УДК 681.14

V. Petsko[1], A. Mitsa[1], F. Geche[1], V. Kotsovsky[1], A. Batyuk[2]
[1] Uzhgorod National University, Uzhgorod, Ukraine
[2] Lviv Polytechnic National University, Lviv, Ukraine

OPTIMAL PARAMETERS OF FOUR-LAYER OPTICAL STRUCTURES WHEN THE LIGHT IS INCIDENT AT AN ANGLE

Наведено результати теоретичних розрахунків можливостей просвітлення чотиришаровою оптичною структурою низькозаломлювальної підкладки під час падіння світла під кутом. Алгоритм розрахунків заснований на використанні методу негладкої оптимізації (r-алгоритму).

Ключові слова: оптичне багатошарове покриття, *r*-алгоритм, матричний метод Абеле, закон Брюстера.

The results of theoretical calculations of the optimal four-layer structure for optical bleaching of the substrate when the light is incident at an angle are given in the paper. Computational algorithm is based on the nonsmooth optimization methods (*r*-algorithm).

Key words: optical multi-layer coating, r-algorithm, Abeles matrix method, law of Brewster.

Interference bleaching multilayer coatings are commonly used in optical systems, space systems, optical instrument, integrated optics, X-ray and neutron spectroscopy, electrodynamics of open systems, generators constructing, and other optical instruments. The aim of this study was to find the optimal parameters of four-layer homogeneous structures in enlightenment of dioptric substrate, dynamics of change with an increase in the spectral range when the light is incident at an angle. In this paper as the substrate was chosen the most popular low dioptric substrate with a refractive index $n_S = 1.51$ (glass).

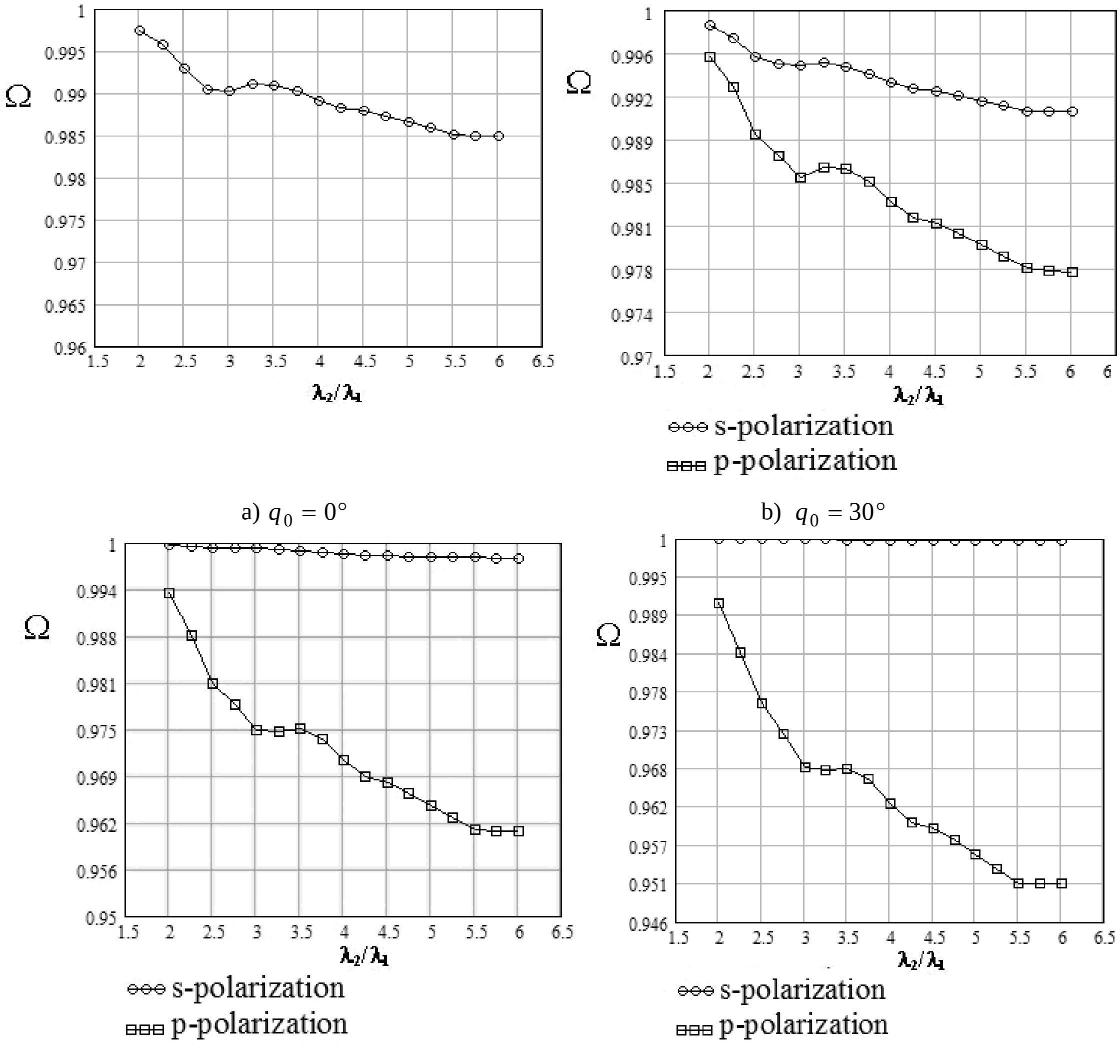
We use Abeles matrix method [1-2] to calculate the spectral characteristics of optical multilayer coatings using. The objective function is the mean square error of the transmittance on spectral band under consideration [3-4]:

