf{K}}$, $\varphi_{\mathbf{K}}$, and $\alpha_{\mathbf{K}}$ are given we can calculate by the formula (54) the output light state $\mathbf{E}_h = (\dot{E}_1^h, \dot{E}_2^h)^{\mathrm{T}}$ for any given input state $\mathbf{E}_0 = (\dot{E}_1^0, \dot{E}_2^0)^{\mathrm{T}}$. So, we can use the triplet $(\theta_{\mathbf{K}}, \varphi_{\mathbf{K}}, \alpha_{\mathbf{K}})$ as measurable characteristic polarized-optical parameter instead of Jones matrix $\mathbf{J}_{\mathbf{K}}$.

Parameter $\theta_{\mathbf{K}}$ defines the absolute phase difference between the light states at the output and input. Parameter $\varphi_{\mathbf{K}}$ defines the phase retardation which light gains on its way in strained body between the input and output points. The third parameter $\alpha_{\mathbf{K}}$ determines change of electric field vector's orientation. Parameters $\varphi_{\mathbf{K}}$ and $\alpha_{\mathbf{K}}$ can be determined with use of polarized-optical technique by measuring the light ellipticity and orientation of polarization ellipse at the body input and output. To determine parameters θ_K more sophisticated interferometric instrumentation is needed, for instance, Mach-Zehnder interferometer [7] can be used for that.

6. Ray integrals of photoelasticity

With the use of the matricant the solutions of the problem (39), (40) can be written in the form

$$\hat{\mathbf{E}} = \cdot \mathbf{E}_0 \cdot \exp\left(-iC \int_0^z \hat{\mathcal{E}}_{\mathbf{K}}(\xi) d\xi\right) \tag{55}$$

where $\mathbf{E}_0 = (E_1^0, E_2^0)^{\mathrm{T}}$ is the input Jones vector

Substituting $z = h_{\mathbf{K}}$ into function (55), and taking into account relations (29) and (38) we obtain:

$$\mathbf{E}_{\mathbf{K}} = \exp\left(-iC\left(\left(h + \int_0^{h_{\mathbf{K}}} \tilde{\varepsilon}_{\mathbf{K}} dz\right)\mathbf{I} + \int_0^{h_{\mathbf{K}}} \hat{\mathcal{E}}_{\mathbf{K}}(z)dz\right)\right) \cdot \mathbf{E}_0, \tag{56}$$

where $\mathbf{E}_{\mathbf{K}}$ stands for output Jones vector.

The trace of the matrix $\hat{\mathcal{E}}_{\mathbf{K}}$ is nonzero: $\mathrm{Trace}(\hat{\mathcal{E}}_{\mathbf{K}}) \equiv \hat{\varepsilon}_{11}^{\mathbf{K}} + \hat{\varepsilon}_{22}^{\mathbf{K}} = -\hat{\varepsilon}_{33}^{\mathbf{K}} \neq 0$, so we can present the matrizant $\int_0^{h_{\mathbf{K}}} \hat{\mathcal{E}}_{\mathbf{K}}(z)dz$ in the form

$$\int_0^{h_{\mathbf{K}}} \hat{\mathcal{E}}_{\mathbf{K}}(z)dz = \int_0^{h_{\mathbf{K}}} \left(-\hat{\varepsilon}_{33}^{\mathbf{K}}\mathbf{I} + \mathcal{E}_{\mathbf{K}}(z)\right) dz, \tag{57}$$

where

$$\mathcal{E}_{\mathbf{K}} = \frac{1}{2}\begin{pmatrix} \tilde{\varepsilon}_{11}^{\mathbf{K}} - \tilde{\varepsilon}_{22}^{\mathbf{K}} & 2\tilde{\varepsilon}_{12}^{\mathbf{K}} \\ 2\tilde{\varepsilon}_{12}^{\mathbf{K}} & -\tilde{\varepsilon}_{11}^{\mathbf{K}} + \tilde{\varepsilon}_{22}^{\mathbf{K}} \end{pmatrix} \tag{58}$$

Using (57), we can rewrite relation (56) in the form

$$\mathbf{E}_{\mathbf{K}} = \exp\left(-iC\left(\left(h + \int_0^{h_{\mathbf{K}}} \varepsilon_{\mathbf{K}} dz\right)\mathbf{I} + \int_0^{h_{\mathbf{K}}} \mathcal{E}_{\mathbf{K}}(z)\, dz\right)\right) \cdot \mathbf{E}_0, \tag{59}$$

where

$$\varepsilon_{\mathbf{K}} = \tilde{\varepsilon}_{\mathbf{K}} - \hat{\varepsilon}_{33}^{\mathbf{K}} = \frac{2\left(\tilde{\varepsilon}_{11}^{\mathbf{K}} + \tilde{\varepsilon}_{22}^{\mathbf{K}}\right) - \tilde{\varepsilon}_{33}^{\mathbf{K}}}{3} \tag{60}$$

