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Experimental research on the influence of the production quality of tooth operation surfaces of cylindrical gearwheels and torque on a gear oscillation amplitude

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Abstract. For carrying out experimental research a proving stand with closed power flow was designed. For the determination of oscillation process parameters (amplitude and frequency) the devices with input signals from strain sensors were used. Planning the experimental research and mathematical treatment of the results obtained was performed with the use of a full factorial experiment. The results of the experiment allowed for obtaining a nonlinear mathematical model for determining the oscillation amplitude and the amplitude-frequency characteristic of gear box oscillations (first harmonic). It then is used for determining the harmonic of the oscillating process caused by the inaccuracy of a tooth operating profile, which influences the smoothness of gear operation. The dependences taking into account internal and external dynamic loads from oscillating processes during the calculations of teeth gear hardness were obtained.

Key words: proving stand, oscillating processes, measuring equipment, full factorial experiment, amplitude-frequency characteristic.

SETTING THE TASK

During the operation process of the tooth gear, dynamic phenomena appear in its mechanical system, that considerably influence the operational characteristics of driving and other constituent elements of a tooth gear transferring relevant torque. Appearing at the process of rotary axis motion, dynamic phenomena cause oscillating processes in a mechanical system. These processes influence the operational characteristics of the structure. The main factor causing oscillations in the tooth gears is the inaccuracy of production of tooth gears, shafts, bearings, boxes, assembly imprecision etc.

The force oscillating frequency in the process of transferring tooth gear torque at some operation speed can coincide with the natural frequency of a whole system. In this case resonant phenomena emerge that quite often lead to the failure of mechanical system units. But in scientific and technical literature the sufficient analysis of these phenomena hasn’t been provided, particularly concerning their effect on the smoothness of tooth gear operation. Therefore, there occurs a need to carry out experimental research on these phenomena using a proving stand and to reveal the amplitude-frequency characteristic of the oscillating the tooth gear greatly depending on the inaccuracy of the tooth operating profile of tooth gearwheels. In addition, while solving this problem, it is necessary to compare the results obtained in practice with the results of theoretical research and to choose optimal parameters for using them in the computations of contact fatigue of tooth gears (namely, of tooth active faces and their bending fatigue).

ANALYSIS OF RECENT ACHIEVEMENTS AND PUBLICATIONS

The examples of the first research in the field of dynamics of reductor tooth gears were shown in the works of such scientists, as A.I. Petrusevych [1, 2] and such foreign authors, as Niman H. and Rettig H. [3].

Existing techniques of investigating the dynamical processes of reductor tooth gears in most cases concern involute gear systems. In [4] the tooth gear oscillations are considered, including high-speed gear pairs, parallel shaft gear units, planetary gearboxes. For each of these groups, dynamical mathematical models in the form of differential equations are given, with instructions on how to investigate them with the help of a computer and with examples of calculating tooth gear external and internal oscillation frequencies. The parameters of the dynamic models and damping forces are shown. In addition, the methods of reducing tooth gear vibration activity at the expense of lowering excitation forces, vibration isolation and avoiding resonant modes are considered. The theoretical part of this work is well elaborated, but such issues of engineering methodology, as determining the influence of dynamical processes on the change of forces and torques, are not considered here.

In [5-8] research on tooth kinematics and dynamics is considered, showing the beat pattern of wheel advance angle in the case of shaft misalignment or shaft obliquity. The analysis of drive operation enabled to draw a conclusion that when the advance angle is positive, it causes radial runout, and when the advance angle is negative, it causes axial runout. A.I. Petrusevych suggested a technique for determining an impact force for the involute gears. The development of tooth gears considering the influence of dynamical forces, including impact ones, is a quite difficult task, because the values and patterns of the change of the dynamic load are influenced by numerous factors. So, the investigation of oscillating processes in the teeth gears of reductors is the important task of the machine-building industry.

The aim of the work is to develop the method of the determining the amplitude-frequency characteristic of tooth gearwheel oscillations, conditioned by the inaccuracy of tooth operating profile, its surface roughness and the torque value on the basis of the result of the research done. Another method is to be determined for taking into consideration the dynamical phenomena of the oscillating process during the calculation of the contact fatigue of tooth gears (namely, of tooth active faces) and their bending fatigue.

SOLVING THE SET TASK

For investigating the oscillating processes the proving stand with closed power flow was designed and constructed [6]. The kinematic diagram of the stand is shown in the Fig.1, and its general view is shown in Fig.2.

