

# RELIABILITY ANALYSIS OF UNSYMMETRICAL RAMIFIED COMPUTING SYSTEMS WITH RAYLEIGH DISTRIBUTED OUTPUT ELEMENTS

© Sydor A., 2006

*Main reliability characteristics of unsymmetrical ramified computing systems with Rayleigh distributed output elements are examined in this paper. Expressions for the failure probability, the failure frequency and the failure rate are worked out in the cases when computing systems ramified to level 2.*

Keywords – reliability characteristics, ramified systems, computing systems.

## 1. Introduction

Reliability as one of main indices of quality deals with products in service. There has developed the intensive study of distribution types in the reliability theory.

Ramified computing systems often have elements which are exposed to aging on output level. Lifetime of such elements is often circumscribed by the Rayleigh distribution. Elements of upper levels have lifetime circumscribed by the exponential distribution [1]. It is necessary to work out methods of reliability prediction with regard for system specific features.

## 2. Reliability Analysis

The unreliable operation of fault-tolerant communication technologies are causing disruptions to thousands of enterprises [2].

Let us consider an unsymmetrical hierarchical system with two nonequivalent branches on level 1, ramified to level 2, with Rayleigh distributed output elements (Fig. 1), where 2 elements of level 1 are subordinate to the element of level 0,  $a_2^{(1)}$  elements of level 2 are subordinate to the first element of level 1, and  $a_2^{(2)}$  elements of level 2 are subordinate to the second element of level 1. Without loss of generality assume that  $a_2^{(1)} < a_2^{(2)}$ .

The exponential distribution displays a no-memory property and is in contrast with the normal distribution which displays a memory property. The importance of the normal distribution lies not only in the fact that many sets of experimental data exhibit the properties of a random sample from this distribution, but also in its key role in the central limit theorem. As a consequence of this theorem it is possible to make inferences about populations on the basis of sample means, even for non-normal populations. Parametric methods of inference rely heavily on the normal distribution.

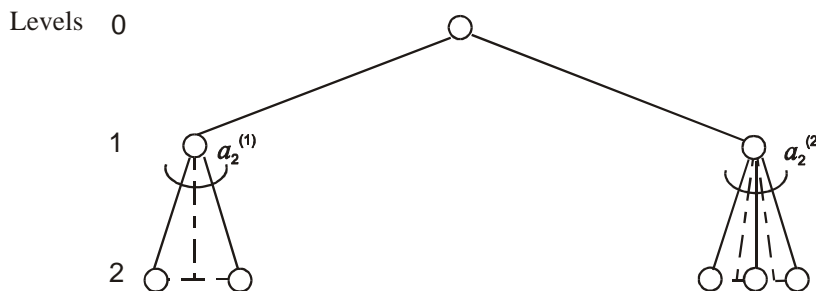


Fig.1. An unsymmetrical system with 2 branches of unequal value

The sum of squares of  $n$  independent standard normal variables has a chi-squared ( $\chi^2$ ) distribution with  $n$  degrees of freedom. The chi-squared distribution belongs to the gamma distribution family and has mean  $n$  and variance  $2n$ .

The Rayleigh distribution is applicable for failure prediction of measuring devices on the stage of intensive wear and aging.

Let  $Q_{2R}(k,t)$  be the failure probability in the prescribed availability condition  $k$ , where  $k$  is the system availability condition (not less than  $k$  operating output elements,  $0 < k \leq a_2^{(1)} + a_2^{(2)}$ ). With regard to [3] we obtain:

$$Q_{2R}(k,t) = 1 - e^{-\lambda_0 t} \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} \sum_{x_2^{(1)}=\max\{0, x_2-a_2^{(2)}\}}^{\min\{x_2, a_2^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{x_2^{(1)}}{a_2^{(1)}}\right)}^1 C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} \sum_{x_1^{(2)}=\text{ceil}\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^1 e^{-\lambda_1(x_1^{(1)}+x_1^{(2)})t} \times \\ \times \left(1 - e^{-\lambda_1 t}\right)^{2-(x_1^{(1)}+x_1^{(2)})} C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} e^{-\frac{x_2 t^2}{2\sigma_2^2}} \left(1 - e^{-\frac{t^2}{2\sigma_2^2}}\right)^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2} \quad (1)$$

