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# Decreasing of error probability in telecommunication access networks by using amplitude modulation of many components

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Abstract. The paper addresses the research of data transmission technologies in telecommunication access networks. These networks are characterized by limited bandwidth and are suitable for high speed data transmission. Particular attention is paid on the data transmission technologies in telecommunication access networks based on symmetrical telecommunication lines. Paper proposes the method of adaptive data transmission in telecommunication access networks using amplitude modulation of many components. The results of mathematical modeling are presented, which allows estimating the bit error rate decrease in telecommunication access networks using this method.

Key words: telecommunication access network, data transmission, bit error rate, quadrature amplitude modulation, amplitude modulation of many components.

## INTRODUCTION

An important characteristic of quality of telecommunication access networks for data transmission is the error probability. Error probability decrease allows improving the quality of telecommunication access networks, in particular to increase the adequacy of received data. Among the factors that have influence on the appearance of errors in telecommunication channels or networks, an important place is occupied by the selected signal modulation methods and correcting coding methods.

Telecommunication access networks based on symmetrical telecommunication lines are rapidly developing among modern telecommunication networks designed for high speed data transmission. In such networks that are characterized by limited bandwidth, in order to achieve maximum data transmission rate, types of multiposition phase and amplitude-phase modulation are used mainly.

In telecommunication access networks based on symmetrical telecommunication lines with limited bandwidth, group of xDSL technologies (Digital Subscribe Line) is used mostly for data transmission. The most high speed type of xDSL is VDSL2 technology (Very High Speed Digital Subscriber Line 2) [1]. It is designed to transmit audio and video messages, data, broadcast high definition television and provide interactive software games.

According to the recommendation G.993.2 of the International Telecommunication Union (ITU), it is possible to transmit data in asymmetrical and symmetrical telecommunication lines with a total rate of downstream and upstream up to 200 Mbps using channel with a bandwidth up to 30 MHz by using VDSL2 technology. When length of telecommunication line is 0.5 km, data transmission rate is reduced to 100 Mbps and when its 1 km – up to 50 Mbps. Thus, VDSL2 reaches higher rate compared with previously developed VDSL technology (Very-high Speed Digital Subscriber Line) [2]. When length of telecommunication line is more than 1.8 km, the data transmission rate using VDSL2 technology is approximately equal to the rate using the most modern version of asymmetric data transmission technology ADSL2+ (Asymmetric Digital Subscriber Line 2) [3].

The data is divided into parts, each of which is transmitted by means of single subchannel using xDSL technology. The number of transmitted information by subchannel depends on the noise level in bandwidth of subchannel and the selected modulation. Binary phase shift keying (BPSK), quadrature phase shift keying (QPSK) and M-ary quadrature amplitude modulation (8-QAM, 16-QAM, 32-QAM, 128-QAM) [1-5] are used mainly.

The feature of multiposition phase and amplitude-phase modulation types is an increasing of the error probability in telecommunication line or network at increasing of number $M$ of symbols that can be transmitted during one informative cycle. However, modulation types with the same number $M$ of transmitted symbols and the same $E_b / N_0$ (ratio of the energy of one bit of information to the power spectral density of white noise) can be characterized by different error probability. So actual task is to develop new or improve known modulation types, which at certain $E_b / N_0$ will provide data transmission with a given rate with the lowest possible error probability. A lot of papers are devoted to this issue [6-17], but it requires further researches.

The aim of this work is to decrease the error probability in the telecommunication access network that are characterized by limited bandwidth by using the proposed amplitude modulation of many components (AMMC).

## THE RESEARCH OF TELECOMMUNICATION ACCESS NETWORKS CAPACITY WITH LIMITED BANDWIDTH CONSIDERING NOISE

The basics of the telecommunication network are telecommunication channels, so data transmission rate over a network will greatly depends on these channels capacity. In general, the channel capacity $C_c$ (bps) is determined by the well-known K. Shannon formula:

$$C_c = F_c \log_2 (1 + P_{s_{av}} / P_{n_{av}}), \tag{1}$$

where: $F_c$ – channel bandwidth, Hz; $P_{s_{av}} / P_{n_{av}}$ – ratio of average signal power to average noise power in channel, dimensionless value.