The main units of this stand (Fig.1) are reductors 1 and 2, which contain testing tooth gearwheels z_1 and z_2, which form a testing reductor, and gearwheels z_3 and z_4 form a closing reductor. Tooth gearwheels z_1 and z_3 are connected together by a shaft 6 with the help of rigid flange couplings 5, and gearwheels z_2 and z_4 are connected together by the shaft 7 with cardan joints. The reductors are operated by a drive from an electric motor *D* with the help of a pin flexible coupling.

The reductor 1 is fixed on a mattress, and the reductor 2 is mounted on the bearings and two supports 8. Supports 8 are also fixed on the mattress. Loading gearwheels is realized by the principle of closed power flow when the redactor 2 is loaded by the moment *M=FL* (*L* = 650 mm), which is created by a lever 4 with a weight 3.

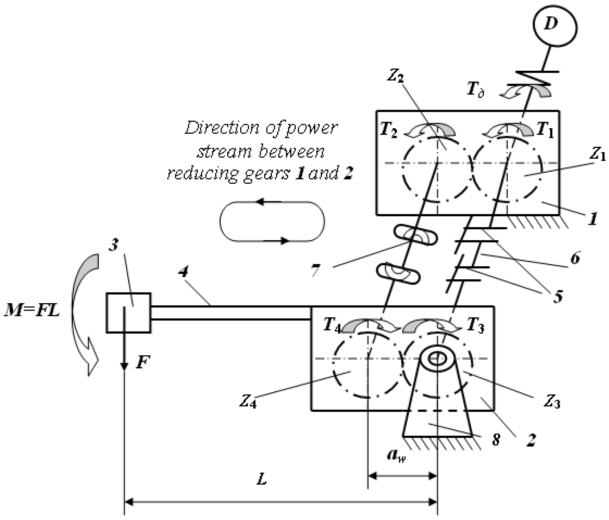

Fig. 1. Kinematic diagram of the testing stand with closed power flow

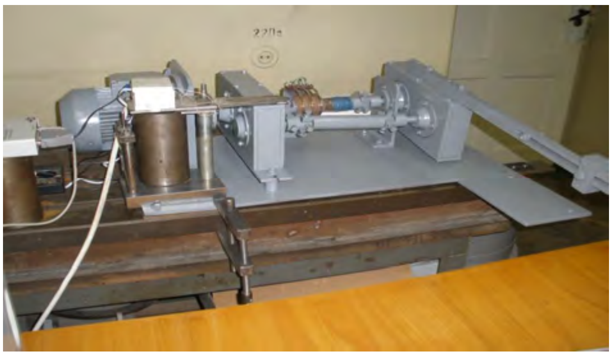

Fig. 2. Testing stand

Tooth gearwheels are loaded by the moment *M=FL*, namely:

$$T_3 = \frac{M}{\left(1 + \frac{z_4}{z_3}\frac{1}{h_{34}}\right) h_{12} h_{34}},$$

$$T_4 = \frac{M}{1 + \frac{z_3}{z_4} h_{34}},$$

$$T_1 = T_3 h_5 ; \quad T_2 = T_4 h_7 ,$$

$$T_{\partial} = \frac{M(1 - h_{12} h_{34})}{\left(1 + \frac{z_4}{z_3}\frac{1}{h_{34}}\right) h_{12} h_{34}},$$

where: $h_{12.}, h_{34}$ – are efficiency factors between the tooth gearwheels of relevant gears; η_5 is the efficiency factor of the rigid flange couplings 5; η_7 – is the efficiency factor of the cardan drive 7.

Rotary motion of the tooth gearwheels is realized by the asynchronous generator A32-4 (*D*) with nominal power P_∂ =1 kW and rotational speed n_∂ = 1410 min^{-1}. Its power is used only for overcoming frictional forces at toothing and in couplings.

While developing the research techniques for carrying out the research on the testing stand, the main attention was paid on following questions.

Firstly it is necessary to determine the influence of the ridge tip height Δ on the tooth operating surface of the driving wheel z_1 (profile error), conditioned by the gear milling technology with the use of hob cutters with a certain tooth number z_ϕ according to its additional angular motion, by a value $\Delta\varphi = f(\Delta)$ in the range 0,00005…0,0004 rad.

Such angular error corresponds to the error of the tooth profile f_{fr} of the wheel in accordance with the state standard GOST 1643-81 from 21,7% to 86,9% of the tolerance band, that is, from wheel accuracy level 6 to 8 at different meanings of the torques T_1 and T_2, which are determined by the moment $M = FL$. In addition, the surface roughness of the tooth operating profile R_a also influences the oscillation processes in the tooth gearwheel.

