COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 2, 2013

PROCESS MODELING IN NONLINEAR ELECTRIC CIRCUITS AND DEVICES USING THE DIFFERENTIAL HARMONIC METHOD

Lev Glukhivskyi

State Institute of Scientific, Technical and Innovative Expertise, Kyiv, Ukraine,
Gl.lev42@gmail.com

Abstract. The article presents the basic principles of a differential harmonic method for the numerical calculation of transient and steady-state periodic processes in nonlinear electric circuits and devices with vibrations caused by periodic external forces. There is the diagram of conversion of differential equations, describing electric circuits and devices and operating with their instant variables, to differential equations operating with the harmonic amplitude vectors, given in the article. It is shown how to obtain the amplitude values of the vectors of harmonics in periodic steady-state processes and their dependences on the time in transients by numerical methods.

Key words: nonlinear electric circuits and devices, calculation of transient and periodic processes, differential harmonic method, harmonic amplitude vector.

1. Introduction

For the first time the differential harmonic method (DHM) was systematically laid out by the author of this article back in 1984 in the monograph [1]. The method principles were outlined there, as well as the application of the method to the numerical simulation of periodic processes in a variety of nonlinear electric circuits and devices with periodic external forces causing vibrations was examined. Subsequently, the DHM was improved and its comprehensive description was presented in the monographs [2, 3] (note that [3] is available in the form of an electronic resource). In recent years, studies were conducted on the applicability of this method for the numerical simulation of transients in nonlinear electric circuits and devices [4, 5].

The purpose of this article is to present the DHM as a method of numerical simulation processes (transient and steady-state periodic ones) in nonlinear electric circuits and devices of the most common items.

2. Presentation of the method essence

The essence of the DHM is that only one type of the replacement of variables in solving differential equations describing processes in nonlinear objects is to be used. If the object is under the influence of an external periodic force causing variations of an instantaneous variable x with frequency ω, this replacement has the form

$$x = X_0 + \sum_{\nu=1}^{n} \left(X_{c\nu} \cos \nu\omega t + X_{s\nu} \sin \nu\omega t \right), \quad (1)$$

In this case, instead of one variable x, new variables $X_0, X_{c\nu}, X_{s\nu}$ $(\nu = 1,..., n)$ are taken into consideration. They are constant in the periodic steady process and are functions of time in transient one. This replacement of variables leads, as shown in [3], to transforming the differential equation with variable x to the system of differential equations with variables $X_0, X_{c\nu}, X_{s\nu}$ $(\nu = 1,..., n)$ according to the scheme

$$x \Rightarrow \vec{X}_G; \quad \frac{dx}{dt} \Rightarrow \frac{d\vec{X}_G}{dt} + \omega D \vec{X}_G, \quad (2)$$

where

$$\vec{X}_G = colon\left(X_0, X_{c1}, X_{s1}, ..., X_{c\nu}, X_{s\nu} \right); \quad (3)$$

$$D = diag\left(0, \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix}, \begin{Vmatrix} 0 & 2 \\ -2 & 0 \end{Vmatrix}, ..., \begin{Vmatrix} 0 & n \\ -n & 0 \end{Vmatrix} \right). \quad (4)$$

In [2, 3] vector $\vec{X}_G$ was named a simple vector of amplitudes.

If the simulated process has two or more instantaneous variables $\left(\text{let}\, x_1, x_2, \dots, x_m\right)$ and if they are presented as a vector variable

$$\vec{x} = colon\left(x_1, x_2, \dots, x_m \right), \quad (5)$$

then the above-mentioned replacement of variables leads to the following transformation

$$\begin{aligned} &\vec{x} \Rightarrow \vec{X}_G^*; \quad \frac{d\,\vec{x}}{dt} \Rightarrow \frac{d\,\vec{X}_G^*}{dt} + \omega D^* \vec{X}_G^*; \\ &D^* = diag\left(D, \dots, D \right); \\ &\vec{X}_G^* = colon\left(\vec{X}_{1G}, \vec{X}_{2G,} ..., \vec{X}_{mG} \right). \end{aligned} \quad (6)$$

In [2, 3] vector $\vec{X}_G^*$ is called a composite vector of amplitudes.

Let the differential equation that relates to the instantaneous variables of the process in the electric circuit or device be on the form

$$\frac{d\vec{y}}{dt}+\vec{z}=\vec{e}\ , \qquad (7)$$

where

$$\vec{y}=colon(y_1, y_2, \dots, y_m);$$
$$\vec{z}=colon(z_1, z_2, \dots, z_m); \qquad (8)$$
$$\vec{e}=colon(e_1, e_2, \dots, e_m);$$

$y_1, \dots, y_m, z_1, \dots, z_m$ are instantaneous variables;

$e_1, \dots, e_m$ are external periodic forces.

