

ANALYTIC SOLUTION FOR THE PROBLEM OF IN-SITU CALIBRATING MAGNETOMETRIC 3D-PROBES BASED ON NON-ORTHOGONALLY PLACED HALL DEVICES

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Запропоновано методику in-situ калібрування 3D-зонтів магнітного поля з лінійними вихідними характеристиками давачів за відомих параметрів кутового розсунення їхніх площин з гранями підкладки зонта. Одержані аналітичні вирази для розрахунку лінійних коефіцієнтів. Експериментально підтверджено ефективність запропонованої методики.

Ключові слова: 3D-зонд магнітного поля, методика калібрування, in-situ калібрування, давачі Холла.

This work presents a method of in-situ calibrating 3D magnetic sensors based on Hall devices that have linear field characteristics and are non-orthogonal, with known position of each Hall device in relation to the 3D sensor's substrate. Formulas for calculation of Hall devices linear coefficients are obtained. The method's efficiency is proved experimentally.

Key words: 3D magnetic sensors, calibration method, in-situ calibration, Hall devices

Introduction

Hall devices are widely used as magnetic sensors. They have a number of advantages including a wide measurement range ($10^{-5} \dots 10^2$ T), low power consumption (<10 mW) and small size (1..3 mm). The output signal of an ordinary Hall device is linearly dependent on the projection of the magnetic-field vector \vec{B} onto the normal N to its sensitive semiconductor layer. So, in order to obtain a measurement device for measuring three orthogonal projections B_x , B_y , B_z of the magnetic-field vector simultaneously, one integrates three Hall devices into a 3D-sensor placing them on the adjacent sides of the sensor's cubic substrate [1, 2]. At best three Hall devices in a 3D sensor are completely orthogonal, so in that case 3D-sensor's calibration comes to finding of the linear coefficient of each Hall device included in this sensor. However, the condition of the Hall devices complete perpendicularity is not practically met because of processing limits of 3D sensor's manufacturing. In case if the condition of strict perpendicularity is not fulfilled, calibration gets more complicated: in this case the parameters of Hall devices angular displacements are to found in addition to the above-mentioned linear coefficients.

A number of important magnetometry problems set requirements that sensors have to be stable when working for a long time in a harsh environment. Such problems include magnetic field measurement in magnetic deflection systems of charged particle accelerators, magnetic confinement systems of thermonuclear reactors, systems of magnetic diagnostics of hardware and pipelines etc. The main difficulty of producing magnetic field measurement devices in such a harsh environment is thermal and radiation instability of a Hall device sensitive layer. The described problem can be solved in two ways: using very stable semiconductors and recalibrating sensors.

Problems of improving stability of semiconductors used in Hall devices intended for operation in a harsh environment including radiation conditions of charged particle accelerators and thermonuclear reactors are covered in a number of publications [3, 4, 5]. A number of publications [6] cover in-situ calibration of semiconductor Hall devices by means of a miniature coil integrated into a magnetometric sensor. The coil periodically establishes a test magnetic field. The drift of Hall coefficient can be

determined by measuring the change of the Hall device output signal caused by test magnetic field established by the coil.

However in some cases test coils are not applicable. First reason for it is the fact that test coils can be impacted by low or high temperatures, so the field that they produce should be considered unstable. The second reason is that the magnitude of test magnetic flux density is rather small (does not exceed several mT), so at high outside magnetic or electromagnetic fields the Hall coefficients cannot be defined with sufficient accuracy. The third reason is the fact that test coils increase the magnetic sensor size significantly. This hamper Hall devices to be placed tightly thus restricting the spatial resolution of a 3D sensor.

That's why ordinary periodical calibration of 3D sensors in a magnetic measure or an electromagnet is still urgent in a number of magnetometry problems. The first calibration may be done by the manufacturer using high-precision mechanical devices for exact positioning the calibrated sensors in the uniform magnetic field. However, in-situ calibration (calibration in work environment) sets the condition of involving few if any additional devices, so use of expensive high-precision positioning devices is undesirable for in-situ calibration.