For determining T_1 on the shaft 6 (Fig.1) four tension sensors were glued on between tooth gearwheels z_1 and z_3 of the right and left redactor. They were stuck at the angle of 45^0. Two of them (R_1 and R_3) were directed right-hand, and another pair (R_2 and R_4) – left-hand, all being connected into a half-bridge circuit. The half-bridge circuit, then, was connected to a strain gauge installation "УТ-4" with the help of a current-collector. The signal from the strain gauge installation was input to microammeters, used for fine balancing of the bridge circuit together with the strain gauge installation. The microammeters were connected directly to the analog-to-digital converter USB300.

For obtaining, processing and storing the results of measurements in a PC, software «PowerGraph» was used.

For determining a value T_1 the calibration of values T_1 was made by the deflection of a cursor, caused by the loading force F, that is, by the moment M. Loading was performed using mass 3, fixed on the lever 4 (Fig.1). Then a force $F = mg$, N (where m is the loading mass, kg; g is free fall acceleration, m/s^2). For creating the loading moment M = 150 Nm, the mass m = 23,5 kg ($M = FL$ = 23,5·9,81·0,65 = 150 Nm) was used. At this load the cursor deflected by 40 mm.

For determining the parameters of the oscillating process (amplitude and frequency), the beam of uniform strength was used, with tension sensors being glued on the beam and connected into a bridge circuit. A signal from the bridge circuit was input to an amplifier and then to a recorder, to the ADC USB300 and the computer. A computer active window is shown in Fig.3.

The beam of uniform strength is fixed on the support, which is connected with the mattress. For determining the parameters of the oscillating process (the amplitude-frequency characteristic) the beam of uniform strength, fixed on the support, was connected to a gear box mounted rigidly on the mattress.

The full factorial experiment (FFE) was used for planning the experimental research and mathematical processing of the results obtained [11-13]. The task is set to determine the influence of the error of the tooth operating profile $\Delta\varphi$, the roughness of the tooth operating surface R_a and torque T_1, which is spent for overcoming the useful load, on the oscillation amplitude value a. The tests were performed with the use of spurs $b = 0^0$, produced of steel 40X and a module m_n = 4 mm with a number of teeth $z_1 = z_3 = 30$ i $z_2 = z_4 = 30$ ($a_t = 20^0$) and a gear face b = 20 mm. Tooth gearwheels after thermal treatment (improvement) had hardness values as follows: for a driving wheel HB$_1$ = 245…280; for a driven one HB$_2$ = 215…235. The gearwheel teeth were toothed using hobbing cutters m_n = 4 mm made of quick-cutting steel P6M5 with a grade of accuracy AA and a number of teeth z_ϕ = 6…10.

The axle spacing of the reductors of the testing stand is a_w = 120 mm, and gear ratio u = 1. During the period of investigations the lubrication of the reductor tooth gearwheels on the testing stand was done using a lubricant CT – 20, being considered the most effective one [14, 15].

The relationship between the amplitude a and the factors influencing its value can be shown in such a way:

$$a = C_a \Delta j^{\,n} R_a^m T_1^p \,, \qquad (1)$$

where: C_a – is a proportionality constant; n, m, p are unknown power exponents.

The dependence (1) is non-linear according to the factors included. For passing to a linear dependence the logarithm of it should be found, and it gets a form:

$$\ln a = \ln C_a + n \ln \Delta f + m \ln R_a + p \ln T_1 .$$

Let us introduce a following notation:

$\ln a = \tilde{y}$; $\ln C_a = b_0$; $n = b_1$; $\Delta\varphi = \tilde{x}_1$; $m = b_2$; $R_a = \tilde{x}_2$; $p = b_3$; $T_1 = \tilde{x}_3$.

Then we obtain:

$$\tilde{y} = b_0 + b_1 \ln \tilde{x}_1 + b_2 \ln \tilde{x}_2 + b_3 \ln \tilde{x}_3 . \qquad (2)$$

The equation (2) is a postulated empirical model of the amplitude dependence on factors influencing its value. For determining the proportionality constant and power exponents let us use the FFE of a 2^3 type.

The levels of the change of the factors influencing the value of the oscillation amplitude are given in the Table 1.

For the reproducibility of measurements, let us set the number of repeated tests r = 2. The conditions of the experimental research and the results obtained are given in the Table 2.

