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The mathematical model of material particle motion

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Abstract. This article is concerned with the problem of high-performance dust-catching equipment creation for various industries, where: fine-dyspersated dust fractions are emitted, in order to make their emissions conform to sanitary norms. This article elucidates new tendencies in the area of the creation of equipment for air from dust cleansing, which are based on the use of centrifugal inertia forces.

Key words: dust collecting , air cleaning, pollution, centrifugal, cyclone.

INTRODUCTION

Ecological aspects are well-known to those, who have a notion of air environment state in the rooms, where:, for instance, welding fumes are given off. The emitted hazardous substances comprises gases and aerosols, some particles of which are so tiny, that can get into the blood, penetrating through the lung tissue. In the most frequent cases welding fume contains the particles of iron, zinc, cadmium, manganese oxides, as well as the particles of fluorine, asbestos, chromium, copper etc [1]. As a result of such particles action, the conjunctivae are traumatised, allergic diseases, silicosis, pulmonary oedema, headaches, stethalgias arise, kidneys destructs and cancerous diseases emerge.

The application of local exhaust ventilation ensures the required MPC level in the worker's breathing zone during various production processes, and that is controlled by the legislation of all countries in the labour protection and ecology sphere.

The Latest Researches Analysis. Electrostatic precipitators are the most widespread and are used in different spheres due to high catching degree of the most hazardous particles from 1 to 0,01 microns and less in size. At this point, such agents of serious diseases as microbes, viruses, bacterium, pathogen fungi and pests simply perish in the filter's electrostatic field. Thus electrostatic filters compare favourably with the mechanic ones, the elements of which accumulate in themselves hazardous particles and with bad care of the filter they can become the pollution sources themselves [2].

Electrostatic filters are effectively used for the cleansing of air from the particles of various fume types, oil mist and fine-dyspersated dust, from 200 to 0.01 microns in size. For this reason the air from dust cleansing efficacy reaches 94%.

Air filters with the mechanic method of air filtration are used for polluted air cleansing from big particles of different dust types, oil mist, welding fume, which is emitted in the process of galvanised steel, aluminium, stainless steel welding and galvanics, and also from the fume, which is emitted in the process of soldering and spot welding. Mechanic filters have high degree of air cleansing from the particles from 200 to 0,1 microns in size.

Dust catching units (dry cyclones) with the mechanic method of air filtration are used for polluted air cleansing from medium- and coarse-dyspersated particles of various dust types. According to the efficacy suggested units refer to the 3 class air filters, which catch the dust particles more than 10 microns in size.

Instrumental and laboratory measurements determined that at initial dust content of the atmosphere of 5,3 g/m^3 the efficacy of the first degree cyclones cleansing is 81-85%, of bag filters – 99%, and general unit's ECE – 99,8%. The application of dusty air dry cleansing in the high-performance apparatuses of the present day design allows us to secure the reduction of atmospheric emissions to 20-40 mg/m^3.

Analysing the above-mentioned, we can precisely determine that currently for the security of sanitaryhygiene requirements of environmental protection there is no apparatus support for the creation of hazardous release norms. The best of the units, existing for this purpose, are not able to cope with this task. Therefore we made it our aim to create the units, which are able to highly effectively catch fine-dyspersated dust.

The development of technology and constantly growing requirements for energy and material saving results in the need for the creation of brand-new air-cleansing devices with higher energy conversion efficiency and simultaneously with lower material expenditures on the cleansing itself. The analysis of dust-air mixture motion within the separator has a considerable influence on the substantiation of new type dust catchers basic parameters. Despite the external simplicity of such apparatuses design, aerodynamic processes, taking place within, are extremely complicated and up to this day there are no full mathematical models of dusty air behavior in separator. Thus, the tasks of newly created apparatuses aerodynamics research have both theoretical and practical values.

The significance of problem lies in the lack of complete scientific theory of dust-cleansing process, which would meet the requirements to air from dust cleansing degree.

RESULTS AND DISCUSSION

The motion of material part M will be referred to the cylindrical coordinate system. Point M coordinates are determined by the radius r, azimuth and applicate z. The azimuth is positive when it is reckoned from the polar axis Ox to the radius r in a counterclockwise direction.

The radius r indicates the distance from the axis Oz to the point M, and obtains only positive values.

The applicate z indicates the point M position in regard to the axis Oz.

