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BASKET PAYMENT COMBINED WITH MARKOWITZ PORTFOLIO APPLIED TO COMMODITY TRADE

Abstract. The article presents a method for expressing the value of commodities on world markets (petroleum products) as a function of a basket of currencies or a basket of precious metals (Gold and Silver) and compares their performance with a Markowitz portfolio of the underlying. The method of basket payments can be used as a tool for diminishing the riskiness of forward transactions on commodities markets. By establishing a model of payment based on the baskets, we propose to liberate the payment rules from disadvantages inherent in the use of official currencies. After minimizing the price variability of individual commodities by using instrumental (basket) prices, we propose to construct Markowitz portfolios of the commodities to reach further reduction of the forward transactions riskiness. Results of applying of the idea to selected petroleum products are shown.

Key words: basket payments, Markovitz portfolio, commodity trade.

1. Introduction – currency baskets and financial stability. The idea of basket payment has been promoted for a decade as a way to stabilize world markets against fluctuations in official currencies' (mainly USD) exchange rates and resultant variations in commodities prices (e.g. see [8]). The main issue is to establish an instrumental currency as a weighted average of leading currencies. Mundell [10] proposed the idea of optimal currency areas as a means to facilitate economic exchanges. There are two options available to a government when it comes to a decision to stabilize its currency: it can link the currency to another currency or to a basket of currencies. The objective of pegging to a currency basket is usually to stabilize the exchange rate between the pegging country and its major trading partners in ways to minimize economic shocks resulting from exchange rate fluctuations [7]. An alternative to basket pegging is pegging to SDR (Special Drawing Rights). However, as Branson and Katseli [3] report, SDRs represent a generic currency basket and are not adapted to the needs of any particular country. In Asia, the crisis of 1997 has shown that a dollar-peg exchange regime was not the optimal mechanism for managing exchange rates [11] as there appeared differences between actual and desirable rates of exchange.

Commodities, such as metals or petroleum products, are priced on open markets in terms of a base currency. Effectively, most commodities are priced and transacted in spot, futures or options contracts in terms of the dollar. The price of an asset does not, however, have to be denominated in a particular currency but can be expressed as a weighted average of a defined number of selected assets. Such basket payments may be less risky due to mutual compensation of variability of particular basket components prices.

In our earlier papers [4-6], we proposed to apply a model of basket payments to metal commodities, by constructing a dedicated (instrumental) currency, minimizing the forward transaction risk. In addition to official currencies, precious metals (i.e. Gold and Silver) can be included into such a basket. This article extends this idea with the Markowitz portfolio theory [9] to select the least risky set of commodities composing the individual basket currency. The goal is to model the behaviour of commodity prices by a suitably chosen portfolio of currencies and commodities and illustrate the usability of basket instruments for either hedging or investment purposes.

In this paper analysis focuses on modeling the values of selected petroleum products: TXPropan; USGulfROil; HeatOil; LCrude1; NYGasF; WTI and Brent. In the first step, the value of each of these products is expressed by a basket mix of global currencies (Euro, British Pound, SDRs, Yen, Ruble, Polish Zloty, Indian Rupees, Brazilian Real, the Australian dollar and the US dollar) as well as precious metals (Silver and Gold). Second, the performance of every basket identified in the first step is compared to an optimal basket of selected currencies based on the Markowitz efficient frontier.

2. Basket payments and portfolios – theoretical background

2.1 Basket optimisation. Let us consider a trade contract made at time n, concerning a commodity k, to be delivered at time $n+p$. One can take the agreement (**Rule I**) the contract amount due may be paid at time n or $n+p$ with a package of quota $V_k=\{V_{kc}, c=1, ..., C\}$ in different currencies

$$V_{kcn} = P_{kn}\beta_c R_{cn};$$
$$\sum_{c=1}^{C}\beta_c = 1; \quad \sum_{c=1}^{C}\frac{V_{kcn}}{R_{cn}} \equiv P_{kn}, \tag{1}$$

where β_c means the fraction of the original price to be paid in c-th currency, agreed at time n or before. The quota V_k are fixed at the time n according to eq.(1), so that $V_{kcn+p} = V_{kcn}$.