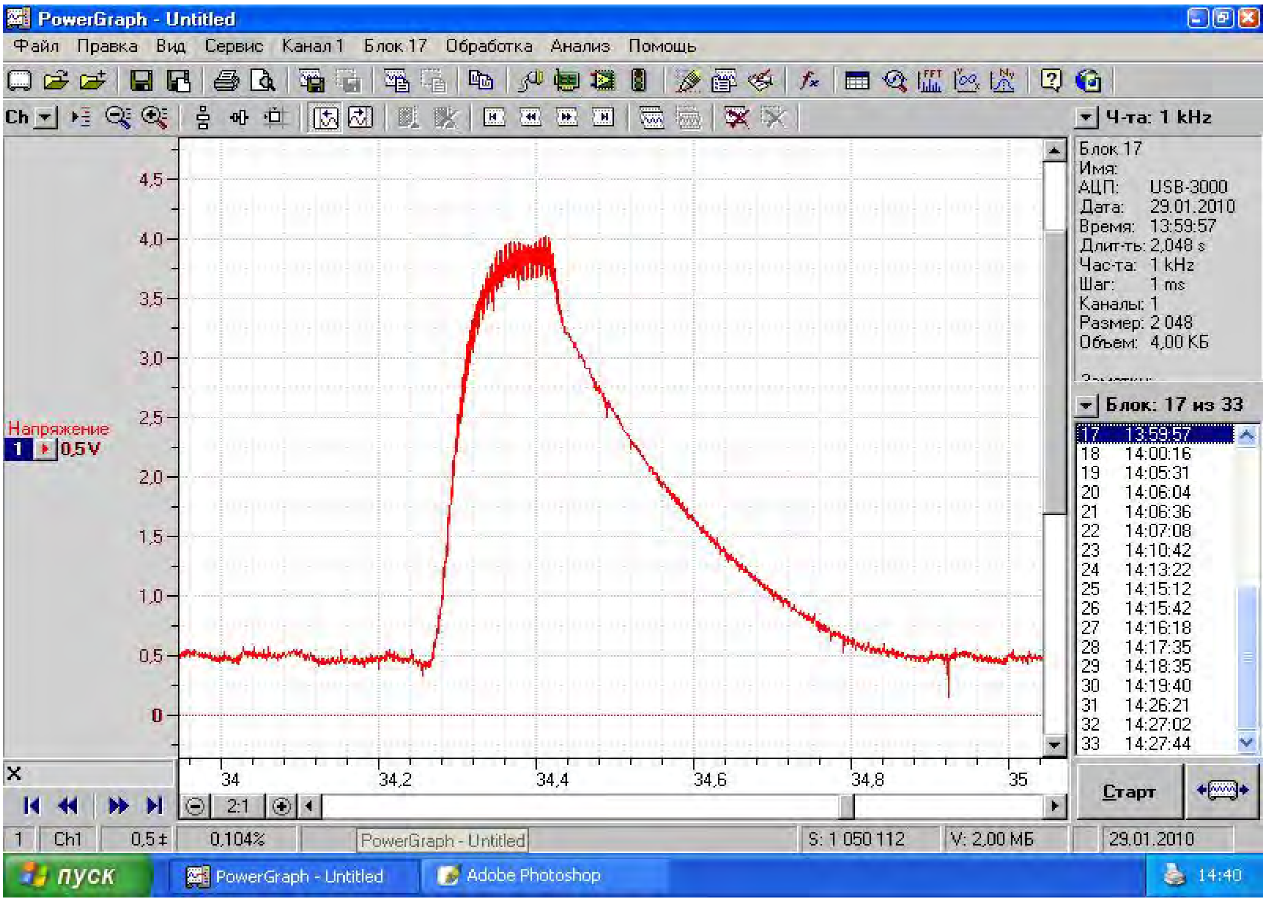

Fig. 3. Computer active window of the calibration process

Table 1. Factor changing levels

Factor levels	Δφ, rad		R_a, μm		T_1, N·m	
	$\tilde{x}_1$	$\ln \tilde{x}_1$	$\tilde{x}_2$	$\ln \tilde{x}_2$	$\tilde{x}_3$	$\ln \tilde{x}_3$
Upper level (+)	0,0004	-7,82	3,2	1,16	150	5,01
Zero (0)	0,000225	–	1,9	–	100	–
Lower level (–)	0,00005	-9,9	0,63	-0,46	50	3,91