Problem statement and the work purpose

In this work we introduce a new method of periodical refinement of linear coefficients in the output signals of Hall devices included in the 3D sensor being calibrated in the case if the strict perpendicularity condition is not met and the parameters of angular displacements are known. The problem of finding the angular displacements is solved in [7] using numerical methods with high computational complexity. The same solution can be used for refinement of the linear coefficients.

However, the growing significance of intelligent sensors cause the problem of finding such a 3d-sensor's in-situ calibration method that would possess the minimal computational complexity and thus can be implemented by microconverters included in intelligent sensors based on magnetometric 3D-sensors.

We've obtained analytic formulas for the linear coefficients of Hall devices combined into a magnetic 3D-sensor. The derived formulas use the parameters of the mentioned angular displacements that are defined once during the first calibration process and sets of Hall devices output signals measured when rotating a 3D sensor in a uniform magnetic field in arbitrary manner. The proposed method requires no high-precision mechanisms that are usually used for positioning 3D-probes in a magnetic field.

Method description

The set problem has concern to in-situ calibrating magnetometric 3D-sensors, but the mathematical tool for its solution can be used for solving the problems of measuring vector quantities in general. That's why all analytic dependencies given below are formulated abstractedly with undisclosed physical meaning of the formulas terms.

Let the vector quantity $\vec{M} = (x \ y \ z)$ being measured have the invariant module:

$$M = \sqrt{x^2 + y^2 + z^2} \quad (1)$$

Let's suppose that the vector quantity is to be measured by a 3D-probe with three sensors on adjacent sides of its cubic substrate and each sensor's output signal linearly depends on the vector quantity's projection normal to its plane:

$$\begin{aligned} S_1 &= R_1 M_{N1} \\ S_2 &= R_2 M_{N2} \\ S_3 &= R_3 M_{N3} \end{aligned} \quad (2)$$

there S_1, S_2, S_3 are output signals, R_1, R_2, R_3 are the linear coefficients of three sensors in a 3D-probe, M_{N1}, M_{N2}, M_{N3} are the projections of the vector \vec{M} which are normal to the planes of three sensors.

If the planes of all sensors combined into a 3D-probe are completely perpendicular then one can calibrate such a 3D-probe (and thus find the linear coefficients R_1, R_2, R_3) rotating it arbitrarily in a uniform test vector field and reading its output signals. This calibration method is given in [8].

Let the condition of strict perpendicularity be broken. Having the direction cosines of the perpendiculars to the planes of sensors combined into a 3D-probe marked as $(t_{11}, t_{12}, t_{13}), (t_{21}, t_{22}, t_{23}), (t_{31}, t_{32}, t_{33})$ in some Cartesian coordinates xyz assigned to this 3D-probe, we obtain the equations:

$$\begin{aligned} R_1 t_{11} x + R_1 t_{12} y + R_1 t_{13} z &= S_1 \\ R_2 t_{21} x + R_2 t_{22} y + R_2 t_{23} z &= S_2 \\ R_3 t_{31} x + R_3 t_{32} y + R_3 t_{33} z &= S_3 \end{aligned} \quad (3)$$

The work [7] introduces the method of calibrating magnetic 3D-sensor with output signals that are described by equations (3). The parameters of sensor's angular displacements (i.e. the direction cosines t_{ij} , $i = \overline{1,3}, j = \overline{1,3}$) remain invariant during sensor's operation but the linear coefficients R_1, R_2, R_3 can change.

Let's find the linear coefficients R_1, R_2, R_3 by arbitrary rotation of a 3D-sensor in a space where the vector quantity has the invariant module M (values t_{ij} ($i = \overline{1,3}, j = \overline{1,3}$) are assumed to be known).

If the set of equations (3) possesses a solution then its matrix A has an inverse matrix C :

$$A = \begin{pmatrix} R_1 t_{11} & R_1 t_{12} & R_1 t_{13} \\ R_2 t_{21} & R_2 t_{22} & R_2 t_{23} \\ R_3 t_{31} & R_3 t_{32} & R_3 t_{33} \end{pmatrix}; \quad C = A^{-1} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \quad (4)$$

where c_{ij} are some unknown coefficients.