We use  $a_{2R}(k,t)$  to denote the failure frequency in the prescribed availability condition. It is determined as the derivative of the failure probability with respect to time  $t$ , that is

$$a_{2R}(k,t) = e^{-\lambda_0 t} \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} \sum_{x_2^{(1)}=\max\{0, x_2-a_2^{(2)}\}}^{\min\{x_2, a_2^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{x_2^{(1)}}{a_2^{(1)}}\right)}^1 C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} \sum_{x_1^{(2)}=\text{ceil}\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^1 C_{a_2^{(2)}}^{x_2-x_2^{(1)}} \times \\ \times \sum_{j_1=0}^{2-(x_1^{(1)}+x_1^{(2)})} C_{2-(x_1^{(1)}+x_1^{(2)})}^{j_1} (-1)^{j_1} \sum_{j_2=0}^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2} C_{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2}^{j_2} (-1)^{j_2} \times \\ \times \left(\frac{x_2 + j_2}{\sigma_2^2} t + \lambda_0 + \lambda_1(x_1^{(1)} + x_1^{(2)} + j_1)\right) e^{-\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)t} e^{-\frac{x_2+j_2}{2\sigma_2^2} t^2} \quad (2)$$

Let  $\lambda_{2R}(k,t)$  be the failure rate of the system in the prescribed availability condition which is determined as a result of division of the failure frequency by the availability function. We obtain:

$$\lambda_{2R}(k,t) = \left( \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} \sum_{x_2^{(1)}=\max\{0, x_2-a_2^{(2)}\}}^{\min\{x_2, a_2^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{x_2^{(1)}}{a_2^{(1)}}\right)}^{a_1-x_1} C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} \times \right. \\ \times \sum_{x_1^{(2)}=\text{ceil}\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^1 C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} \sum_{j_1=0}^{2-(x_1^{(1)}+x_1^{(2)})} C_{2-(x_1^{(1)}+x_1^{(2)})}^{j_1} (-1)^{j_1} \sum_{j_2=0}^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2} C_{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2}^{j_2} \times \\ \left. \times (-1)^{j_2} \left(\frac{x_2 + j_2}{\sigma_2^2} t + \lambda_0 + \lambda_1(x_1^{(1)} + x_1^{(2)} + j_1)\right) e^{-\lambda_1(x_1^{(1)}+x_1^{(2)}+j_1)t} e^{-\frac{x_2+j_2}{2\sigma_2^2} t^2} \right) / \\ / \left( \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} \sum_{x_2^{(1)}=\max\{0, x_2-a_2^{(2)}\}}^{\min\{x_2, a_2^{(1)}\}} \sum_{x_1^{(1)}=\text{ceil}\left(\frac{x_2^{(1)}}{a_2^{(1)}}\right)}^1 C_{a_2^{(1)}x_1^{(1)}}^{x_2^{(1)}} \sum_{x_1^{(2)}=\text{ceil}\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^1 e^{-\lambda_1(x_1^{(1)}+x_1^{(2)})t} \times \right. \\ \left. \times \left(1 - e^{-\lambda_1 t}\right)^{2-(x_1^{(1)}+x_1^{(2)})} C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} e^{-\frac{x_2 t^2}{2\sigma_2^2}} \left(1 - e^{-\frac{t^2}{2\sigma_2^2}}\right)^{a_2^{(1)}x_1^{(1)}+a_2^{(2)}x_1^{(2)}-x_2} \right). \quad (3)$$

The exponential distribution is a special case of the Weibull distribution just as it is of the gamma distribution. The Weibull distribution has different failure rates depending on a shape parameter. The Raileigh distribution is a special case of the Weibull distribution which demarcate two different types of increasing failure rate behavior.