$$\Omega(l_1, l_2) = \max_{\bar{n},\bar{d}} F(\bar{n},\bar{d}) = \max_{\bar{n},\bar{d}} \left(\frac{1}{L} \sum_{i=1}^{L} T^2 \left(\bar{n}, \bar{d}, l_{(i)} \right) \right)^{1/2}, \tag{1}$$

where $\bar{n} = (n_1, n_2, ..., n_{k-1}, n_k)$ − is a vector of the values of refraction indices of layers; $\bar{d} = (d_1, d_2, ..., d_{k-1}, d_k)$ − is a vector of geometrical thicknesses of layers; L − is the number of points of the spectral band from l_1 to l_2 in case of its uniform division with an step Δl : $\Delta l = (l_2 - l_1)/L$. Bounding values of the parameters were following $1.35 \le n_j \le 2.6$, $50\ nm \le d_j \le 750\ nm$ $\left(j = \overline{1, 4}\right)$, $l_1 = 200\ nm$, $300\ nm \le l_2 \le 1200\ nm$.

The objective function has many local maxima. Therefore, we chose for our optimization algorithm searching optimal four-layer structures 512 initial approximations. They were chosen as follows: the whole area of all possible values of parameters was divided into 512 subregions. Then starting points of the algorithm was selected in each area. To find the optimal parameters we used *r*-algorithm, described in [5-6]. The software is written in Delphi environment. The study used special techniques of [1], which provided that the observable parameters lie in mentioned limits.

We studied the optimal value of the functional (1) for four-layer structures in the case of different angles of incidence. First consider the case when the beam falls perpendicularly to the layered surface (see Fig. 1a). It is well known that for a four-layer surfaces, as well as for two-layer and three-layer the optimal refraction parameter values of the upper layer for all spectral bands is constant and equal to the lower limit for the refractive index — 1.35 [1]. By increasing the right boundary λ_2 from 400 to 600 nm values of the functional decreases from 0.997464 about 0.8%, with an increase of λ_2 from 600 to 650 nm — increased by 0.1%, while further enlargement of right boundary from 650 to 1200 nm causes fall by about 0.6%. Thus, the functional $\Omega(l_1, l_2)$ is not strictly decreasing with increasing right border, but has a tendency to decrease and this decrease is not significant.



c) $q_0 = 45°$ d) $q_0 = 50°$

Fig. 1. The dynamics of the change of functional $\Omega(l_1, l_2)$ for four-layer homogeneous structure in cases $q_0 = 0°$, $q_0 = 30°$, $q_0 = 45°$, $q_0 = 50°$

When the angle is not equal to $q_0 = 0°$ light beam can be decomposed into TE-wave (*s*-polarization) and TM-wave (*p*-polarization). These two cases are considered separately.

Then we studied the case when the angle of incidence is equal to $q_0 = 30°$. The Fig. 1b shows that the functional $\Omega(l_1, l_2)$ in the *s*-polarization case decreases from 0.995880 by about 1%, while the right boundary increases from 400 to 600 nm, and in the case of *p*-polarization decreases from 0.998646 to about 0.3 %. By increasing the right boundary l_2 from 600 to 650 nm the functional value increases by 0.1 % for *s*-polarization and by 0.03% for *p*-polarization. The further decreasing of the right border up to 1200 nm causes the growth of the functional (1) by 0.9% for *s*-polarization and 0.4% for *p*-polarization. As in case $q_0 = 0°$ the functional Ω in both cases is not strictly decreasing, but has a tendency to decrease. We note that in case of *s*-polarization the decreasing is more rapid than in the case of *p*-polarization.