The following equation is used to research the selected telecommunication channel capacity $C_c$ (bps) [18]:

$$C_c = \int_{f_l}^{f_h} \log_2 \left( 1 + \left| \hat{S}_s(f) / \left[ \hat{S}_n(f) \cdot 10^{[|\alpha(f)|/10]} \right] \right| \right) df, \tag{2}$$

where: $f_l$, $f_h$ – the lowest and highest frequencies of channel bandwidth, Hz; $\hat{S}_s(f)$ – power spectral density of signal at the channel input, W/Hz; $\hat{S}_n(f)$ – power spectral density of white noise at the channel output, W/Hz; $|\alpha(f)|$ – modulus of loss in channel, dB; $f$ – current frequency, Hz.

Modulus of loss $|\alpha(f)|$ in the researches channel mainly depends on the modulus of loss $|\alpha_l(f)|$ of the symmetrical or coaxial telecommunication line, which can be calculated using the following equation:

$$|\alpha_l(f)| = 10 \lg \left( 1 / \left[ a (1 + b \cdot f)^c \right]^{1/l_{max}} \right), \tag{3}$$

where: $l$ – length of the telecommunication line; $l_{max}$ – maximal length of telecommunication line for which coefficients $a$, $b$ and $c$ are determined.

This formula allows decreasing of the computer simulation time in comparison with the use of complex mathematical models of telecommunication line. Such formula is suitable for use under condition $l \le l_{max}$.

To calculate the capacity of telecommunication access network based on xDSL technologies with $n$ subchannels, the following formula is proposed:

$$C_{TAN} = \sum_{i=1}^{n} F_i \log_2 \left[ 1 + \frac{P_{s_i}}{P_{n_i}} 10^{-|\alpha_{av_i}|/10} \right], \tag{4}$$

where: $n$ – the number of subchannels; $F_i$ – bandwidth of $i$-th subchannel, Hz; $P_{s_i}$ – transmitter signal power of $i$-th subchannel at the telecommunication line input, W; $P_{n_i}$ – noise power in bandwidth of the $i$-th subchannel at the telecommunication line output, W; $|\alpha_{av_i}|$ – modulus of average loss in telecommunication line in bandwidth of the $i$-th subchannel, dB.

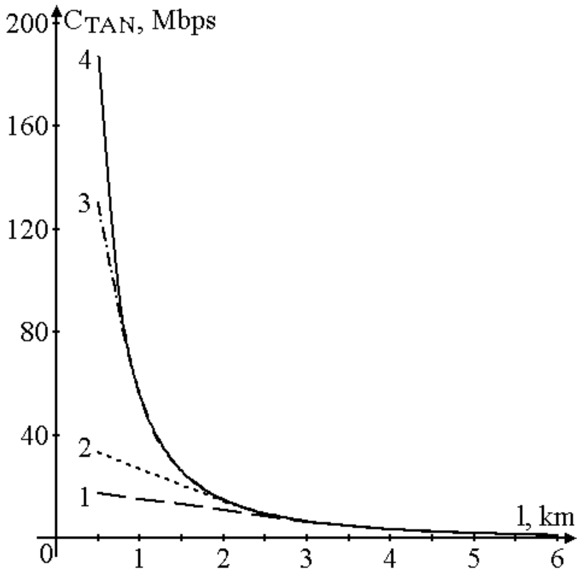


**Fig. 1.** Dependence of telecommunication access network capacity $C_{TAN}$ from the symmetrical line length $l$ in bandwidth: 1 – 1,104 MHz; 2 – 2,208 MHz; 3 – 12 MHz; 4 – 30 MHz

**Table 1.** Modern types of xDSL technologies

| Technology | Bandwidth, MHz | Numberof subcannals | Maximum total rate in both directions, Mbps / length of the telecommunication line, km |
|---|---|---|---|
| ADSL | 1.104 | 256 | 8.875 |
| ADSL2 | 1.104 | 256 | 15.5 |
| ADSL2+ | 2.208 | 512 | 25.216 / 1 |
| ADSL2+M | 2.208 | 512 | 27.5 / 1 |
| VDSL | 12 | 2783 | 64 |
| VDSL2 (12b) | 12 | 2783 | 68 |
| VDSL2 (17a) | 17.664 | 4096 | 100 / 0.5; 50 / 1 |
| VDSL2 (30a) | 30 | 3479 | 200 |

Mathematical models [18] and Eqs. (1)-(4) was used for development of an improved model of telecommunication access network based on symmetrical and coaxial lines. The influence of the symmetrical line length on capacity of telecommunication access network based on symmetrical lines using the group of xDSL technologies was researched by means of such an improved model. Mathematical modeling is performed at the constant power spectral density of signal $S_s(f) = 1 \cdot 10^{-8}$ W/Hz and the constant power spectral density of white noise $S_n = 2,25 \cdot 10^{-14}$ W/Hz. Research is performed in bandwidths 1.104 MHz (for ADSL and ADSL2), 2,208 MHz (for ADSL2+ and ADSL2+M), 12 MHz (for VDSL) and 30 MHz (for VDSL2). Specifications for modern types of xDSL technologies are given in Table 1. The calculation results are shown on Fig. 1.