The coordinates origin, point O, is located on the symmetry axis of the frame at the top of it [3-5].

The point M velocity r, j, z n cylindrical coordinate system is determined as the sum of three components $\vec{V}_z, \vec{V}_j, \vec{V}_z$ i.e.:

$$\vec{V} = \vec{V}_z + \vec{V}_j + \vec{V}_z, \quad (1)$$

where: $eV_r = \frac{dr(t)}{dt}$ – the point M (r, j, z), radial velocity, which is directed along the radius r, i.e. along the line the point M (r, j, z), transverse velocity, which is directed perpendicular to the line O_1M and the vector $\vec{V}_j$ is perpendicular to the axis *Oz*; [6-8]:

$$V_z = \frac{dz(t)}{dt}, \quad (2)$$

– the point M (r, j, z), velocity about the axis *Oz* and it is directed parallel to the axis ***Oz***.

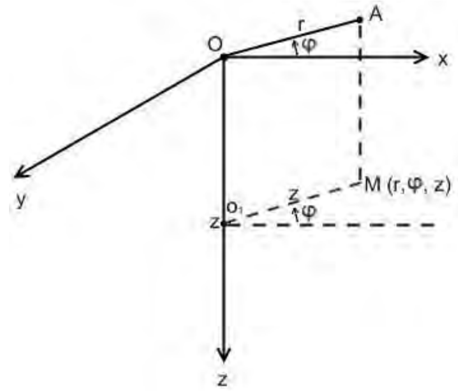

Fig. 1. The axis symmetry of the case

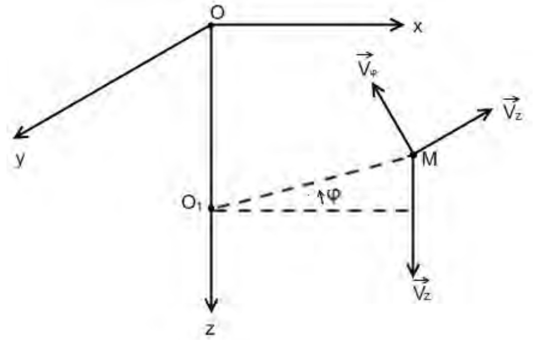

Fig. 2. Point M in cylindrical coordinate system

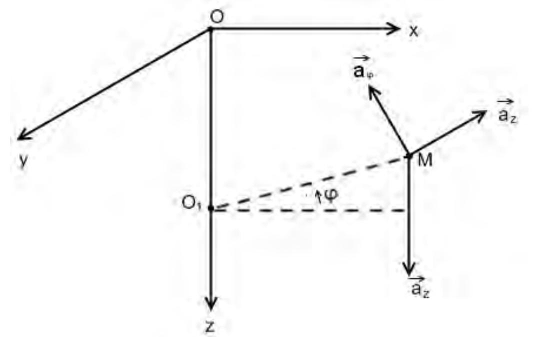

Fig. 3. The point M (r, j, z), acceleration about the axis *Oz*

The point M (r, j, z) acceleration in cylindrical coordinate system is equal to the sum of three vector components: $\vec{a} = \vec{a}_r + \vec{a}_j + \vec{a}_z$,

where:
$$a_r = \frac{d^2r(t)}{dt^2} - r\left(\frac{dj}{dt}\right)^2, \quad (3)$$

radial acceleration component, which is directed along the line O_1M:

$$a_j = r\frac{d^2j(t)}{dt^2} + 2\frac{dj(t)}{dt} \cdot \frac{dr(t)}{dt}, \quad (4)$$

– transverse acceleration component, which is directed perpendicular to the line O_1M and the vector $\vec{a}_j$ is perpendicular to the axis *Oz*; [9-11]:

$$a_z = \frac{d^2z(t)}{dt^2}. \quad (5)$$

In the process of material particle motion inside the apparatus frame it is exerted by: $\vec{P}$ – the material particle weight and $\vec{R}$ – the force, caused by air-dust mixture. According to the Second Law Of Dynamics we shall write:

$$m\vec{a} = \vec{P} + \vec{R}. \qquad (6)$$

Projecting this equation on the axis of cylindrical coordinate system, we shall obtain:

$$ma_r = -R_r,$$
$$ma_j = +R_j \cdot sign(w_0 - \dot{j}), \qquad (7)$$
$$ma_z = P - R_z,$$

where: m – material particle mass.

