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EFFECTIVE ALGORITHM OF CALCULATING THE STATIC MODES OF NONLINEAR ELECTROMAGNETIC CIRCUITS

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Abstract: Existing electromagnetic circuits are mostly non-linear, however known methods of analysis of the processes for those circuits do not meet modern requirements and it is the one of least developed parts of Theoretical Electric Engineering. Therefore the development of methods of analysis is an important issue. The article discusses the problem of developing the algorithm of calculation for static modes in nonlinear electromagnetic circuits with reactive elements under the influence of periodical disturbances. Those modes are dynamic and are described by the system of differential equations (DE). The solution of the system consists of periodic non-harmonic dependencies. The problem is solved by a numerical method as the boundary value problem of DE of the first order with periodic boundary conditions so that it is possible to get dependencies on the period of state variables without solving the problem in time domain.

Key words: nonlinear electromagnetic circuit, reactance, static process, periodical mode, algorithm of calculation, boundary value problem, spline approximation.

1. Introduction

Steady state electrical devices with nonlinear elements occupy a prominent place in nonlinear electrical engineering. They are described by nonlinear systems of DEs, and therefore their analysis is only possible on the basis of the theory of nonlinear DEs. In the case of a periodic disturbance, steady states are dynamic and characterized by periodic non-harmonic changes of coordinates, and the problem of their calculation is rather a complicated task. Hence the creation of new methods and algorithms of calculation and improvement of existing methods has great theoretical and practical significance.

A mathematical basis for calculation of the steady state for a nonlinear electromagnetic circuit is detected as the periodical solution of the nonlinear system of DEs that describes the circuit. It can be obtained by solving the Cauchy problem in the time domain for some initial conditions, namely on the basis of the calculation of the transition process to its completion. Thus, the application of the method to the determination of the periodic solution is ineffective for many reasons, but the most effective way of solving the problem is to consider it as a boundary value problem for the system of DEs, which describes the electromagnetic state of a circuit in a timeless region.

For the analysis of periodic modes of nonlinear electrical circuits graph-analytical, analytical, numerical and combined methods are used, but in the terms of mathematical modeling only the last one is noteworthy. The most common analytical methods of the investigation of periodic modes are asymptotic methods [1], the method of small parameter [2] and the method of harmonic balance, which are based on the periodic solution approximation by trigonometric series. On that basis, numerical methods [3, 4] are constructed, and adapted to solve nonlinear problems. The comparative analysis of numerical and analytical methods is implemented in [5].

Asymptotic methods, including the method of a small parameter are effective for the investigation of quasilinear systems. However, the analysis of vitally nonlinear objects causes problems related to a separation into linear and nonlinear parts of equations and a decomposition with the index of the small parameter.

One of the ways of solving a two-point boundary value problem for the DE that describes periodic processes is the method of finite differences [6, 7], the essence of which is in approximation of the initial system of DEs with difference equations for aggregation of nodes on the period. Nevertheless, to provide the convergence and stability of the computational process, the number of nodes on the period should be large enough. Convergence problems also appear in the method [8], which is based on searching for initial conditions under which the integration of the DE system in the time domain during the period leads up to receiving the periodical dependences of coordinates.

The purpose of the paper is to describe an algorithm for solving the problem of finding the periodic solution in a timeless region, namely on the basis of solving it as a boundary value problem [7].

2. Mathematical task formulation

Processes in nonlinear electrical circuit with a periodic disturbance are described by the nonlinear DE system, that can be represented as follow

$$\frac{d\vec{y}(\vec{x},t)}{dt} = \vec{z}(\vec{y},\vec{x},t) + \vec{u}(t), \qquad (1)$$

where $\vec{y}, \vec{x}$ – vector T-periodical time functions; $\vec{u}(t)$ – defined vector function of periodical disturbance (EMF and currents of power supplies). Note that the system (1) may include finite equations. In this case, the left sides of these equations written in the form (1) are equal to zero.