Table 2. Conditions and results of the tests

Test	x_1		x_2		x_3		First test	Second test	Average value	$\ln \bar{a}$
	Код	Δφ, rad	Код	R_a, μm	Код	T_1, N·m	a_1, μm	a_2, μm	$\bar{a}$, μm	
1	+	0,0004	+	3,2	+	150	10	9	9,5	2,25
2	–	0,00005	+	3,2	+	150	8	9	8,5	2,14
3	+	0,0004	–	0,63	+	150	9	8	8,5	2,14
4	–	0,00005	–	0,63	+	150	7	8	7,5	2,01
5	+	0,0004	+	3,2	–	50	13	14	13,5	2,60
6	–	0,00005	+	3,2	–	50	12	11	11,5	2,44
7	+	0,0004	–	0,63	–	50	12	13	12,5	2,53
8	–	0,00005	–	0,63	–	50	11	10	10,5	2,35

During the process of testing the oscillation processes of the system related to the error of the tooth operating profile $\Delta\varphi$, the roughness of the tooth operating surface R_a and torque T_1 were recorded. They were recorded as oscillograms, which are shown in Fig.4 and Fig.5. The amplitude of the system oscillation a is related to the value of voltage changing in Volts (V) and the periodicity of the oscillation is given in microseconds (µs).

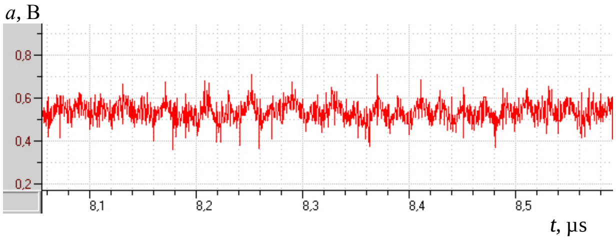

Fig. 4. Results of test 1 (Table 2)

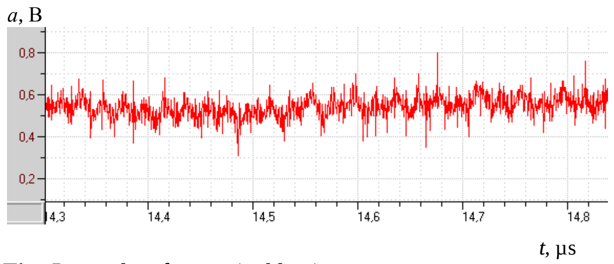

Fig. 5. Results of test 5 (Table 2)

After processing the results of FFE with the use of PC, we can obtain the average value of the amplitude $\bar{a}$ of the gear box oscillation in µm (the 1st harmonic component):

$$\bar{a} = \frac{67{,}16\Delta j^{\,0{,}07} R_a^{0{,}06}}{T_1^{0{,}31}}, \qquad (3)$$

where: $\Delta\varphi$ – is the error of the tooth operating profile, rad; R_a – is the roughness of the tooth operating surface, mm; T_1 – is the torque on the driving wheel, N·m.

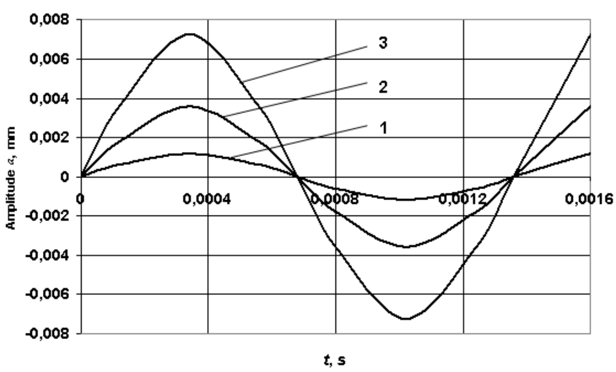

Fig. 6. Dependence of the oscillation amplitude on the error of the tooth profiles of driving and driven wheels: *1 – Δφ* = 0,0001 rad; *2 – Δφ* = 0,0004 rad; *3 – Δφ* = 0,0006 rad

For checking the correspondence of the results obtained during the experimental research they were compared with the results of theoretical research. The latter were obtained by the computing the common nonlinear differential equation system of motion with the help of the Runge-Kutta method, the software being developed in programming language Fortran-6. The results of the theoretical investigations are shown in Fig.6.