When incidence angle is equal to $q_0 = 45°$ (Fig. 1c) or $q_0 = 50°$ (Fig. 1d) the functional $\Omega(l_1, l_2)$ in *p*-polarization case slowly decreases from 0.999 to 0.996.

In the case that incidence angles equal to $q_0 = 55°$ (Fig. 2a) or $q_0 = 60°$ (Fig. 2b) the value of functional $\Omega(l_1, l_2)$ in *p*-polarization case decreases to 0.995 and 0.918 respectively.

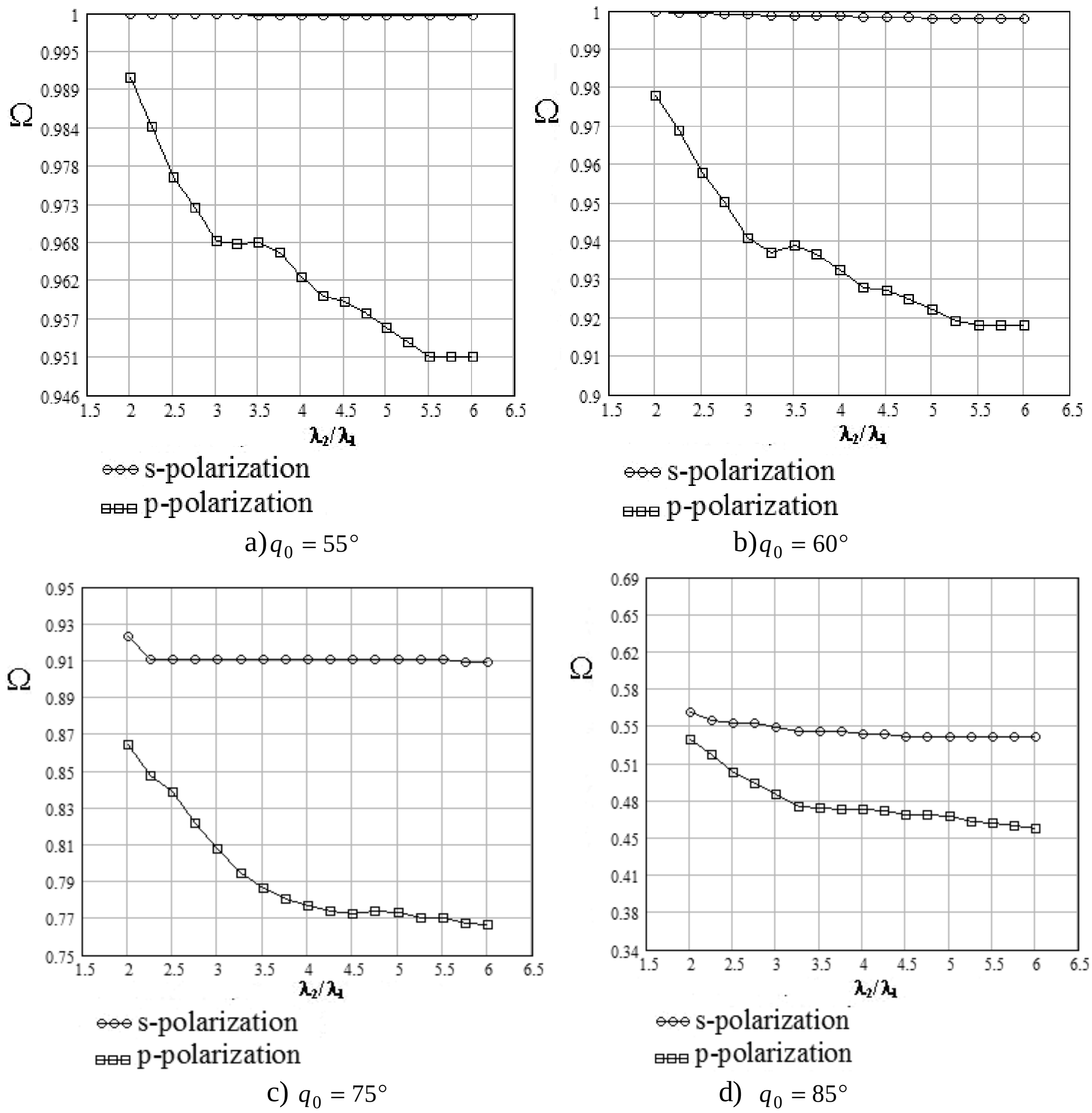
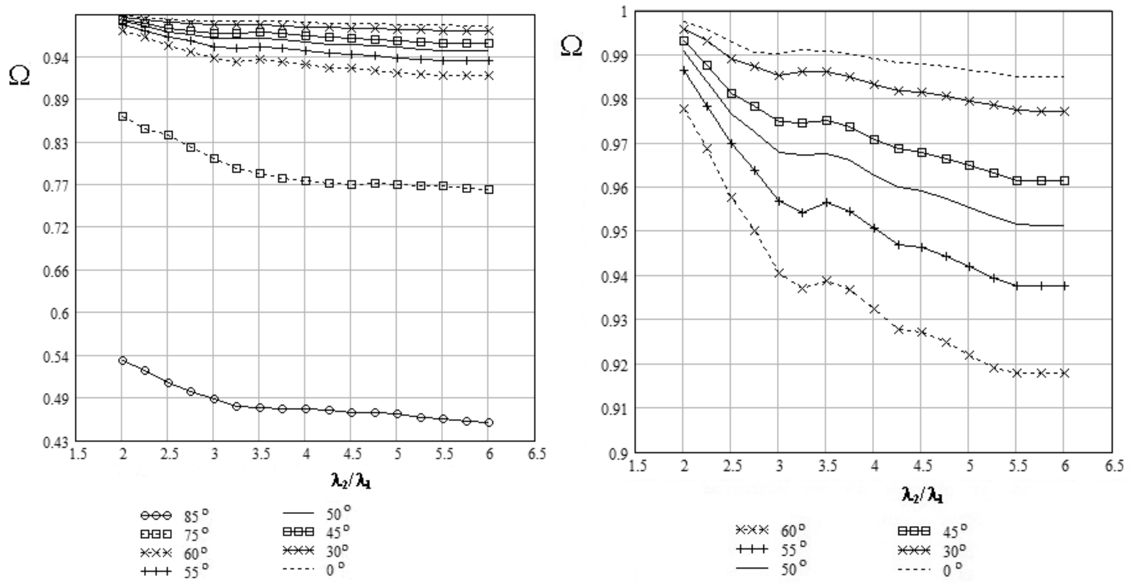