The solution of the system of DEs (1) is not a set of coordinates, but their functional dependencies during the period that could be one or more, and, moreover, some of them may be unstable. The problem of calculation of a steady state with the periodic disturbance is to find the periodical dependencies of vector coordinates

$$\vec{x}(t) = \vec{x}(t+T).$$

3. Problem solution algorithm.

The major place in solving boundary value problems is taken by projection methods. The main point of those methods is that the approximate solution of a DE, describing the periodical mode, is obtained as the projection of infinite functional space into a finite subspace defined by the linear combination of basic functions. These functions can be trigonometric or ordinary polynomials, spline functions, etc. The method for obtaining approximation formulas in which the basic functions are splines of third order [9], is considered on the example of a scalar equation in a following form

$$\frac{dy(x,t)}{dt} = f(y,x,t). \qquad (2)$$

Let us apply a grid with $n+1$ nodes on the period *T*, the first of the nodes being at the beginning of the period and the last one at the end and forming *n* sections. On each of them the coordinates of a vector $\vec{y}$ are approximated with a cubic spline of a following form

$$y(t) = a_j + b_j(t_j - t) + c_j(t_j - t)^2 + d_j(t_j - t)^3, \qquad (3)$$

where $j = \left(\overline{1,N}\right)$ – number of section which is determined by the node number on its right boundary; a_j, b_j, c_j, d_j – spline coefficients. The first and second derivatives of the function (3) are determined by formulas

$$\frac{dy(t)}{dt} = -b_j - 2c_j(t_j - t) - 3d_j(t_j - t)^2, \qquad (4a)$$

$$\frac{d^2y(t)}{dt^2} = 2c_j + 3d_j(t_j - t), \qquad (4b)$$

The conditions of spline continuity and its first and second derivatives have the following form

$$a_{j-1} = a_j + b_jh_j + c_jh_j^2 + d_jh_j^3;$$
$$b_{j-1} = b_j + 2c_jh_j + 3d_jh_j^2; \qquad (5)$$
$$\tilde{n}_{j-1} = \tilde{n}_j + 3d_jh_j,$$

where $h_j = t_j - t_{j-1}$.

In general, the grid of nodes on the period can be irregular, condensing nodes on time intervals where the radical change of coordinates is expected. However, since generally the character of their changes is not known beforehand, the grid can be considered regular. The increase of nodes on the period does not cause the significant increase of calculations, as the Jacobi matrix of the obtained system of algebraic equations is poorly filled, and it allows working only with nonzero elements.

In the case of an even step $h_j = h$, a connection between vectors $\vec{a} = (a_1,\ldots,a_n)^*$ and $\vec{c} = (c_1,\ldots,c_n)^*$, whose components are the corresponding spline coefficients (an upper index (*) denotes transposition), is determined by the equation

$$H_1\vec{a} = H_2\vec{c} \qquad (6)$$

taking into account the conditions of periodicity

$$a_{j+1} = a_j, \; c_{j+1} = c_j,$$

where

$H_1 = \frac{1}{h} \times$

3	−6	3			
	3	−6	3		
		3	−6	3	
			3	−6	
3				3	−6
−6	3				3

$H_2 = h \times$

1	4	1			
	1	4	1		
		1	4	1	
			1	4	1
1				1	4
4	1				

Similarly, connection between vectors $\vec{b} = (b_1,\ldots,b_N)^*$ and $\vec{c}$ is defined as follows

$$H_3\vec{b} = H_4\vec{c}, \qquad (7)$$

where

$H_3 =$

−1			⋯			1
1	−1⋮		⋯			
⋮	⋮	⋮		⋮	⋮	⋮
			⋯	1	−1	
			⋯		1	−1

$H_4 = h \times$

1			⋯			1
1	1		⋯			
⋮	⋮	⋮		⋮	⋮	⋮
			⋯	1	1	
			⋯		1	1

From (6), (7) the equation is obtained

$$H_4 H_2^{-1} H_1 \vec{a} = H_3 \vec{b}\ , \qquad (8)$$

defining the connection between vectors $\vec{a}$ and $\vec{b}$.