It is evidently from Table 1 and Fig. 1, that VDSL2 technology is the most perspective for use in telecommunication access networks.

## THE RESEARCH OF THE BIT ERROR RATE IN THE TELECOMMUNICATION ACCESS NETWORKS

Output power of telecommunication transmission equipment is regulated by international organizations (in particular International Telecommunication Union). Noise level in the telecommunication channel can be decreased to a certain value by using shielded cable lines. Therefore, modern telecommunication access networks operate under certain ratio $P_s / P_n$ that is very difficult to increase. This influences on the achieved bit error rate in telecommunication access networks for data transmission.

It is advisable to apply new modulation methods and correcting coding methods in order to decrease the error probability. These methods provide data transmission with a certain rate and less bit error rate in telecommunication access network at the certain ratio $P_s / P_n$ (or ratio $E_b / N_0$).

In modern telecommunication access networks the amplitude-phase modulation types are actively used for high rate data transmission, including QAM. Traditionally, QAM signal is represented as a sum of two orthogonal components – in-phase and quadrature [19]:

$$\begin{aligned} u_{QAM}(t) &= U_0 a_I u_{mI}(t) \cos(\omega_0 t + \varphi_0) + \\ &+ U_0 a_Q u_{mQ}(t) \cos\left(\omega_0 t + \varphi_0 - \frac{\pi}{2}\right), \end{aligned} \quad (5)$$

where: $U_0$, $\omega_0$, $\varphi_0$ – amplitude, angular frequency and initial phase of the carrier oscillation; $a_I$, $a_Q$ – proportional coefficients for in-phase I and quadrature Q channels; $u_{mI}(t)$ – modulating signals on in-phase I and quadrature Q inputs of modulator.

Euclidean distance between adjacent signal points in the signal constellation for M-QAM (Eq. (5)) in case of amplitude modulation of each component by the modulating signals with maximal possible amplitude $U_{s_{max}}$ (V) and number $M_U$ of equally distant levels of amplitude is calculated using the following equation:

$$d = \sqrt{2} U_{s_{max}} / (M_U - 1). \quad (6)$$

By the implementation of such M-positional modulation $\log_2(M)$ bit of information is transmitted during the continuance of one informative symbol.

To increase data transmission rate or to decrease bit error rate in telecommunication access networks, which use QAM, new modulation group based on amplitude modulation of many components (AMMC) is proposed to use. Modulated AMMC signal is formed as a sum of $N$ harmonic components that differ in initial phases $\varphi_n$. It is [20]:

$$u_{AMMC}(t) = \sum_{n=1}^{N} U_0 a_n u_{m_n}(t) \cos(\omega_0 t + \varphi_0 + \varphi_n), \quad (7)$$

where: $a_n$ – proportional coefficients for the $n$-th channels of modulator; $u_{m_n}(t)$ – modulating signals at the $n$-th inputs of modulator.

We obtained equation to calculate the Euclidean distance d (B) between adjacent signal points in the signal constellation for AMMC with $N = 3$, the initial phases of components $\varphi_1 = 0$ radian, $\varphi_2 = \pi/3$ radian, $\varphi_3 = 2\pi/3$ radian and the maximal possible amplitude of the modulated signal $U_{s_{max}}$ (V) in case of amplitude modulation of each component by modulating signals with number $M_U$ of equally distant levels of amplitude:

$$d = U_{s_{max}} / (M_U - 1). \quad (8)$$

The number of unduplicated AMMC signals that can be unambiguously demodulated (hence the effective number of symbols) is:

$$M_{eff} = 3 M_U (M_U - 1). \quad (9)$$

The total number of symbols that can be obtained using all possible combinations of modulating informative signals at arbitrary initial phases of components is calculated using the following equation:

$$M_{all} = (M_U)^N. \quad (10)$$

During one informative symbol $\log_2 M_{eff}$ bit of information can be transmitted.