Using the inverse matrix (4) we can write unknown components of the vector \overline{M} in the way:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \cdot \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad (5)$$

So, the components of the vector being measured can be written in the following way:

$$x = c_{11} \cdot S_1 + c_{12} \cdot S_2 + c_{13} \cdot S_3, \quad y = c_{21} \cdot S_1 + c_{22} \cdot S_2 + c_{23} \cdot S_3, \quad z = c_{31} \cdot S_1 + c_{32} \cdot S_2 + c_{33} \cdot S_3.$$

Since the measured vector's module (1) is invariant in a uniform field, for each 3D-sensor's position in this field we can write:

$$(c_{11} S_1 + c_{12} S_2 + c_{13} S_3)^2 + (c_{21} S_1 + c_{22} S_2 + c_{23} S_3)^2 + (c_{31} S_1 + c_{32} S_2 + c_{33} S_3)^2 = M^2 \quad (6)$$

Having opened the brackets and combined similar terms we obtain:

$$\begin{aligned} &(c_{11}^2 + c_{21}^2 + c_{31}^2) S_1^2 + (c_{12}^2 + c_{22}^2 + c_{32}^2) S_2^2 + (c_{13}^2 + c_{23}^2 + c_{33}^2) S_3^2 + \\ &2(c_{11} c_{12} + c_{21} c_{22} + c_{31} c_{32}) S_1 S_2 + 2(c_{11} c_{13} + c_{21} c_{23} + c_{31} c_{33}) S_1 S_3 + \\ &+ 2(c_{12} c_{13} + c_{22} c_{23} + c_{32} c_{33}) S_2 S_3 = M^2 \end{aligned} \quad (7)$$

Having measured the output signals S_1, S_2, S_3 for different sensor's position in a uniform field (the measured vector's components x, y, z are unknown) we compose a set of equations of form (7). This set is a set of linear algebraic equations in 6 unknowns: $(c_{11}^2 + c_{21}^2 + c_{31}^2), (c_{12}^2 + c_{22}^2 + c_{32}^2), (c_{13}^2 + c_{23}^2 + c_{33}^2), (c_{11} c_{12} + c_{21} c_{22} + c_{31} c_{32}), (c_{11} c_{13} + c_{21} c_{23} + c_{31} c_{33}), (c_{12} c_{13} + c_{22} c_{23} + c_{32} c_{33})$. The least number of the output signals set is 6, as follows from the amount of unknowns. The solutions of the set of at least 6 equations of form (7) are some numbers $c_1, c_2, c_3, v_1, v_2, v_3$ which are connected with the matrix C 's elements by the relationship:

$$\begin{aligned} c_{11}^2 + c_{21}^2 + c_{31}^2 &= c_1^2; \\ c_{12}^2 + c_{22}^2 + c_{32}^2 &= c_2^2; \\ c_{13}^2 + c_{23}^2 + c_{33}^2 &= c_3^2; \\ 2c_{11} c_{21} + 2c_{12} c_{22} + 2c_{13} c_{23} &= v_1 \\ 2c_{11} c_{31} + 2c_{12} c_{32} + 2c_{13} c_{33} &= v_2 \\ 2c_{21} c_{31} + 2c_{22} c_{32} + 2c_{23} c_{33} &= v_3 \end{aligned} \quad (8)$$

The matrix $A^{-1} = C$ can be written using the matrix A elements:

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} R_2 R_3 (t_{22} t_{33} - t_{23} t_{32}) & R_1 R_3 (t_{13} t_{32} - t_{12} t_{33}) & R_1 R_2 (t_{12} t_{23} - t_{13} t_{22}) \\ R_2 R_3 (t_{23} t_{31} - t_{21} t_{33}) & R_1 R_3 (t_{11} t_{33} - t_{13} t_{31}) & R_1 R_2 (t_{13} t_{21} - t_{11} t_{23}) \\ R_2 R_3 (t_{21} t_{32} - t_{22} t_{31}) & R_1 R_3 (t_{12} t_{31} - t_{11} t_{32}) & R_1 R_2 (t_{11} t_{22} - t_{12} t_{21}) \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \quad (9)$$