Thereby instead of a scalar DE (2) the system (8) of algebraic equations of *n*-th order is obtained. If $\vec{a} = \vec{y}$, $\vec{b} = -\vec{f}$, where $\vec{y} = (y_1, ..., y_n)^*$, $\vec{f} = (f_1, ..., f_n)^*$ – vectors, whose components are the corresponding node values, equation (8) takes the form

$$H_4 H_2^{-1} H_1 \vec{y} = -H_3 \vec{f}\ , \qquad (9)$$

or in a reduced form

$$H\vec{y} = D\vec{f}\ . \qquad (10)$$

4. Calculation algorithm.

Let us consider the problem of the calculation of a steady-state mode for the electric circuit shown on Fig. 1. The circuit consists of two identical capacitors, three nonlinear inductive elements with mutually inductive connections and identical nonlinear Weber-Amper characteristics $\psi = \psi(i)$; it is fed by a sinusoidal current power supply J .

Compiled by the Kirchhoff laws, the equation of electric balance have the form

$$\frac{d\psi_a}{dt} - \frac{d\psi_b}{dt} = -r_a i_a + r_b i_b + u_{kab}$$

$$\frac{d\psi_a}{dt} - \frac{d\psi_c}{dt} = -r_a i_a + r_c i_c - u_{kca}\ ;$$

$$\frac{du_{kab}}{dt} = \frac{i_{kab}}{C}\ ; \qquad (11)$$

$$\frac{du_{kCA}}{dt} = \frac{i_{kca}}{C}\ ;$$

$$i_a + i_b + i_c = 0;$$

$$i_a + i_{kab} - i_{kca} = 0\ ;$$

$$i_c + i_{kca} = J\ .$$

where $\psi_a, \psi_b, \psi_c, r_a, r_b, r_c$ – flux linkage and active resistances of corresponding inductors.

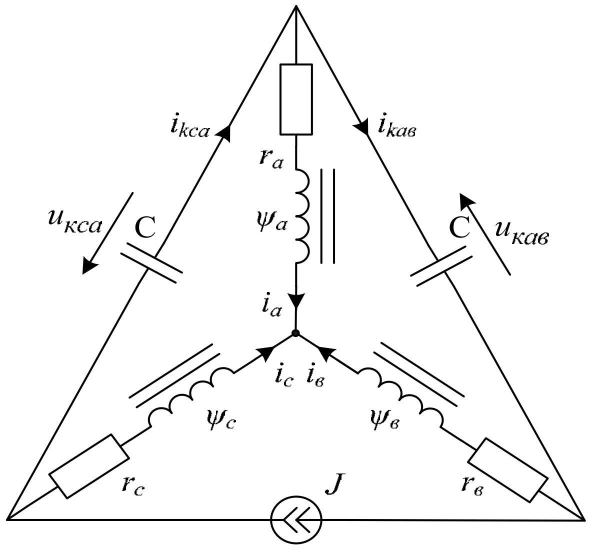

Fig. 1. Electric circuit diagram.

Designed according to the above algorithm, the algebraic analogue of DE system (11) consisting of the vector equations of the form (10) can be presented as a matrix equation

$$S\vec{Y}_c = P\vec{F}_c\ , \qquad (12)$$

where *S* and *P* – block-diagonal matrices, whose blocks are matrices *H* and *D*, and $\vec{Y}_c = (\vec{y}_1, ..., \vec{y}_n)^*$, $\vec{F}_c = (\vec{f}_1, ..., \vec{f}_n)^*$ – vectors consisting of the node values of vectors $\vec{y}_j$ and $\vec{f}_j$ accordingly.