After replacing all the instantaneous variables in the formula (1), equation (7) considering (6) takes the form

$$\frac{d\vec{Y}_G^*}{dt}+\omega D^*\vec{Y}_G^*+\vec{Z}_G^*=\vec{E}_G^*, \qquad (9)$$

where

$$\vec{Y}_G^*=colon\left(\vec{Y}_{1G}, \vec{Y}_{2G}, \dots, \vec{Y}_{mG}\right);$$
$$\vec{Z}_G^*=colon\left(\vec{Z}_{1G}, \vec{Z}_{2G}, \dots, \vec{Z}_{mG}\right); \qquad (10)$$
$$\vec{E}_G^*=colon\left(\vec{E}_{1G}, \vec{E}_{2G}, \dots, \vec{E}_{mG}\right).$$

Let us illustrate the above-mentioned change of variables and transformations of differential equations with instantaneous variables to differential equations with amplitudes of harmonics appearing in R-L-C circuit with a nonlinear resistance, nonlinear inductance and nonlinear capacitor connecting in series, as well as electromotive force, which is a periodic function of time. The behavior of the instantaneous variables (current i, flux linkage inductance Ψ, voltage $u_{\hat{e}}$ and charge q of the capacitor, voltage u_r on active resistance) is described by a system of two differential equations

$$\frac{d\Psi}{dt}+u_r+u_{\hat{e}}=E_m\sin(\omega t+\alpha);\frac{dq}{dt}-i=0, \quad (11)$$

where

$$\Psi=\Psi(i); \quad u_r=u_r(i); \quad q=q(u_{\hat{e}}) \quad (12\ \text{a,b,c})$$

are non-linear dependences of the flux inductor on current; of the voltage on the active resistance on current; of the capacitor charge on its voltage, respectively.

If the system (11) is represented as a vector differential equation (7), we obtain formulae

$$\vec{y}=\begin{Vmatrix}\Psi\\ q\end{Vmatrix};\ \ \vec{z}=\begin{Vmatrix}u_r+u_{\hat{e}}\\ -i\end{Vmatrix};\ \vec{e}=\begin{Vmatrix}E_m\sin(\omega t+\alpha)\\ 0\end{Vmatrix}, \quad (13)$$

where independent variables i and $u_{\hat{e}}$ form a vector

$$\vec{f}=colon(i, u_{\hat{e}}). \qquad (14)$$

In this case equation (7) after replacing the variables is transformed to equation (9), in which

$$\vec{Y}_G^*=colon\left(\vec{\Psi}_G, \vec{Q}_G\right); \qquad (15)$$

$$\vec{Z}_G^*=colon\left(\vec{U}_{rG}+\vec{U}_{\hat{e}G}, -\vec{I}_G\right); \qquad (16)$$

$$\vec{E}_G^*=colon\left(\vec{E}_G, 0\right), \qquad (17)$$

and its independent variable is a drawn vector of amplitudes of the form (10)

$$\vec{F}_G^*=colon\left(\vec{I}_G, \vec{U}_{\hat{e}G}\right), \qquad (18)$$

formed from the vector of the amplitudes of current and voltage harmonics of the capacitor. In (15) - (17) the vector of flux amplitudes

$$\vec{\Psi}_G=colon\left(\Psi_0, \Psi_{c1}, \Psi_{s1}, \dots, \Psi_{cn}, \Psi_{sn}\right) \quad (19)$$

and the vector of voltage on the active resistance

$$\vec{U}_{rG}=colon\left(U_{r0}, U_{rc1}, U_{rs1}, \dots, U_{rcn}, U_{rsn}\right) \quad (20)$$

are nonlinear functions

$$\vec{\Psi}_G=\vec{\Psi}_G(\vec{I}_G); \qquad (21)$$

$$\vec{U}_{rG}=\vec{U}_{rG}(\vec{I}_G) \qquad (22)$$

of the vector of current amplitudes

$$\vec{I}_G=colon\left(I_0, I_{c1}, I_{s1}, \dots, I_{cn}, I_{sn}\right). \qquad (23)$$

The vector of amplitudes of capacitor charge

$$\vec{Q}_G=colon\left(Q_0, Q_{c1}, Q_{s1}, \dots, Q_{cn}, Q_{sn}\right) \qquad (24)$$

is the nonlinear function

$$\vec{Q}_G=\vec{Q}_G\left(\vec{U}_{\hat{e}G}\right) \qquad (25)$$

of the vector of capacitor voltage amplitudes

$$\vec{U}_{rG}=colon\left(U_{\hat{e}0}, U_{\hat{e}c1}, U_{\hat{e}s1}, \dots, U_{\hat{e}cn}, U_{\hat{e}sn}\right). \quad (26)$$

Entries (21), (22), and (25) imply that non-linear relationships between instantaneous variables (12) lead to the fact that each of the amplitudes of the harmonics of any dependent variable is a function of the amplitudes of all harmonics of all orders of the respective independent variable .