For obtaining the amplitude-frequency characteristics, which can take into account the results of the experimental research, let us follow guidelines from [16]. In this work (while considering trigonometric function-series) it is shown that the sums of the trigonometric functions make the possibility for simulations the great variety of excitations and the reactions of dynamical systems. It is determined, that a function $a = a(\varphi)$, where a is the oscillation amplitude and φ is the rotational angle of the tooth gearwheel, can be shown as:

$$a_{(k)} = c_k \sin \Omega_k t_i, \qquad (4)$$

with the satisfactory accuracy level, where:
$a_{(k)}$ – is an amplitude of the *k-th* harmonic component; c_k – is the proportionality constant; $\Omega_k = k\omega$; ω – is the angular velocity of the considered tooth gearwheel, s^{-1}; t_i – is time, s;

$$w = \frac{p\,n}{30},\ s^{-1},\quad t = \frac{2p}{z},\ \text{rad},\quad t_t = \frac{t}{w},\ s,$$

n – is a shaft speed, min^{-1}; τ – is the period of changing the amplitude; z – is a number of gearwheel teeth; t_τ – is the duration of one period.

For the analysis of the results of theoretical and experimental research the harmonics of the frequencies were considered. They are allocated as follows:

– the first harmonic is related to the gear box,

– the second harmonic refers to a bearing bushing, fixed to the gear box together with an external racer, pressed in the bushing,

– the third harmonic is related to the rolling elements of the bearing,

– the forth harmonic concerns an inner racer of the bearing,

– the fifth harmonic is related to the shaft, on which the tooth gearwheel is fixed,

– the sixth harmonic concerns the tooth gearwheel, whose amplitude-frequency characteristic is being determined,

– the seventh harmonic is related to the tooth operating profile of the gearwheel.

In our case it is necessary to consider the first, the sixth and the seventh harmonic. The first harmonic is analyzed with the use of dependences (3) and (4) at the nominal value T_1 = 150 N·m of the testing stand torque and the roughness of the tooth operating surfaces of the tooth gearwheels R_a = 0,63 µm (according to the requirements of the working drawings considering the tooth gearwheels of the general-purpose reductors) at two values of the tooth profile error: 1) $\Delta\varphi$ = 0,0001 rad (gear milling was performed by means of the hobbing cutter $z_ф$ = 10); 2) $\Delta\varphi$ = 0,0004 rad (gear milling was performed by means of the hobbing cutter $z_ф$ = 6).

For determining the amplitude-frequency characteristic of the first harmonic on the basis of the dependences (3) and (4) we obtain:

$$a_{(k)} = \frac{67{,}16\Delta j^{\,0{,}07} R_a^{0{,}06}}{T_1^{0{,}31}} \sin(kwt_i),$$
$$a_{(1)} = \frac{67{,}16\Delta j^{\,0{,}07} R_a^{0{,}06}}{T_1^{0{,}31}} \sin(1wt_i), \quad (5)$$

where: $\omega = \pi n_д/30$ = 3,14·1410/30 = 147,58 s^{-1}; t_i – is time, s; let us consider this time as the period from t_i = 0 till $t_i = t_\tau = \tau/\omega = 2\pi/(z\omega)$ = 2·3,14/(30·147,58) = = 0,0014 s.

In Fig.7 the amplitude-frequency characteristic of the first harmonic is given at the relevant values of the error of the tooth gearwheel profile.

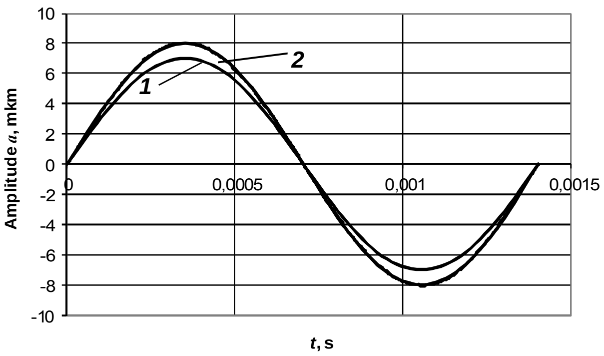

Fig. 7. Amplitude-frequency characteristic of the first harmonic (oscillations of the gear box) at the error of the tooth gearwheel profile: 1 – $\Delta\varphi$ = 0,0001 rad; 2 – $\Delta\varphi$ = 0,0004 rad

Using the curves (Fig. 7) and trend models, let us present the amplitude-frequency characteristic of the first harmonic as a 3rd degree polynomial:

$$a_{(1)} = c_1 t_i^3 + c_2 t_i^2 + c_3 t_i + c_4 ,$$

where: for $\Delta\varphi$ = 0,0001 rad: c_1 = 5·10^{10}; c_2 = –10^8; c_3 = 5469; c_4 = –0,5826,

for $\Delta\varphi$ = 0,0004 rad: c_1 = 6·10^{10}; c_2 = –10^8; c_3 = 62303; c_4 = –0,6614.