The transaction risk [12] could be expressed as the change $\Delta_I P_{kp}$ of the commodity price recalculated to US dollars at the time $n+p$:

$$\Delta_I P_{kp} = P_{kn+p} - \sum_{c=1}^{C}\frac{V_{kcn}}{R_{cn+p}} = = P_{kn+p} - \sum_{c=1}^{C}\beta_c \frac{R_{cn}}{R_{cn+p}}P_{kn}. \tag{2}$$

One can take also another rule (**Rule II**): at the time n we define only a currency basket $W_n=\{W_{cn}: c=1, ..., C\}$ where $W_{cn} = b_c R_{cn}$ is the quota of c-th currency to be paid for 1 USD, either at time n or $n+p$. The transaction risk may be expressed as the difference $\Delta_{II} P_{kp}$ of the commodity price paid at n and $n+p$, recalculated to US dollars at the time $n+p$:

$$\Delta_{II} P_{kp} = (P_{kn+p} - P_{kn})\sum_{c=1}^{C}\beta_c \frac{R_{cn}}{R_{cn+p}} \tag{3}$$

Nevertheless, the risk measures (2) and (3) may be misleading, as they do not take into account changeable position (appreciation/depreciation) of the USD itself. Moreover the risk assessment involves the ratio of two random variables R_{cn}/R_{cn+p} that makes it more uncertain. Hence, to eliminate the above drawbacks, in the paper [6] we propose to use for the trade risk assessment an instrumental price Π_{kn}, based on the currency basket composed of the currencies $c=1, \ldots, C$, recalculated to USD with constant exchange ratios R_{cref}:

$$\Pi_{kn} = P_{kn}\sum_{c=1}^{C} b_c \frac{R_{cn}}{R_{cref}}; \qquad \sum_{c=1}^{C} b_c = 1, \tag{4}$$

where $\{b_c$: c=1, …,$C\}$ are the factors (the basket coefficients) partitioning the transaction risk onto the currencies c.

The contract can be made according to the rules I or II with $\beta_c=b_c$, but its risk may be evaluated as the instrumental price change:

$$\Delta_I \Pi_{kp} = \sum_{c=1}^{C}\left(\frac{b_c}{R_{cref}}(P_{kn+p}R_{cn+p} - P_{kn}R_{cn})\right) \tag{5}$$

or weighted change of the original price (like in eq.(3):

$$\Delta_{II} \Pi_{kp} = (P_{kn+p} - P_{kn})\sum_{c=1}^{C}\frac{b_c}{R_{cref}} R_{cn}(\ ). \tag{6}$$

The above measures express better the contract risk than eqs (2,3), particularly when the quota V_{kcn} to be paid at time n had been acquired in a longer time interval (not bought at time n), which is rather typical case. Hence, the most suitable reference exchange rate R_{cref} seems to be the mean value R_{cNL} in a presumed time interval containing N historical samples of R_{ci} and ending at L-th sample ($i=\tilde{L}N+1, \ldots, L$), with L taken arbitrarily (NL interval)

$$R_{cref} = R_{cNL} \stackrel{def}{=} \frac{1}{N}\sum_{i=1}^{N} R_{mL-i+1}. \tag{7}$$

The currency basket coefficients b_c may be adjusted in such a way, to minimize the overall trade risk, expressed by the variance of $\Delta_I\Pi_{kp}$ or $\Delta_{II}\Pi_{kp}$ in NL interval, averaged over the set of the commodities to be sale/buy with the same basket. To this aim the linear quadratic optimization tools may be applied, minimizing one of the above performance measures:

$$J_{I\,NL} = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=0}^{N-1}\left(\sum_{c=1}^{C}\left(\frac{b_c}{R_{cNL}}(P_{kL-i+p}R_{cL-i+p} - P_{kL-i}R_{cL-i})\right)\right)^2 - \left(\sum_{c=1}^{C}\left(\frac{b_c}{R_{cNL}}\frac{1}{N}\sum_{i=0}^{N-1}(P_{kL-i+p}R_{cL-i+p} - P_{kL-i}R_{cL-i})\right)\right)^2\right) \tag{8}$$

$$J_{II\,NL} = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=0}^{N-1}\left(\sum_{c=1}^{C}\left(\frac{b_c}{R_{cNL}}R_{cL-i}(P_{kL-i+p} - P_{kL-i})\right)\right)^2 - \left(\sum_{c=1}^{C}\left(\frac{b_c}{R_{cNL}}\frac{1}{N}\sum_{i=0}^{N-1}R_{cL-i}(P_{kL-i+p} - P_{kL-i})\right)\right)^2\right) \tag{9}$$