The solution of the nonlinear system (12) is a vector of nodal coordinate values $\vec{X}_c = (\vec{x}_1, ..., \vec{x}_n)^*$, where $\vec{x}_j = (i_{aj}, i_{bj}, i_{cj}, u_{kabj}, u_{kcaj}, i_{kabj}, i_{kcaj})^*$.

Determining the vector $\vec{X}$ requires the use of iterative methods, the most effective of which is the iterative Newton method with a quadratic convergence. However, it is necessary to have the initial value of the vector $\vec{X}$, located in the neighborhood of the iterative method convergence, and it is a separate task to be solved.

One of the most reliable ways to obtain the initial value is a differential method, the essence of which is in following.

Let us select the vector of electromotive forces $\vec{U}_c = \left(\vec{u}_1,...,\vec{u}_n\right)^*$, causing the flow of currents in a circuit, in the right part of equation (12) and consider them as a sum of vectors

$$\vec{F}_c = \vec{Z}_c\left(\vec{Y}_c, \vec{X}_c\right) + \vec{U}_c .$$

As a result, equation (13) takes the form

$$S\vec{Y}_c = P\left(\vec{Z}_c + \vec{U}_c\right). \qquad (13)$$

Obviously, each value of vector $\vec{U}_c$, which consists of node values of the rms in the circuit of current source, corresponds to the value of the node vector $\vec{X}_c$ determining the steady periodical mode. To find the solution of the system (13), which corresponds to a given value of electromotive force (current *J*), using differential method, the vector $\vec{U}_c$ is multiplied by scalar parameter ε

$$S\vec{Y}_c - P\vec{Z}_c = \varepsilon P\vec{U}_c \qquad (14)$$

and a resulting equation is differentiated by ε. In this case, let's consider $\vec{Y}_c = \vec{Y}_c\left(\vec{X}_c, \varepsilon\right)$, as well as $\vec{Z}_c = \vec{Z}\ \left(\vec{Y}_c, \vec{X}_c, \varepsilon\right)$. As a result the DE is obtained

$$\left(\left(S - P\frac{\partial\vec{Z}_c}{\partial\vec{Y}_c}\right)\frac{\partial\vec{Y}_c}{\partial\vec{X}_c} - \frac{\partial\vec{Z}_c}{\partial\vec{X}_c}\right)\frac{\partial\vec{X}_c}{\partial\varepsilon} = P\vec{U}_c, \qquad (15)$$

where $\frac{\partial\vec{Y}_c}{\partial\vec{X}_c}$, $\frac{\partial\vec{Z}_c}{\partial\vec{Y}_c}$, $\frac{\partial\vec{Z}_c}{\partial\vec{X}_c}$ – block-diagonal matrices in which the blocks are matrices, whose elements are determined by the values of the parameters in the *j*-th node and do not depend on their values in other nodes:

$$\left.\frac{\partial\vec{y}}{\partial\vec{x}}\right|_j = $$

$L_{aaj} - L_{baj}$	$L_{abj} - L_{bbj}$	$L_{acj} - L_{bcj}$				
$L_{baj} - L_{caj}$	$L_{bbj} - L_{cbj}$	$L_{bcj} - L_{ccj}$				
			1			
				1		

$$\left.\frac{\partial\vec{z}}{\partial\vec{x}}\right|_j = $$

$-r_a$	r_b		1			
	$-r_a$	r_c		-1		
					$1/C$	
						$1/C$
1	1	1				
1					1	-1
		1				1

$$\left.\frac{\partial\vec{z}}{\partial\vec{y}}\right|_j = $$

			1			
				-1		

After integrating the system of DE (15) by ε by a numerical method, a set of vectors $\vec{X}_c$ is obtained (each of them is specified by the Newton method). They correspond to the increase of vector $\vec{U}_c$ proportionally to ε. Differential inductances $L = \partial\psi / \partial i$ for the corresponding node values of currents are determined by the characteristics $\psi = \psi(i)$.