For determining the amplitude of the certain oscillation harmonic $a_{(k)}$ depending on the duration of the oscillation period on the basis of the polynomial obtained we use the Fourier series at the time interval [$-t_\tau$, $+t_\tau$] [16]. Then:

$$a_{(k)} = \sqrt{a_k^2 + b_k^2} ,$$

where: a_k – is the cosine index of the k-th harmonic / (k = 1; 2; 3; …); b_k – is the sine index of the k-th harmonic.

For the sixth harmonic (k = 6) the indexes a_6 and b_6 can be determined by the dependences [16]:

$$a_6 = \frac{1}{t_t} \int_{-t_t}^{t_t} (c_1 t_i^3 + c_2 t_i^2 + c_3 t_i + c_4) \cos \frac{6p}{t_t} t_i dt , \quad (6)$$

$$b_6 = \frac{1}{t_t} \int_{-t_t}^{t_t} (c_1 t_i^3 + c_2 t_i^2 + c_3 t_i + c_4) \sin \frac{6p}{t_t} t_i dt . \quad (7)$$

After the integration of the dependences (6) and (7) we obtain:

for $\Delta\varphi$ = 0,0001 rad

$$a_6 = \frac{c_2 t_t^2}{9p^2} = \frac{-10^8 \cdot 0{,}0014^2}{9 \cdot 3{,}14^2} = -2{,}21,$$

$$b_6 = -\frac{c_1 t_t^3}{3p} + \frac{c_1 t_t^3}{18p^3} - \frac{c_3 t_t^3}{3p} =$$
$$= -\frac{5 \cdot 10^{10} \cdot 0{,}0014^3}{3 \cdot 3{,}14} + \frac{5 \cdot 10^{10} \cdot 0{,}0014^3}{18 \cdot 3{,}14^3} -$$
$$-\frac{54691 \cdot 0{,}0014^3}{3 \cdot 3{,}14} =$$
$$= -14{,}56 + 0{,}25 - 0{,}000016 = -14{,}31.$$

In this case the oscillation amplitude in µm will be:

$$a_{(6)} = \sqrt{a_6^2 + b_6^2} = \sqrt{(-2{,}21)^2 + (-14{,}31)^2} = 14{,}5 .$$

A phase angle φ_k can be determined by the dependence:

$$j_6 = arctg \frac{a_6}{b_6} = arctg \frac{-2{,}21}{-14{,}5} = arctg(0{,}1526) = 8^0 40' ,$$

and frequency in Hz is proportional to the rotation frequency of the tooth gearwheel and to the number of the teeth:

$$w_6 = \frac{nz}{60} = \frac{1410 \cdot 30}{60} = 705 .$$

Accordingly, for $\Delta\varphi$ = 0,0004 rad we obtain: a_6 = –2,21; b_6 = –17,16; $a_{(6)}$= 17,3 µm; $\varphi_6 = 7^0 20'$.

Similar calculations are done for the seventh harmonic, that is, for oscillations conditioned directly by the error of the tooth operating profile. For performing them, let us determine the periods of the oscillations, caused directly by the ridges on the tooth operating surface. Then it should be mentioned, that the number of the ridges on the operating surface of each tooth depends on the tooth number of the hob cutter $z_ф$. In this case the time of the period of oscillation will be:

1) for $\Delta\varphi$ = 0,0001 rad ($z_ф$ = 10)

$$t = \frac{2p}{zz_ф} = \frac{2 \cdot 3{,}14}{30 \cdot 10} = 0{,}021, \text{ rad},$$

$$t_t = \frac{t}{w} = \frac{0{,}021}{147{,}58} = 0{,}00014 , \text{ s},$$

2) for $\Delta\varphi$ = 0,0004 rad ($z_ф$ = 6): τ = 0,035 rad; t_τ = 0,00024 s.

For the seventh harmonic (k = 7) the indexes a_7 and b_7 can be determined by the dependences:

$$a_7 = \frac{1}{t_t} \int_{-t_t}^{t_t} (c_1 t_i^3 + c_2 t_i^2 + c_3 t_i + c_4) \cos \frac{7p}{t_t} t_i dt ,$$

$$b_7 = \frac{1}{t_t} \int_{-t_t}^{t_t} (c_1 t_i^3 + c_2 t_i^2 + c_3 t_i + c_4) \sin \frac{7p}{t_t} t_i dt .$$

After the integration for $\Delta\varphi$ = 0,0001 rad:
a_7 = –0,016; b_7 = –1,16; $a_{(7)}$= 1,16 µm; $\varphi_7 = 0^0 47'$.
For $\Delta\varphi$ = 0,0004 rad: a_7 = –0,016; b_7 = –4,01;
$a_{(7)}$= 4,01 µm; $\varphi_7 = 0^0 13'$