If the delivery delay p is differentiated or varying, the adequate averaged measure of the trade risk is simply the standard deviation (variance) of Π_{ki} in the NL interval. Thus, the third alternative performance index may calculated as follows:

$$J_{\Pi\,NL} = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{i=0}^{N-1}\left(\sum_{c=1}^{C}\frac{b_c}{R_{cNL}}P_{kL-i}R_{cL-i}\right)^2 - \left(\sum_{c=1}^{C}\left(\frac{b_c}{R_{cNL}}\frac{1}{N}\sum_{i=0}^{N-1}P_{kL-i}R_{cL-i}\right)\right)^2\right) \tag{10}$$

The following constraints must be satisfied:

$$b_c \geq 0 \quad \text{for } c=1, \ldots, C, \quad \text{and} \quad \sum_{c=1}^{C} b_c = 1 \tag{11}$$

2.2 Markovitz portfolio and its application to the basket payments. Using instrumental currencies Π_k (calculated for the individual baskets b_{ck}), one may consider the construction of a portfolio of the given commodities to be bought/sold in such a way to minimize the overall transaction risk.

The optimal portfolio idea [9] is to construct a portfolio composed of a set of commodities k=1, ..., K, and find an optimal set of portfolio weights a_k for each of them. The portfolio coefficients should be adjusted in such a way to satisfy a compromise between two criteria: maximize the expected transaction return and minimize a risk measure of the transaction. The optimization is based on series of historical data from a presumed time interval containing N samples, recorded at the same time instants for all the commodities. These are expressed as the series of return rates w_{kn} for $n=n_0$, ..., $N+n_0-1$. In the Markowitz portfolio the risk is measured by the variance of the portfolio return, assuming that the statistics of the historical returns are representative of future transactions. Referring to the basket payments, the returns w_{kn} have to be calculated for the instrumental prices Π. Let W_{kn0} denote the averaged value of k-th commodity returns in the window starting with n_0 sample, $C_{km\,m0}$ – the covariance coefficient of the k-th and m-th returns in this window. For transactions concerning the prices p samples ahead, the above quantities are calculated in the following way:

$$w_{kn} \overset{def}{=} 1 - \frac{\Pi_{kn-p}}{\Pi_{kn}}, \qquad W_{kn0} \overset{def}{=} \frac{1}{N}\sum_{n=n0}^{N+n0-1} w_{kn}, \qquad C_{km\,n0} \overset{def}{=} \frac{1}{N}\sum_{n=n0}^{N+n0-1} w_{kn}w_{mn} - W_{kn0}W_{mn0}. \tag{12}$$

The Markowitz portfolio optimization task may be expressed in the following form:

find the portfolio coefficients a_k, k=1, ..., K, minimizing the performance index:

$$J_M = -(1-\lambda)\sum_{k=1}^{K} a_k W_{kn0} + \lambda\sum_{k=1}^{K}\sum_{m=1}^{K} a_k a_m C_{kmn0} \tag{13}$$

subject to the constrains:

$$\sum_{k=1}^{K} a_k = 1, \qquad a_k \geq 0 \quad \text{for} \quad k = 1, \ldots\ldots, K, \tag{14}$$

where $\lambda \in \langle 0, 1\rangle$ denotes the aversion to risk coefficient taken arbitrarily.

It should be noticed that the above formulation involves the price return ratios w_{kn}, while the baskets currency Π is optimized for the price increments $\Delta\Pi$ defined in eq.(5) or eq.(6). Thus the portfolio optimization is not consistent with the basket currency application. Nevertheless, the basket optimizing the increments $\Delta_I\Pi$ (see eq.5) may be used in our approach, as it produces consistent price time series, for which the return ratios w_{kn} may calculated (hence it is more appropriate than the basket related to $\Delta_{\Pi}\Pi$ – see eq.6).

2.3 Method and results of calculations. The method proposed in the paper is applied to the pricing of basket contracts on six petroleum products: TXPropane; USGulfROil; HeatOil; LCrude1; NYGasF and WTI Brent. All data were recorded in the time interval from 01.01.1998 to 26.11.2010 from various Internet sources [12] Data sources. Table 1 presents the data used in the study. One year forward transactions are considered and optimized with the discussed method: first by constructing the optimal currency basket related to the instrumental price increments defined in eq.(5), then by optimizing the commodity portfolio by following the steps described in eqs.(12-14). The proposed basket elements are the world's principal currencies (Euro, British Pound, SDRs, Yen, Rouble, Polish Zloty, Indian Rupees, Brazilian Real, the Australian dollar and the US dollar) as well as Gold and Silver. For each petroleum commodity an individual currency basket is constructed using these elements. The performance of the basket and Markowitz portfolio method is then contrasted to the Markowitz portfolio computed with market prices of the commodities by comparing the risk of transacting in the basket against transacting in the US dollar, first at the time interval of four years used in the optimization tasks, then in the one year validation interval.