As far as matrix $\mathcal{E}_{\mathbf{K}}$ is real, symmetric, and its trace equals zero, we can compare now equalities (59) and (54). Taking into account (52), we obtain

$$\begin{aligned} &\int_0^{h_{\mathbf{K}}} \left(\tilde{\varepsilon}_{11}^{\mathbf{K}} - \tilde{\varepsilon}_{22}^{\mathbf{K}}\right) dz = 2\frac{\lambda}{\pi}\cos\left(2\alpha_{\mathbf{K}}\right)\left(\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}\right), \\ &\int_0^{h_{\mathbf{K}}} \varepsilon_{12}^{\mathbf{K}}\, dz = \frac{\lambda}{\pi}\sin\left(2\alpha_{\mathbf{K}}\right)\left(\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}\right). \end{aligned} \tag{61}$$

Hence, we reduce the model (21), (22) of interaction of polarized light with slightly anisotropic inhomogeneous medium to integral relations (61). The left sides of these relations are line integrals of linear combination of permittivity components. Their right sides are expressed by two independent characteristic optical parameters $\alpha_{\mathbf{K}}$, and $\varphi_{\mathbf{K}}$, that can be measured by polarized-optical method.

Using relations (11) or (12), we can express the permittivity components $\tilde{\varepsilon}_{kl}$ in (61) in terms of strain or stress components and come to corresponding integral relations for strain or stress fields. In term of strain components they take the form

$$\begin{aligned} &\int_0^{h_{\mathbf{K}}} \left(e_{11}^{\mathbf{K}} - e_{22}^{\mathbf{K}}\right) dz = 2\frac{\lambda}{\hat{P}_e\pi}\cos\left(2\alpha_{\mathbf{K}}\right)\left(\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}\right), \\ &\int_0^{h_{\mathbf{K}}} e_{12}^{\mathbf{K}}\, dz = \frac{\lambda}{\hat{P}_e\pi}\sin\left(2\alpha_{\mathbf{K}}\right)\left(\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}\right). \end{aligned} \tag{62}$$

If strain is elastic, we are able to apply relations (12), and reduce (62) to the form:

$$\begin{aligned} &\int_0^{h_{\mathbf{K}}} (\sigma_{11} - \sigma_{22})\, dz = 2\frac{\lambda}{\hat{P}_\sigma \pi} \cos(2\alpha_{\mathbf{K}}) (\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}), \\ &\int_0^{h_{\mathbf{K}}} \sigma_{12}^{\mathbf{K}}\, dz = \frac{\lambda}{\hat{P}_\sigma \pi} \sin(2\alpha_{\mathbf{K}}) (\varphi_{\mathbf{K}} + 2\pi n_{\mathbf{K}}). \end{aligned} \tag{63}$$

Relations (62) and (63) are integral photoelasticity relations for arbitrary inhomogeneous elastic 3-d stress-strained state. It is easy to see, that, when stress component $\sigma_{12}^{\mathbf{K}}$ equals zero, parameter $\alpha_{\mathbf{K}}$ also equals zero (the principal axes of the strain tensor are non-changeable along the direction $\mathbf{K}$). In this case the second relation (63) becomes identity and the first one change into relation (2).

7. Conclusions

Induced by strain optical inhomogeneity of the dielectric body is rather weak — the permittivity is changed on distances considerably exceeding the wavelength of sounding light. This enables to consider light rays propagating in strained medium as straight lines, neglect by their reflection in the body volume and by the influence of material inhomogeneity in directions, normal to the rays. With the use of these assumptions two integral elasticity relations were obtained. The relations connect line integrals of strain or stress deviators' components along any direction crossing the body with two independent characteristic optical parameters, which can be measured by polarized-optical method. Probing the body by polarized light beams in N different directions and measuring their polarization states on the output of the body one can obtain values for $2N$ line integrals. These data can be used to formulate inverse problems for non-destructive determination of body's stress-strained state.

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Інтегральні співвідношення фотопружності для неоднорідно деформованих діелектриків

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Розглянуто модель взаємодії поляризованого світла із неоднорідно деформованим немагнітним діелектричним тілом. Встановлені променеві інтеграли фотопружності, які пов'язують розподіли компонент тензора деформації у будь-якому напрямку в об'ємі тіла з вимірювальними поляризаційно-оптичними параметрами світлового променя, що поширюється у цьому напрямку. Модель можна використати для розроблення математичних методів для поляризаційно-оптичної обчислювальної томографії напружено-деформованого стану діелектричних тіл.

Ключові слова: *деформовані діелектрики, явище фотопружності, томографія тензорних полів, поляризаційно-оптичні методи*

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