With the modulator and demodulator AMMC it is possible to form and process AMMC signals (Eqs. (7)-(10)) with a various signal constellations. A signal constellation of AMMC signal with $N$ components is inserted in polygon with $2N$ angles. From a practical point of view constellation of AMMC signals with three and six components are interesting. In particular, the signal constellation of AMMC signals with $N = 3$,

$M_U = 3$ equally distant levels of amplitude of modulating signals with displacement of amplitude levels using 16 points (16-AMMC with displacement of amplitude levels of the modulating signals) and $M_U = 4$ equally distant levels of amplitude of modulating signals with displacement of amplitude levels using 32 points (32-AMMC with displacement of amplitude levels of the modulating signals) are shown on Fig. 2, d and Fig. 2, e. Also, the signal constellation of AMMC signal with $N = 3$, $M_U = 4$ and 36 points (36-AMMC) is shown on Fig. 2, f. For comparison, there are shown the signal constellation of widely used modulation types 16-QAM (Fig. 2, a), 32-QAM (Fig. 2, b) and 36-QAM (Fig. 2, c).

Let us compare the properties of 36-QAM modulation (Fig. 2б c) and 36-AMMC modulation (Fig. 2, f) with the same number 36 of signal points. Using both types of modulation it is possible to transmit the same number of information during one informative symbol.

The Euclidean distance between adjacent signal points in the signal constellation of 36-QAM signal with the maximal possible amplitude $U_{s_{max}} = 1$ V and $M_U = 6$ equally distant levels of amplitude of modulating signals according to Eq. (6) is $d = 0.28$ V. By the implementation of 36-AMMC at $U_{s_{max}} = 1$ V and $M_U = 4$ the Euclidean distance according to Eq. (8) is $d = 0.33$ V. Thus, provide the same robustness during data transmission period using 36-AMMC it is needed 1,43 dB less ratio $P_s / P_n$ in the telecommunication channel compared to using 36-QAM.

For implementation of AMMC it is necessary to use offered AMMC modulator and demodulator [20]. Their feature is in a possibility to perform modulation and demodulation BPSK, QPSK, M-PSK, M-QAM and modulation group based on AMMC. Thus, for implementation of BPSK by AMMC modulator it enough to use only one multiplier, and for QPSK, 8-PSK, M-QAM – two multipliers and one phase return device on $\pi / 2$. For demodulation of BPSK by AMMC demodulator it is enough to use only one multiplier and one low-pass filter, and for QPSK, 8-PSK, M-QAM – two multipliers, two low-pass filters and one phase return device on $\pi / 2$.