The examined petroleum products and currencies making the baskets are described in Table 1.

Time series of the examined raw material prices and exchange rates are presented in Fig. 1, 2.

As shown, all studied series are non-stationary and highly varying in the last three years (during the crisis of 2008–2010). It made one year forward transactions very risky

Numerical treatment of the data with software tools used in our research faces two technical problems: the first one is incoherency of the data registration period, the second one comes from deficiency of data (for example weekends, holidays). Weekends are synchronous interruptions and that’s the reason why we can ignore them and regard as continuous period. Asynchronous deficiencies (holidays or global incidents such as terrorist attack on WTC or U.S. intervention in Iraq) cause mainly interruptions which lasts couple days or more and effects the work of stock exchange. Such deficiencies were removed through linear interpolation.

The calculations were performed with our own program working on the MATLAB software platform, employing MATLAB *fmincon*() function as the solver of the optimization tasks (8-11) and (12–14). The coefficient λ has been taken in such an interval to produce non-dominated compromise solutions (Pareto curves – see Figure 3), and finally its mean value in this interval has been accepted as the best compromise solution.

Table 1

List of raw commodities and exchange rates used in calculations

Commodity Prices	
TXPropan	Mont Belvieu, TX Propane Spot Price FOB (Cents/Gallon)
USGulfROil	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB (Cents per Gallon)
HeatOil	New York Harbor No. 2 Heating Oil Spot Price FOB (Cents per Gallon);
NYGasF	NY Harbor Conventional Gasoline Regular Spot Price FOB (Cents per Gallon)
WTI	Cushing, OK WTI Spot Price FOB (Dollars per Barrel)
Brent	Europe Brent Spot Price FOB (Dollars per Barrel)
Silver	London Bullion Market Association, held each working day at 12.00 PM in the City of London, Dollars per Troy Ounce
Gold	London Bullion Market Association, Gold prices Day 3:00 PM, USD per Troy Ounce
Exchange rates	
AUD/USD, BRL/USD, GBP/USD, EUR/USD, INR/USD, JPY/USD, PLN/USD, RUB/USD, SDR/USD	

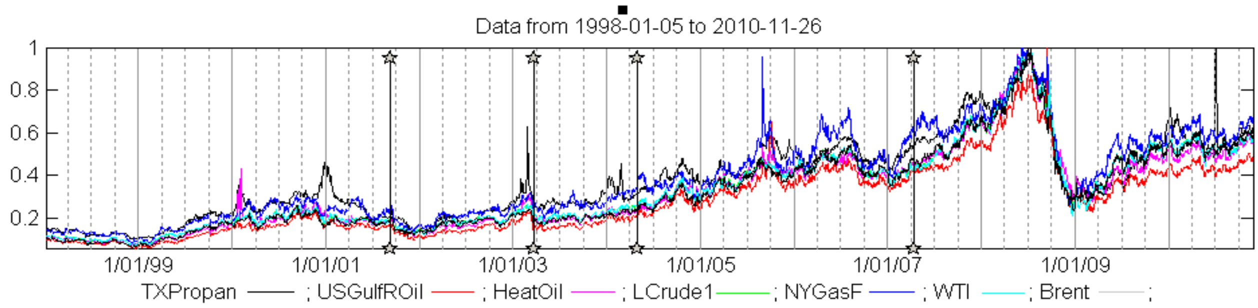

Fig. 1. The time series of the considered commodities (in USD). The values in each series are proportional to their maximal value. Vertical dotted lines – three months and 1-year (bold) intervals