Substituting c_{ij} expressed through relevant elements of the matrix A^{-1} we obtain:

$$\begin{aligned} & 2R_1 R_2 R_3^2 (t_{22} t_{33} - t_{23} t_{32})(t_{13} t_{32} - t_{12} t_{33}) + 2R_1 R_2 R_3^2 (t_{23} t_{31} - t_{21} t_{33})(t_{11} t_{33} - t_{13} t_{31}) + \\ & + 2R_1 R_2 R_3^2 (t_{21} t_{32} - t_{22} t_{31})(t_{12} t_{31} - t_{11} t_{32}) = v_1 (\det A)^2 \\ & 2R_1 R_2^2 R_3 (t_{22} t_{33} - t_{23} t_{32})(t_{12} t_{23} - t_{13} t_{22}) + 2R_1 R_2^2 R_3 (t_{23} t_{31} - t_{21} t_{33})(t_{13} t_{21} - t_{11} t_{23}) + \\ & + 2R_1 R_2^2 R_3 (t_{21} t_{32} - t_{22} t_{31})(t_{11} t_{22} - t_{12} t_{21}) = v_2 (\det A)^2 \\ & 2R_1^2 R_2 R_3 (t_{13} t_{32} - t_{12} t_{33})(t_{12} t_{23} - t_{13} t_{22}) + 2R_1^2 R_2 R_3 (t_{11} t_{33} - t_{13} t_{31})(t_{13} t_{21} - t_{11} t_{23}) + \\ & + 2R_1^2 R_2 R_3 (t_{12} t_{31} - t_{11} t_{32})(t_{11} t_{22} - t_{12} t_{21}) = v_3 (\det A)^2 \end{aligned} \quad (10)$$

The determinant of A is as follows:

$$\begin{aligned} \det A &= R_1 R_2 R_3 (t_{11} t_{22} t_{33} - t_{11} t_{23} t_{32}) + R_1 R_2 R_3 (t_{12} t_{23} t_{31} - t_{12} t_{21} t_{33}) + R_1 R_2 R_3 (t_{13} t_{21} t_{32} - t_{13} t_{22} t_{31}) = \\ &= R_1 R_2 R_3 (t_{11} t_{22} t_{33} - t_{11} t_{23} t_{32} + t_{12} t_{23} t_{31} - t_{12} t_{21} t_{33} + t_{13} t_{21} t_{32} - t_{13} t_{22} t_{31}) \end{aligned} \quad (11)$$

Taking into consideration equations (10) and (11) we obtain:

$$\begin{aligned} R_1 R_2 &= \frac{2((t_{22} t_{33} - t_{23} t_{32})(t_{13} t_{32} - t_{12} t_{33}) + (t_{23} t_{31} - t_{21} t_{33})(t_{11} t_{33} - t_{13} t_{31}) + (t_{21} t_{32} - t_{22} t_{31})(t_{12} t_{31} - t_{11} t_{32}))}{v_1 (t_{11} t_{22} t_{33} - t_{11} t_{23} t_{32} + t_{12} t_{23} t_{31} - t_{12} t_{21} t_{33} + t_{13} t_{21} t_{32} - t_{13} t_{22} t_{31})^2} \\ R_1 R_3 &= \frac{2((t_{22} t_{33} - t_{23} t_{32})(t_{12} t_{23} - t_{13} t_{22}) + (t_{23} t_{31} - t_{21} t_{33})(t_{13} t_{21} - t_{11} t_{23}) + (t_{21} t_{32} - t_{22} t_{31})(t_{11} t_{22} - t_{12} t_{21}))}{v_2 (t_{11} t_{22} t_{33} - t_{11} t_{23} t_{32} + t_{12} t_{23} t_{31} - t_{12} t_{21} t_{33} + t_{13} t_{21} t_{32} - t_{13} t_{22} t_{31})^2} \\ R_2 R_3 &= \frac{2((t_{13} t_{32} - t_{12} t_{33})(t_{12} t_{23} - t_{13} t_{22}) + (t_{11} t_{33} - t_{13} t_{31})(t_{13} t_{21} - t_{11} t_{23}) + (t_{12} t_{31} - t_{11} t_{32})(t_{11} t_{22} - t_{12} t_{21}))}{v_3 (t_{11} t_{22} t_{33} - t_{11} t_{23} t_{32} + t_{12} t_{23} t_{31} - t_{12} t_{21} t_{33} + t_{13} t_{21} t_{32} - t_{13} t_{22} t_{31})^2} \end{aligned} \quad (12)$$