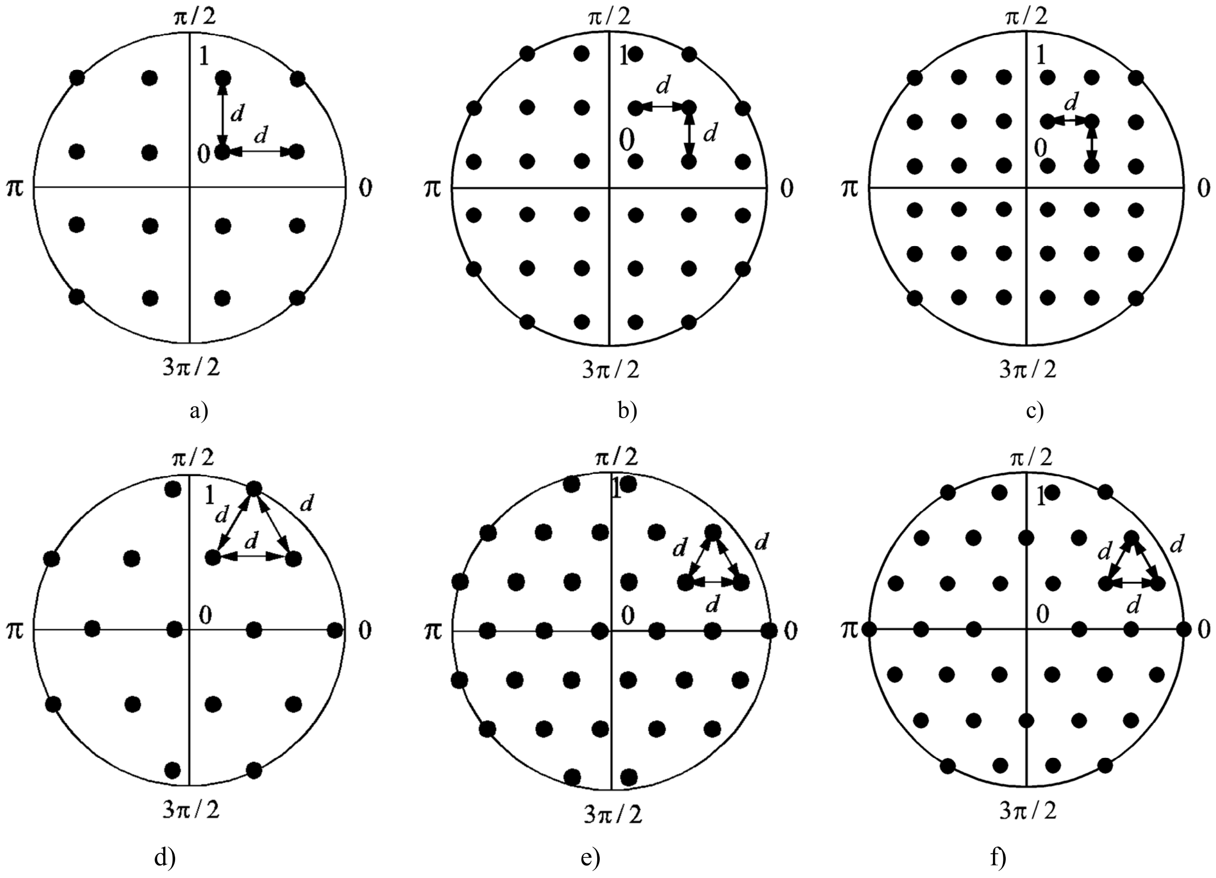
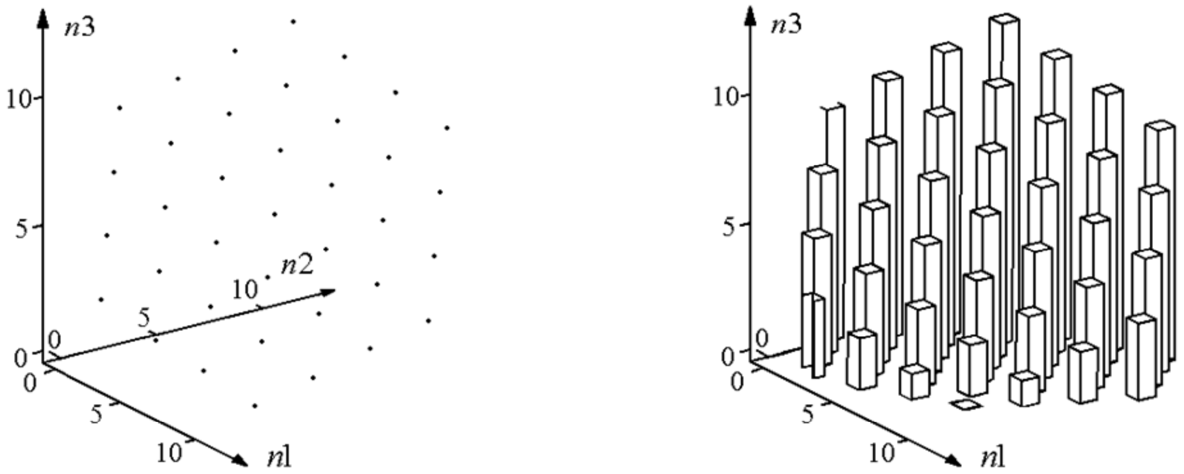


**Fig. 2.** Signal constellations: a) 16-QAM; b) 32-QAM; c) 36-QAM; d) 16-AMMC with displacement of amplitude levels; e) 32-AMMC with displacement of amplitude levels; f) 36-AMMC



a) b)