Fig. 2. .The dynamics of the change of functional $\Omega(l_1, l_2)$ for four-layer homogeneous structure in cases $q_0 = 55°$, $q_0 = 60°$, $q_0 = 75°$, $q_0 = 85°$

In *s*-polarization case the decreases of the functional $\Omega(l_1, l_2)$ is significantly greater. Thus, when the incidence angle equals to $q_0 = 45°$ the functional $\Omega(l_1, l_2)$ decreases approximately by 3.5 % starting from 0.993200 (Fig. 1c); in the case $q_0 = 50°$ — from the value 0.990823 decreasing by about 4 % (Fig. 1d); in the case $q_0 = 55°$ — from 0.986419 decreasing by about 4.5% (Fig. 2a); in the case $q_0 = 60°$ — from the value 0.977784 decreasing by about 6 % (Fig. 2b).

If the angle of incidence equals to $q_0 = 75^\circ$ (Fig. 2c) the functional $\Omega(l_1, l_2)$ in *s*-polarization case decreases from 0.864027 to 0.772539 with increasing right border l_2 from 400 to 900 nm. If the right border l_2 increases from 900 to 950 nm the functional (1) increases up to 0.773879. Further increase of the right border l_2 from 950 to 1200 nm causes that the value of our objective functional slowly decreases and comes to 0.766204, i.e. in *s*-polarization case $\Omega(l_1, l_2)$ has the tendency to decrease. In the case of *p*-polarization the functional (1) decreases from 0.923152 to 0.910704, while l_2 grows from 400 to 450 nm. Further increase of the right border l_2 to 1200 nm causes the slowly decrease of objective functional to 0.910499, that is the functional $\Omega(l_1, l_2)$ in *p*-polarization case is decreasing.