Let's mark the right sides of all equations (12) as P_1 , P_2 , P_3 respectively in order to make them more concise. Using the third equation of the equations set (12) R_3 can be written in this way: $R_3 = \frac{P_3}{R_2}$.

Substituting this expression in the second equation of the set (12) we can express R_2 in the following way:

$R_2 = \frac{P_3 R_1}{P_2}$. After substitution of the obtained value in the first equation in the set (12) we derive the

following expression: $R_1^2 = \frac{P_1 P_2}{P_3}$, i.e. $R_1 = \pm \sqrt{\frac{P_1 P_2}{P_3}}$.

Hence we can know the module of R_1 . Its sign is to be defined by the sign of the sensor's output signal (the vector \overline{M} direction must be known). The values of R_2 and R_3 can be derived from the second and the third equations of the set (12).

Method approbation

The proposed in-situ calibration method was experimentally approved using a 3D probe based on Hall devices with output signals that can be approximately described by functions (3). In the case of Hall devices the sought values R_1 , R_2 , R_3 have the following physical meaning:

$$R_i = R_{Hi} \frac{I_i}{d_i} \quad (13)$$

where R_{Hi} is the Hall coefficient of the Hall sensor numbered i , I_i is the current through its current electrodes, d_i is the thickness of its sensitive layer.

The direct experiments with 3D magnetic sensors calibration in a harsh environment can be hardly conducted because they can be too time-consuming and require a year or more. So in order to perform an approbation of the proposed calibration method we used the approach based on the imitation of the Hall coefficients instability by means of arbitrary changing the feed current of the Hall devices. As follows from (13), the Hall coefficient R_{Hi} of a sensor and its feed current I_i influence the sought coefficient R_i in the same way. So if we change the current I_i within certain limits and consider this change random and unknown, thus we conduct experiments with imitation of instability of the linear coefficient R_i . During the conducted experiments we changed the feed current of each Hall device in the 3D magnetic sensor independently by $\pm 5\%$ of its nominal value (20 mA).

At the first calibration stage the 3D magnetic sensor was placed into a magnetic shield in order to measure the offset voltages of all Hall devices included into it. Then we measured the 3D probe's output signals S_1, S_2, S_3 for its 100 arbitrary positions in a test uniform magnetic field with magnetic flux density $B_{REF} = 127$ mT and subtracted the corresponding offset voltages measured previously from the obtained values. The Fig. 1 shows results of measuring the magnetic-field vector B_{ER} at the following changes of the feed currents: $dI_1 = -2.9\%$, $dI_2 = 4.2\%$, $dI_3 = -0.8\%$ without correction of the linear coefficients (B_{ER}) and with applied correction in accordance to the proposed method (B_{COR}).

It can be seen that with no correction the deviation of the calculated values of the magnetic-field vector B was in range from 123 to 133 mT, which means the error $\pm 4.6\%$ while refinement of the linear coefficients in accordance with the proposed method provided deviation of the calculated magnetic-field vector's module that does not exceed 0.3%. This error up to 0.3% is caused by the fact that Hall devices output signals are described by equations (3) approximately.