**Fig. 3.** Demodulation space for 37-AMMC

**Table 2.** Bit error rate in telecommunication access network using different types of signal modulation

| $E_{b_{av}} / N_0$, dB | 12.95 | 14.68 | 15.93 | 16.91 | 17.72 |
|---|---|---|---|---|---|
| Modulation | Bit error rate $P_b$ | | | | |
| 16-QAM | $2.662 \cdot 10^{-5}$ | $4.728 \cdot 10^{-7}$ | $8.170 \cdot 10^{-9}$ | $1.385 \cdot 10^{-10}$ | $2.317 \cdot 10^{-12}$ |
| 16-AMMC with displacement of amplitude levels | $1.128 \cdot 10^{-5}$ | $1.160 \cdot 10^{-7}$ | $1.149 \cdot 10^{-9}$ | $1.109 \cdot 10^{-11}$ | $9.291 \cdot 10^{-14}$ |
| 32-QAM | $5.474 \cdot 10^{-4}$ | $4.148 \cdot 10^{-5}$ | $3.135 \cdot 10^{-6}$ | $2.362 \cdot 10^{-7}$ | $1.776 \cdot 10^{-8}$ |
| 32-AMMC with displacement of amplitude levels | $3.840 \cdot 10^{-4}$ | $2.098 \cdot 10^{-5}$ | $1.137 \cdot 10^{-6}$ | $6.123 \cdot 10^{-8}$ | $3.283 \cdot 10^{-9}$ |
| 36-QAM | $1 \cdot 10^{-3}$ | $1 \cdot 10^{-4}$ | $1 \cdot 10^{-5}$ | $1 \cdot^{x} 10^{-6}$ | $1 \cdot 10^{-7}$ |
| 36-AMMC | $7.576 \cdot 10^{-4}$ | $5.732 \cdot 10^{-5}$ | $4.325 \cdot 10^{-6}$ | $3.253 \cdot 10^{-7}$ | $2.433 \cdot 10^{-8}$ |

During demodulation of QPSK or QAM signals each signal point can be represented on the proposed demodulation plane in a Cartesian coordinate system with coordinates equal to the signals amplitude at the outputs of the first and second low-pass filters. During demodulation of AMMC signals with $N$ components we should use the proposed $N$-dimensional demodulation space. All signal points of AMMC signal in such space have $N$ coordinates, each of which is proportional to the signal amplitude at one of N outputs of AMMC demodulator. It is possible to use normalized coordinates, which are equal to possible amplitude levels at the outputs of the AMMC demodulator. Signal constellation in three-dimensional demodulation space with axes $n1$, $n2$, $n3$ is presented in case of 37-AMMC signal with three components as shown on Fig. 3. At each output of AMMC demodulator signal may be present with one of 13 possible levels of signals amplitude.

It is found that Eq. (12) is suitable for using amplitude phase keying (APK), QAM and AMMC to calculate the bit error rate that does not exceed 0.1.

To evaluate AMMC advantages in comparison with other well-known signal modulation varieties, a research of bit error rate in telecommunication access network depending on the applied modulation type was conducted. The results of error probability in the telecommunications access network with limited bandwidth in case of noise when using different types of signal modulation for different values $E_{b_{av}} / N_0$ (ratio of the average energy of one bit of information to the power spectral density of white noise) are shown in Table 2.

The AMMC application decreases the bit error rate at the output of deciding device of telecommunication channel or telecommunication access network compared to the use of QAM at the same number of symbols $M$ that can be transmitted during one informative cycle as can be seen from Table 2. This phenomena cause by distance increasing between signal points on the signal constellation. For example, using 16-AMMC with displacement of amplitude levels of the modulating signals, bit error rate is 2.36 times less compared to 16-QAM when ratio $E_{b_{mid}} / N_0 = 13,95$ dB and 24.94 times less when ratio $E_{b_{mid}} / N_0 = 17,72$ dB.

According to obtained results, we determined that bit error rate in telecommunication access networks may be decreased by application the proposed method of adaptive data transmission in telecommunication access networks based on symmetrical and coaxial telecommunication lines using AMMC. The method bases on the adaptive selection of modulation types, which at appropriate ratio of signal power to noise power will provide the lowest bit error rate for a given transmission rate. We should apply one of the modulation types – BPSK, QPSK and M-positional varieties modulation 8-PSK, M-QAM or M-AMMC. Correcting coding should be used in order to decrease the bit error rate.

## CONCLUSIONS

To increase data transmission rate or to decrease bit error rate in telecommunication access networks, which use QAM, new modulation group based on AMMC is proposed to use.

The proposed AMMC increases the distance between the signal points on the signal constellation, thereby decreasing bit error rate in the telecommunication access network to 25 times compared to the use QAM with the same maximum power of modulated signal and the same number of symbols M that can be transmitted during one informative cycle.

The proposed method of adaptive data transmission in telecommunication access networks based on symmetrical and coaxial telecommunication lines using AMMC is an adaptive selection of modulation types that ensures the lowest possible bit error rate during data transmission at some ratio of signal power to noise power. The modulation types BPSK, QPSK, 8-PSK, M-QAM, M-AMMC and correcting coding should be used in order to decrease the bit error rate.

The proposed AMMC and adaptive data transmission method is useful in design of modern telecommunication access networks for data transmission.

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