Let us compare the results of the experimental research with the results of the theoretical research (Fig. 6) determining the relative error (relative to the experimental results). At the tooth profile error $\Delta\varphi$ = 0,0001 rad the theoretical result is $a_т$ = 0,0012 mm =1,2 µm, and the experimental result is a = 1,16 µm. Then the relative error is:

$$d = \frac{a - a_T}{a} 100\% = \frac{1{,}16 - 1{,}2}{1{,}16} 100 = -3{,}45\%.$$

At the tooth profile error $\Delta\varphi$ = 0,0004 rad: $a_т$ = 3,6 µm; a = 4,01 µm:

$$d = \frac{4{,}01 - 3{,}6}{4{,}01} 100 = 10{,}2\%.$$

Numerous calculations of the relative error have shown, tha it doesn't exceed 11,3%. This value is acceptable, and the results obtained can be used for the calculations of the of contact fatigue of tooth gears (namely, of tooth active faces and their bending fatigue) [17-19].

While doing the calculations of tooth gearwheel strength, internal dynamic loads are taken into consideration with the use of indexes K_{Hv} (contact strength of tooth active faces) and K_{Fv} (tooth folding strength), which are determined in the state standard ГОСТ 21354-87 [20] depending on the degree of accuracy and the smoothness standard of the gear operation, the hardness of the gearwheel teeth and tooth angle velocity. In addition, during the determining the indexes K_{Hv}, K_{Fv}, external dynamic loads are taken into account by introducing the index K_A, whose value depends on the loading mode of an engine.

On the basis of the results of the theoretical and experimental research the dependences have been obtained, taking into consideration internal and external dynamic loads during the calculations of the tooth gearwheel strength.

In this case:

$$K_{Hv} = 1{,}1 \exp\left[\frac{77{,}7\, l \cos b}{m_n z_1}\right] \cdot \exp\left[3 \cdot 10^{-6} \frac{w_1}{u} d_2\right],$$

$$K_{Fv} = 1{,}4 \exp\left[\frac{77{,}7\, l \cos b}{m_n z_1}\right] \cdot \exp\left[3 \cdot 10^{-6} \frac{w_1}{u} d_2\right],$$

where: $\lambda = \Delta + f_{pbr}$ – the error, conditioned by the profile error depending on the ridge tip height Δ on the tooth operating surface, and a tooth pitch error which also causes the dynamical loads, mm.

The ridge tip heights Δ (mm) are determined by the dependence:

$$\Delta = \frac{p^2 r_1 k^2}{2 z_\phi^2 z_1^2} + \frac{p^2 r_2 k^2}{2 z_\phi^2 z_2^2},$$

where: ρ_1, ρ_2 – are the radii of the involute curves of driving and driven wheels in the point under consideration, mm; z_1, z_2 – are tooth numbers of driving and driven wheels correspondingly; k – is a number of hob cutter passes; z_ϕ – is a number of the hob cutter teeth; β – is the gradient angle of the gearwheel teeth; m_n – is a normal module of the tooth gearwheel, mm; ω_1 – is the angular velocity of the driving tooth gearwheel with the number of the teeth z_1, s^{-1}; u – is the gear ratio of the tooth gearwheel; d_2 – is the reference diameter of the driven tooth gearwheel with the number of the teeth z_2 ($d_2 = m_n z_2/\cos\beta$), mm.

The tooth pitch error f_{pbr} is taken into consideration during the calculations of the value λ replacing it by the acceptable value f_{pb} in accordance with the necessary accuracy level of the tooth gear and the index of operating smoothness.

CONCLUSIONS

1. The results of the research with the help of the full factorial experiment on the testing stand enabled us to obtain non-linear mathematical model for determining the oscillation amplitude depending on the operating profile error of the gearwheel teeth, their roughness and the torque value.

2. On the basis of the non-linear mathematical model for determining the oscillation amplitude the amplitude-frequency characteristic of the gear box oscillations (the first harmonic) has been obtained.

3. With the use of the amplitude-frequency characteristic of the first harmonic the harmonics of tooth gear oscillations have been determined, including the harmonic of the oscillations caused by the tooth gearwheel errors, influencing on the gear operation smoothness.

4. The research results made it possible to obtain the dependences for determining the influence of the treatment of the tooth operating surfaces of the gearwheels on the internal and external dynamic loads of the tooth gearwheel while calculating its fatigue strength.

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