Fig. 2d provides an illustration of behavior of objective functional in the case of *s*-polarization for the angle of incidence $q_0 = 85^\circ$. If the right border l_2 increases from 400 to 650 nm, the value of $\Omega(l_1, l_2)$ rapidly decreases from 0.537593 to 0.475012. Further increase of the right border λ_2 from 650 to 1200 nm causes a slowly decrease of (1) to 0.454174. In the case of *p*-polarization one can observe the decrease of the functional $\Omega(l_1, l_2)$, from 0.563425 to 0.539841, while the right border λ_2 changes from 400 to 1200 nm. For both type of polarization the objective functional is decreasing, and for *s*-polarization its decrease is less than one in the case of *p*-polarization.



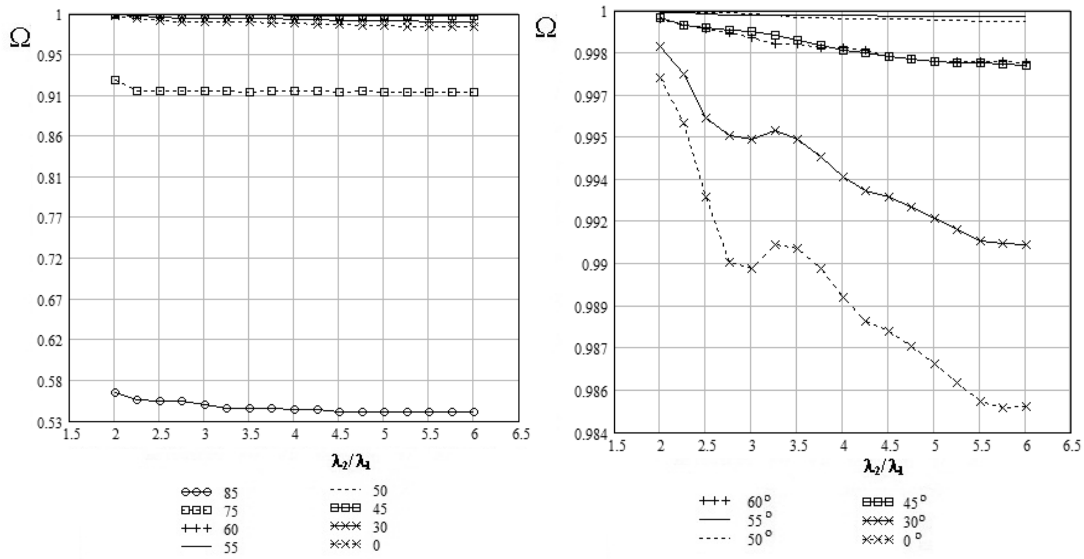
a) angle of incidence from 0° to 85°

b) angle of incidence from 0° to 60°

Fig. 3. Dynamics of the functional $\Omega(l_1, l_2)$ changes for a four-layer homogeneous structure in s-polarization case

From Fig. 3 it is possible to detect the dynamics of the changes of the functional (1) in the case of a four-homogeneous structure with *s*-polarization depending on the angle: for all angles within the right border λ_2 range from 400 nm to 1200 nm the objective functional tends to decrease. If $q_0 = 0^\circ$ or $q_0 = 30^\circ$, then for values of λ_2 between 600 nm and 650 nm a small growth of $\Omega(l_1, l_2)$ can be observed. In the cases $q_0 = 45^\circ$, $q_0 = 50^\circ$, $q_0 = 55^\circ$, $q_0 = 60^\circ$ this growth occurs in the range from 650 nm to 700 nm. For the angle $q_0 = 75^\circ$ a growth observed in the range from 900 nm to 950 nm, and for the angle $q_0 = 85^\circ$ the objective functional is everywhere decreasing. The figure shows that in the case of *s*-polarization the value of functional $\Omega(l_1, l_2)$ decreases.