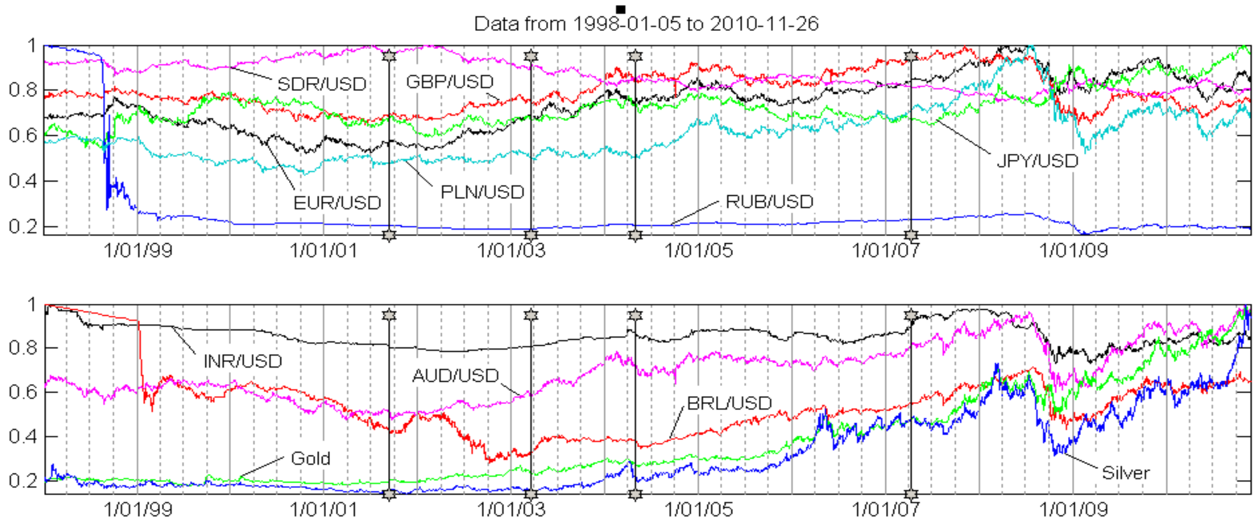

Fig. 2. Time series of exchange rates of currencies used in calculations. The values in each series are proportional to their maximal value. Vertical dotted lines – three months and 1-year (bold) intervals

The currency basket has been optimized in four-years intervals (1044 samples). The interval length corresponds to cyclic properties of World economy [5]. Significant contribution of four-year cycles to financial time series is often suggested in literature (see [1, 2]). In our earlier papers [4,5] we have shown that eight-year cycles in leading Stock Market indices are also present, but that four-year oscillations are of significance too. Thus the interval covering four years data seems to be a good compromise between filtering (averaging) and flexibility properties of the numerical analysis.

In the same time interval we have calculated two Markovitz portfolios: first based on the optimised basket currency, and second for the original prices. Then the both portfolios (with constant basket and portfolio coefficients) were applied in one year validation interval (in fact it covers two years, as the one year ahead transactions are examined during one year). The procedure has been repeated for consecutive years, since 2003 to 2010, in the intervals shifted ahead by one year. Typical Pareto curves found for the both portfolios are shown in Figure 3.

The expected (average) portfolio return ratio of the basket portfolio W_B and the standard deviation σ_{wB} of the returns, related to the same quantities, W_P and σ_{wP}, reached with the portfolio based on original prices were used as efficiency measures of the basket-Markowitz portfolio in the optimization intervals (W_{Bopt}/W_{Popt}, $\sigma_{wBopt}/\sigma_{wPopt}$) and in the validation intervals (W_{Bval}/W_{Pval}, $\sigma_{wBval}/\sigma_{wPval}$).

Two basket types were employed: currencies and currencies plus Gold and Silver.

The results of calculations are summarized in Tables 2 and 3. They show that the basket portfolio is usually much more effective than that based on the original prices ($\sigma_{wB}/\sigma_{wP}<1$), although in some intervals it is worse, both in the optimization and validation intervals ($\sigma_{wB}/\sigma_{wP}>1$). Notice that the basket portfolio was strongly advantageous during the last two years, mainly in the validation intervals. It is also noteworthy that the basket excluding Gold and Silver produces significantly better results.