Fig. 2 displays the results of calculating the periodical dependencies of currents i_a and i_b, and Fig. 3 shows the voltages on capacitors u_{kab} and u_{kca} with $J = 33{,}8\sin(\omega t - \pi/2)$, $C = 20\mu kF$.

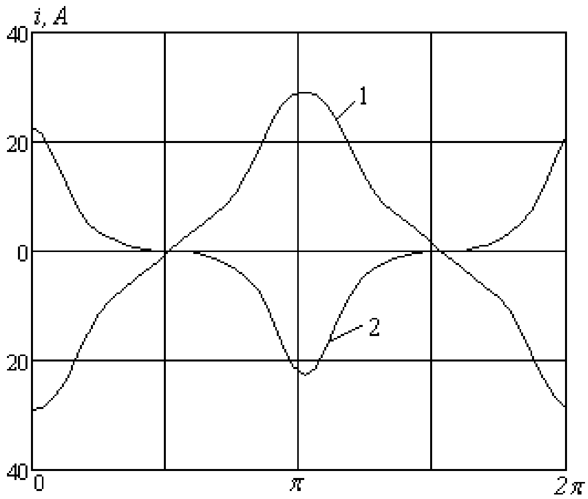

Fig. 2. Periodical dependencies of currents i_a (1) and i_b (2).

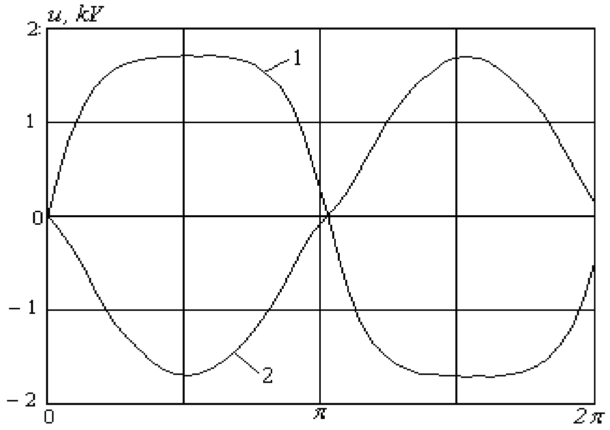

Fig. 3. Periodical dependencies of voltages u_{kab} *(1) and* u_{kca} *(2).*

5. Conclusion

The described algorithm allows us to perform the calculation of static periodical electromagnetic modes in nonlinear circuits in the timeless area. Moreover, the problem is solved as the boundary value problem for a first-order nonlinear DE system, which describes steady dynamic states in the electrical circuit. The algebraic representation of the DE is made through the approximation of state variables of cubic splines. In contrast to the steady method, whereby the problem is solved in the time domain, this algorithm allows finding all the possible periodic modes for given circuit and investigating the effects of changing any parameter of the circuit on the nature of processes in it.

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ЕФЕКТИВНИЙ АЛГОРИТМ РОЗРАХУНКУ СТАЦІОНАРНИХ РЕЖИМІВ НЕЛІНІЙНИХ ЕЛЕКТРОМАГНІТНИХ КІЛ

Петро Стахів, Василь Маляр, Ірина Добушовська

Розглянуто проблему розрахунку стаціонарних режимів у нелінійних електромагнетних колах з реактивними елементами, які перебувають під дією періодичних вимушувальних сил. Задача розв'язується як крайова для системи диференціальних рівнянь першого порядку з періодичними крайовими умовами, які описують динамічний усталений режим, що дає змогу отримати залежності змінних стану на періоді, не вдаючись до розв'язування задачі в часовій області.

Petro Stakhiv – Ph.D., D. Sc., Professor, born in 1948 in Lviv region, Ukraine. In 1970 he graduated from Lviv State University, Department of Physics, Ukraine, and received a M.Sc. degree in Radio Physics and Electronics. From 1970 to 1973 he was a postgraduate student at the Department of Theoretical Electro and Radio Engineering.

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