Algorithms for calculating the values of the amplitudes of the vectors (19) and (20) by the value of the vector of amplitudes (23) and for calculating the value of the vector of amplitudes (24) by value of the vector of amplitudes (26) using the dependence (12) are given in [3]. There is also the software implementation of this algorithm in the form of procedures written in FORTRA there.

3. Transients

In order to determine changes in the transients the values of the vectors of amplitudes of harmonics $\vec{I}_G$ of the current and $\vec{U}_{\hat{e}G}$ of the capacitor voltage using one of the numerical methods, let's solve the equation (9) with respect to the composite vector of amplitudes (18):

$$\frac{d\vec{F}_G^*}{dt} = \left(S_{YG}^*\right)^{-1}\left(\vec{E}_G^* - \omega D^*\vec{Y}_G^* - \vec{Z}_G^*\right), \qquad (27)$$

where

$$S_{YG}^* = \frac{d\vec{Y}_G^*}{d\vec{F}_G^*} = diag\left(L_G, C_G\right) \qquad (28)$$

is a composed matrix of differential harmonic parameters;

$$L_G = \frac{d\vec{\Psi}_G}{d\vec{I}_G}; \qquad (29)$$

$$C_G = \frac{d\vec{Q}_G}{d\vec{U}_{\hat{e}G}} \qquad (30)$$

are matrices of differential harmonic inductances and differential harmonic capacitances, respectively. These matrices are functions of vectors of amplitudes $\vec{I}_G$ and $\vec{U}_{\hat{e}G}$

$$L_G = L_G(\vec{I}_G); \quad C_G = C_G(\vec{U}_{\hat{e}G}). \qquad (31)$$

The values of the matrices L_G and C_G can be calculated using algorithms and computer procedures given in [3]. Along with their use, the differential parameter dependencies should be used

$$l^{\partial} = \frac{d\Psi}{di} = l^{\partial}(i); c^{\partial} = \frac{dq}{du_{\hat{e}}} = c^{\partial}(u_{\hat{e}}), \qquad (32)$$

the differential inductance and differential capacitance, respectively, obtained by differentiating (analytical or numerical) dependencies (12 a, c).

At the each step of numerical integration of the differential equation (27) with the known value of the composite vector of amplitudes $\vec{F}_G^*$ using procedures given in [3], the values of the composite vectors of amplitudes $\vec{Y}_G^*$ and $\vec{Z}_G^*$, value of the matrix S_{YG}^* are computed and actions are conducted, which are contained in the right side of equation (27).

The usage of the DHM for the numerical calculation of transients in nonlinear electric circuits and devices is advisable, of course, only in cases when it has significant advantages over other methods. Attention is drawn to some of these benefits [4, 5], in particular, there is a possibility to significantly increase the integration step, since the intensity of harmonic amplitude changes over time is much lower than the intensity of changes of instantaneous variables of the process.

4. Periodic processes

The periodic process of an electric circuit or device can be obtained as the end of the transient. It is necessary to carry out the numerical integration of equation (9) over the period of the periodic external force until the extinction of aperiodic components. Determination accuracy of the initial data, as well as accumulated errors of the numerical integration will have influence on the accuracy of the periodic process defined in such a way. The need to model the entire transition process for opbtaining periodic process may be proper when you consider that in the periodic process the amplitudes of harmonics are constant in time, and then in equation (9) $\frac{d\vec{Y}_G^*}{dt} = 0$. With this in view, the differential equation (9) is transformed into a finite equation

$$\omega D^*\vec{Y}_G^* + \vec{Z}_G^* - \vec{E}_G^* = 0, \qquad (33)$$

whose solution is the value of the composite vector of amplitudes (18). It gives the approximation of the periodic process, in which the instantaneous variables are determined by the formula (1).