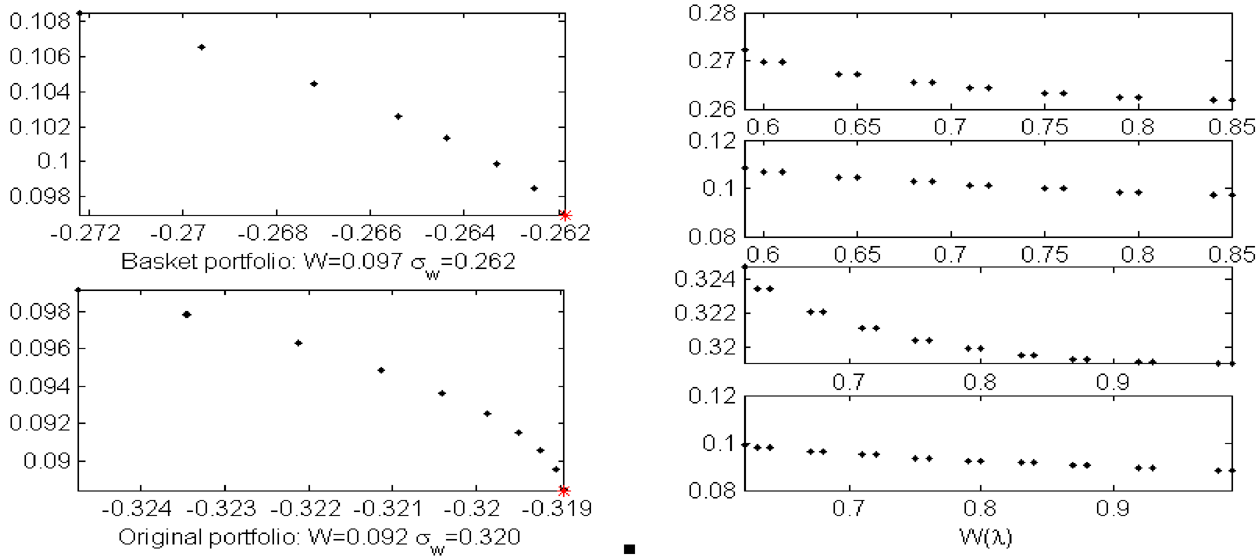

Fig. 3. Pareto curves $W(\sigma_w)$, and dependencies $W(\lambda)$ and $\sigma_w(\lambda)$ in the interval 1998-2001

Table 2

Efficiency assessment of the currency basket containing gold and silver, combined with Markovith portfolio optimisation, used for one year forward transaction on six petroleum products: TXPropan, USGulfROil, HeatOil, LCrude1, NYGasF, WTI and Brent (see Table 1 for explanations)

End of validation interval	30.12.03	30.12.04	30.12.05	30.12.06	30.12.07	30.12.08	30.12.09	26.11.10
W_{Bopt}/W_{Popt} %	109.02	107.03	-52.47	38.71	31.14	45.87	16.90	68.32
W_{Bval}/W_{Pval} %	76.33	95.89	-114.76	-54.55	204.86	74.84	57.81	93.32
$\sigma_{wBopt}/\sigma_{wPopt}$ %	82.53	89.53	87.51	106.46	137.33	118.17	88.16	54.36
$\sigma_{wBval}/\sigma_{wPval}$ %	122.88	97.01	141.65	136.31	56.42	61.02	83.10	44.83

Table 3

Efficiency assessment of the currency basket excluding gold and silver, combined with Markovith portfolio optimisation, used for one year forward transaction on six petroleum products

End of validation interval	30.12.03	30.12.04	30.12.05	30.12.06	30.12.07	30.12.08	30.12.09	26.11.10
W_{Bopt}/W_{Popt} %	104.83	116.07	-27.40	75.04	65.46	48.19	50.94	52.69
W_{Bval}/W_{Pval} %	77.00	105.78	84.59	-7.96	167.26	65.30	88.48	100.18
$\sigma_{wBopt}/\sigma_{wPopt}$ %	81.87	86.34	94.36	91.62	122.90	116.88	74.83	54.56
$\sigma_{wBval}/\sigma_{wPval}$ %	122.19	100.08	133.07	94.82	62.39	56.87	52.04	42.05

Conclusions. The article proposes a new method for expressing the value of petroleum commodities as a function of a basket of currencies and precious metals. It then compares the performance of these basket portfolios to an optimal Markowitz portfolio. Results show that variance of basket portfolios computed with the proposed method is proportionally lower (by about 46 %) to that of unstructured baskets (lines 3 and 4 in Tables 2 and 3). Portfolio performance in the out-of-sample validation period is superior for the optimized basket portfolios which proves the usefulness of the method. Although portfolio and basket optimizations are not consistent, the proposed method yields significantly positive results. Markowitz portfolio theory is used to verify that the performance of the commodity baskets is significantly superior to alternatives. An added advantage of the method featured in the article is its relative ease of computation.

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