In the case of *p*-polarization (Fig. 4) with increasing angle to $q_0 = 55^\circ$ the value of the functional (1) firstly grows, and then decreases. This effect is explained by the law of Brewster [7-9]. For *p*-polarization, as well as for *s*-polarization the objective functional decreases when λ_2 changes from 400 nm to 1200 nm for all angles, except the angles $q_0 = 0^\circ$ and $q_0 = 30^\circ$ for which the functional significantly increases in the range from 600 nm to 650 nm). In the case of *p*-polarization fluctuations of the objective function are less noticeable than ones in the case of *s*-polarization, and the functional $\Omega(l_1, l_2)$ value changes very slowly.



a) angle of incidence from 0° to 85° b) angle of incidence from 0° to 60°

Fig. 4. Dynamics of the functional $\Omega(l_1, l_2)$ changes for a four-layer homogeneous structure in the case of p-polarization

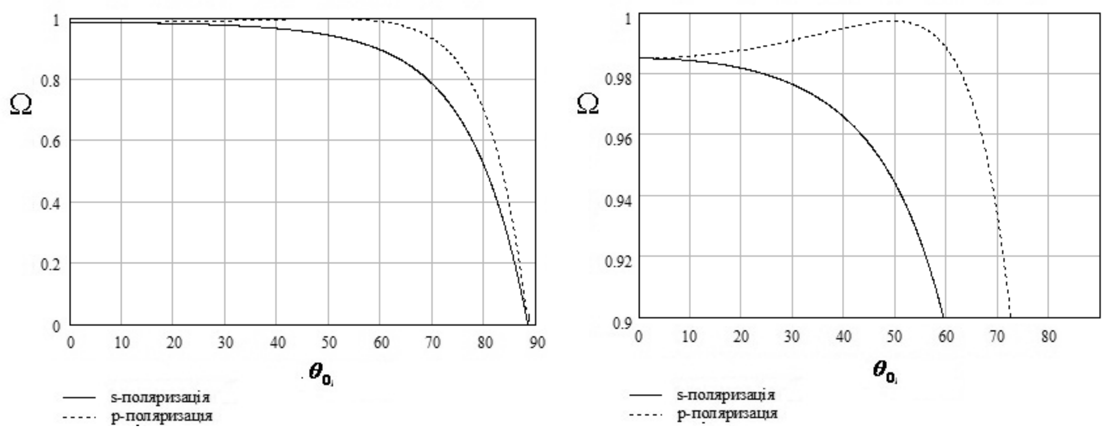


Fig. 5. Dynamics of changes of the functional $\Omega(l_2, l_1)$ for a four-homogeneous structure depending on q_0

The value of the functional (1) on the angle $q_0 = 55^\circ$ has smaller fluctuations than on other angles, and with a further increase of the angle the value of the functional decreases. The plots of functional (1) for the angles $q_0 = 50^\circ$ and $q_0 = 55^\circ$ intersect (Fig. 4b).

It is possible to see the following dependence of the objective function of the angle of incidence: in the case of *s*-polarization the objective function is decreasing, and in the case of *p*-polarization the objective function first increases and then decreases (Fig. 5), which is also explained by the Brewster law [7–9].

Calculations have shown relatively high performance of software developed on the base of the ***r***-algorithm for solving the problem of optimization of multilayer optical structures. The solving of our optimization problem on Intel (R) Core (TM) i5-3230@3.30 GHz takes a few minutes for each spectral band. Dynamics of changes of the functional (1) for a four-layer homogeneous structure as a function, depending on the angle of incidence is consistent with the Brewster law.

1. Furman Sh. Basics of optics of multiplayer systems / Sh.Furman., A.V. Tikhonravov Editions Frontiers, Gif-sur Yvette, 1992. – 242 p. 2. Abeles F. Matrix method / Ann.de Physique. – 1950. – V.5. – P. 596-640. 3. Stetsyuk P.I., Mitsa A. V. Parameters optimization for multilayer optical coatings // Kibernetika i sistemnyy analiz. – Kiev, 2005. – pp. 107-115 (in Russian). 4.Mitsa A.V., Stetsyuk P.I. Search problem of optimal parameters for homogenous optical coating // Teoriya optymalnyh rishen. – № 2. – Kiev, 2003. – pp. 127-134 (in Ukrainian). 5. Shor N.Z., Zhurbenko N.G. Minimization methods on base of spaces stretch in direction of the difference of two successive gradients // Kibernetika. – 1971. – #3. – pp. 51–59 (in Russian). 6. Shor N.Z.. Nondifferentiable Minimization and Application. – Kiev: Naukova Dumka, 1979 (In Russian). 7. Krylova T.N. Interference Coating. – Leningrad: Mashinostroyeniye, 1973 (In Russian). 8.Putilin E. S. Optical Coating. Textbook. – St. Petersburg: St. Petersburg ITMO, 2010 (In Russian). 9. Ritter E. Film dielectric for applications in optics // Physics of thin films. − Moskva: Mir, 1978. − vol. 8. − pp. 7-27 (In Russian).