To solve the finite equation (33), any of the known numerical methods can be applied, in particular – Newton's iterative method [6]. Using this method, the refinement of the solution to equation (33) in the j iteration is performed according to the formula

$$\vec{F}_{G(j+1)} = \vec{F}_{G(j)} - W_{(j)}^{-1}\vec{H}_{(j)}, \qquad (34)$$

where

$$\vec{H}_{(j)} = \omega D^*\vec{Y}_G^* + \vec{Z}_G^* - \vec{E}_G^* \qquad (35)$$

is the value of the residual vector of equation (33) with $\vec{F}_G^* = \vec{F}_{G(j)}^*$;

$$W_{(j)} = \omega D^* S_{YG}^* + S_{ZG}^* \qquad (36)$$

is the value of the Jacobi matrix of the left side of the equation (33) in the j iteration when $\vec{F}_G^* = \vec{F}_{G(j)}^*$; S_{YG}^* is the matrix (28) with $\vec{F}_G^* = \vec{F}_{G(j)}^*$;

$$S_{ZG(j)}^* = \frac{d\vec{Z}_G^*}{d\vec{F}_G^*} = \begin{Vmatrix} R_G & \breve{E} \\ -\breve{E} & \end{Vmatrix}; \qquad (37)$$

$\breve{E}$ is an identity matrix;

$$R_G = \frac{d\vec{U}_{rG}}{d\vec{I}_G} \qquad (38)$$

is the matrix of harmonic differential resistance, which can be calculated using algorithms and computer procedures provided in [3], together with the use of dependency of differential resistance on the current i

$$r^{\partial} = \frac{du_r}{di} = r^{\partial}(i), \qquad (39)$$

obtained by the differentiation of the dependency (12b).

The application of Newton's method for calculating periodic processes in nonlinear electric circuits and devices on the basis of the DHM is detailed in [2, 3].

The examples of numerical modeling using the DHM of periodic processes in complex electric circuits and devices (in particular, different kinds of electric machines) are presented in [1 - 3], where the bibliography of journal articles on this topic can be found.

5. Conclusion

1. Researchers who model the transient and periodic processes in nonlinear electric circuits and devices are invited to note the differential harmonic method (DHM), tested while many problems in this area being solved.

2. The package of procedures developed for the DHM (written in FORTRAN), which implements all the operations of the method and is free to use, is shown in [3].

3. The procedures for implementing the DHM operations can be performed with the use of the MATLAB software package, which includes special interfaces. It provides the access to subprograms written in other programming languages, including FORTRAN.

4. MATLAB software developers are recommended to consider the procedures implementing the DHM operations and shown in [3] in terms of their inclusion to the MATLAB package.

References

[1] L. Glukhivskyi, *Calculation of periodic processes of electric devices (differential harmonic method).* Lviv, Ukraine: Vyshcha shkola, 1984. – 164 p. (Russian)

[2] L. Glukhivskyi, *Periodic Nonlinear Processes in Electric Engineering (differential harmonic method and its software)*. Kyiv, Ukraine: Alfa P&K, 2005. (Ukrainian)

[3] L. Glukhivskyi, *Nonlinear oscillations: numeri-cal polyharmonic simulation.* Kyiv, Ukraine: Alfa P&K, 2008. (Ukraine)

[4] L. Glukhivskyi, "Simulation of transients in nonlinear electric circuits by harmonic differential method," *Electrychni na elektromekhanichni systemy,* no. 736, p. 21, Lviv, Ukraine: Lviv Polytechnic National University, 2012. (Ukrainian)

[5] L. Glukhivskyi, "The issue of calculating transients in nonlinear electric circuits by harmonic differential method," *Electrychni ta elektromekhanichni systemy,* no. 763, p. 19-26, Lviv, Ukraine: Lviv Polytechnic National University, 2012. (Ukrainian)

[6] V. Perkhach, Mathematical problems of electric power. – Lviv, Ukraine: Lviv State Uuniversity, 1989. (Ukrainian)

[7] *http://www.nbuv.gov.ua/books/2008/08gljcpm.pdf* (Ukrainian)

МОДЕЛЮВАННЯ ПРОЦЕСІВ У НЕЛІНІЙНИХ ЕЛЕКТРИЧНИХ КОЛАХ І ПРИСТРОЯХ З ВИКОРИСТАННЯМ ДИФЕРЕНЦІЙНОГО ГАРМОНІЧНОГО МЕТОДУ

Лев Глухівський

Викладено основні засади диференційного гармонічного методу для чисельного розрахунку перехідних і усталених періодичних процесів у нелінійних електричних колах і пристроях, які містять зовнішні періодичні сили, що зумовлюють коливання. Наведено схему перетворення диференціальних рівнянь, які описують електричні кола і пристрої і оперують їх миттєвими змінними, до диференціальних рівнянь, які оперують векторами амплітуд гармонік. Показано способи отримання чисельними методами значень векторів амплітуд гармонік в усталених періодичних процесах і їх залежностей від часу у перехідних процесах.

Lev Glukhivskyi – Doctor of technical sciences, professor.

Research interests: calculation of transient and steady-state periodic processes in nonlinear electrical circuits and electrical machines.