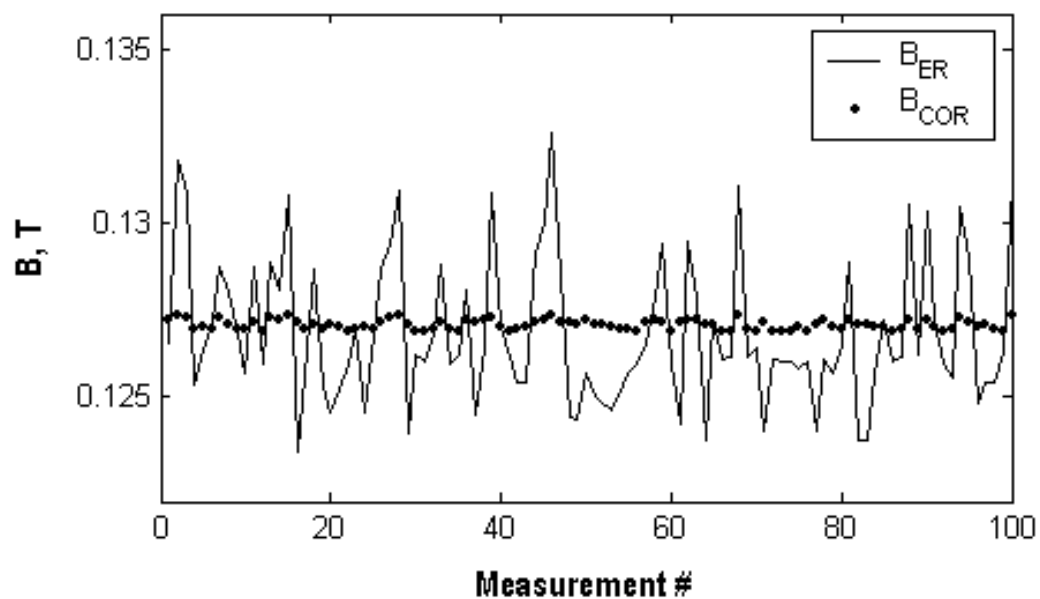


Fig. 2 The results of calculations of the magnetic-field vector module with no correction of linear coefficients (B_{ER}) and after such a correction (B_{COR}).

Conclusion

The work introduces a new method of in-situ calibrating a 3D-probe that is intended for vector quantities measurement and composed from sensors non-orthogonally placed on adjacent sides of its cubic substrate. The parameters of sensors angular displacements are assumed to be known. Each sensor's output signal is linearly dependent on the measured vector's projection normal to its plane. Analytic formulas for the linear coefficients of the 3D-probe's output signals were obtained. The proposed method is based on measurement of the 3D-sensor's output signals by means of its arbitrary rotation in a uniform field (3D-probe's current position is unknown). The method requires no tools for positioning calibrated 3D-sensors in

a test field. In the case if need for development of an intelligent sensor based on 3D-sensors for vector quantities measurement arises the proposed method of refining linear coefficients can be implemented in software of intelligent sensors.

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РОЗПІЗНАВАННЯ ПРОВІДНИХ ТА ВИСОМНИХ ВКЛЮЧЕНЬ У КУСКОВО-ОДНОРІДНОМУ ПІВПРОСТОРИ ПРИ МАТЕМАТИЧНОМУ МОДЕЛЮВАННІ УСТАЛЕНИХ КОЛИВАНЬ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ

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Для кусково-однорідного провідного півпростору розглянуто аналітично-числову методику знаходження компонент векторів напруженості електромагнітного поля, збудженого стороннім джерелом струму. Проведено числові експерименти для повної, з урахуванням струмів зміщення, та квазістаціонарної моделей, досліджено вплив включення з вищою та нижчою, ніж у геосередовищі, провідністю на розподіл поля на денній поверхні.

Ключові слова: система рівнянь Максвелла; система рівнянь Гельмгольца; усталені коливання електромагнітного поля; непрямий метод граничних елементів.

The numerical-analytic technique for finding electric and magnetic components of electromagnetic field in a piecewise homogeneous conductive half-space is suggested. Electromagnetic field is excited by a horizontal contour with current harmonically changing in time. The problem is formulated and solved by means of the boundary element method.

Key words: Maxwell's equation system, the Helmholtz equation system, established EMF fluctuations; indirect boundary element method.

Вступ

Сьогодні гармонічно змінні в часі електромагнітні поля (ЕМП) в однорідних та горизонтально-шаруватих моделях земної кори за дії природних та штучних джерел досліджені за допомогою добре розвинутої теорії спектрального аналізу. Це особливо стосується широкого класу