In case of failure of an element of level 0 the system will fail completely, therefore probability of failure-free operation of this element is maximal. Operation of all elements on other system's levels depends on operation

of this element. Elements of the lowest level influence on the system's operation the least, therefore probability of failure-free operation of them may be the least.

With increase of time of the system's operation, elements of the system begin to fail owing to increase of probability of failure of elements on different levels. With decrease of count of operating output elements, average duration of the system's stay in correspond state increase. When time of the system's operation is large enough, it will fail completely, i.e. no output elements operate.

If the system is unrestorable, every system's element is not restored after its failure and the system remains in the same state till failure of the next element.

### 3. Conclusion

The main thrust of this paper is to reduce the computational time and complexity when evaluating reliability characteristics of unsymmetrical ramified computing systems.

Quality control is the use of statistical methods, including control charts, cusum charts and acceptance sampling, to determine whether processes of goods produced are meeting certain specifications, and to indicate when corrective action should be taken if standards are not being met. Reliability theory is a subset of quality control; in it the characteristics studied is the length of life of the item. Reliability as one of main indices of quality deals with products in service. There has developed the intensive study of distribution types in the reliability theory.

Prediction of reliability parameters at stages of design of complicated systems makes possible to evaluate probabilistic and time characteristics of systems, to compare reliabilities of possible variants of systems' structures depending on requirements of production process.

Thus, analytical expressions are worked out for evaluation of three main reliability characteristics of compound unsymmetrical systems: the failure probability in the prescribed availability condition, the failure frequency in the prescribed availability condition, the failure rate in the prescribed availability condition.

### References

- [1] Sydor A. "A choice of lifetime distribution functions for elements of hierarchical ramified systems", *Proceedings of the VII th International Conference CADSM 2003 "The Experience of Designing and Application of CAD Systems in Microelectronics"*. Lviv – Slavske, 2003, pp. 262-263.
- [2] Snow A.P. "Network reliability: the concurrent challenges of innovation, competition, and complexity", *IEEE Transactions on Reliability*, vol. 50, № 1, pp. 38-40, 2001.
- [3] Sydor A. "Recurrent expressions for probabilistic reliability characteristics of unsymmetric ramified systems", *Proceedings of the VII th International Conference CADSM 2003 "The Experience of Designing and Application of CAD Systems in Microelectronics"*. Lviv – Slavske, 2003, pp. 393-394.
- [4] Grosh D.L. *A Primer of Reliability Theory*. N.Y.: John Wiley & Sons, 1989.
- [5] Mohammed Zayed Raqab. "Optimal prediction-intervals the exponential distribution, based on generalized order statistics", *IEEE Transactions on Reliability*, vol. 50, pp. 112-115, 2001.
- [6] Jong-Wuu Wu, Tzong-Ru Tsai, Liang-Yuh Ouyang. "Limited failure-censored life test for Weibull distribution", *IEEE Transactions on Reliability*, vol. 50, № 1, pp. 107-111, 2001.
- [5] Marunchak D., Sydor A. "Reliability characteristics of ramified systems", *Proceedings of the VI-th International Conference "The Experience of Designing and Application of CAD Systems in Microelectronics" (CADSM2001)*. Lviv - Slavsko, 2001, pp. 130-131.
- [6] Sydor A. "Evaluation of reliability characteristics for ramified computing systems", *Proceedings of the Second IEEE International Workshop on Intelligent Data Acquisition and Advanced Computing Systems*. Lviv, 2003, pp. 80-83.
- [7] Marunchak D., Sydor A. "Estimation of reliability for hierarchical symmetric systems with Weibull distributed output elements", *Proceedings of International Conference on Modern Problems of Telecommunications, Computer Science and Engineer Training*. Lviv - Slavsko, 2000, pp. 1-2.