In the process of material particle motion it is affected by air-dust mixture, its affecting is characterised by the force $\vec{R}(R_r; R_j; R_z)$ [11-14].

The component R_r is determined by using a formula:

$$R_r = C_r \cdot V_r^2 \cdot F_r \cdot \rho, \qquad (8)$$

where: C_r – the coefficient, which accounts for material particle aerodynamic properties due to the component of the motion along the radius r; V_r – radial component of the material particle motion velocity; F_r – maximum material particle sectional area of a plane, perpendicular to its radial velocity direction; r – air-dust mixture density:

$$R_\varphi = C_\varphi \left(\frac{\upsilon_0}{r_0} - \dot{\varphi} \right)^2 \cdot r^2 \cdot F_\varphi \cdot \rho, \qquad (9)$$

where: C_φ – the coefficient, which accounts for material particle aerodynamic properties due to the component of the motion towards $\vec{V}_\varphi$ [15]; V_0 – the velocity of material particle when entering the inlet fitting; r_0 – the distance from material particle at that instant of time, when it is situated at the entry to the inlet fitting, to the apparatus symmetry axis; F_φ – maximum material particle sectional area of a plane, perpendicular to its transverse velocity component direction:

$$R_z = C_z \left(\frac{dz}{dt} \right)^2 \cdot F_z \cdot \rho, \qquad (10)$$

where: C_z – the coefficient, which accounts for material particle aerodynamic properties due to the component of the motion about the axis Oz; F_z – maximum material particle sectional area of a plane, perpendicular to the axis Oz.

The function sign $(\omega_0 - \dot{\varphi})$ is depicted by the following dependence:

$$\sin g(\omega_0 - \dot{\varphi}) = \begin{cases} +, \text{якщо}\ \omega_0 - \dot{\varphi} > 0 \\ -, \text{якщо}\ \omega_0 - \dot{\varphi} > 0 \end{cases}. \qquad (11)$$

Taking into consideration (3) – (11), the material particle motion is depicted by the differential equation system [16]:

$$m\left(\frac{d^2 r(t)}{dt^2} - r\left(\frac{d\varphi}{dt} \right)^2 \right) = -C_r \left(\frac{dr}{dt} \right)^2 \cdot F_r \cdot \rho, \qquad (12)$$

$$m\left(r\frac{d^2\varphi(t)}{dt^2} + 2\frac{d\varphi(t)}{dt} \cdot \frac{dr(t)}{dt} \right) =$$
$$= C_\varphi \left(\frac{\upsilon_0}{r_0} - \dot{\varphi} \right)^2 \cdot r^2 \cdot F_\varphi \cdot \rho \cdot sign(\omega_0 - \dot{\varphi}), \qquad (13)$$

$$m\frac{d^2 z(t)}{dt^2} = mg - C_z \left(\frac{dz}{dt} \right)^2 \cdot F_z \cdot \rho. \qquad (14)$$

Let us specify the initial conditions for material particle: $\varphi(0) = 0$ – at the initial instant of time the material particle was situated on the polar axis Ox; $\dot{\varphi}(0) = \frac{v_0}{z_o}$ – the material particle rotational velocity about the axis Oz at the initial instant of time; r(0) – the material particle initial distance to the axis Oz; $\dot{r}(0) = 0$ – at the initial instant of time the material particle radial velocity is equal to zero; z(0)=0 – the material particle was situated at the upper casing; $\dot{Z}(0)=0$ – at the initial instant of time the material particle vertical velocity is equal to zero;

The differential equation system (12), (13) is solved by means of software.

The differential equation (14) is solved by using the separation of variables method. Let us write this equation as [17-18]:

$$m\frac{d\dot{z}}{dt} = mg - C_z F_z \rho \cdot \dot{z}^2; \quad \frac{d\dot{z}}{dt} = g - \frac{C_z F_z \rho}{m} \cdot \dot{z}^2.$$

Let us introduce the notation: $K^2 = \frac{C_z F_z \rho}{m}$ then

$$\frac{d\dot{z}}{dt} = g - k^2 \dot{z}^2, \frac{d\dot{z}}{g - k^2 \dot{z}^2} = dt : \frac{1}{2\sqrt{g}} \left(\frac{d\dot{z}}{\sqrt{g} - k\dot{z}} + \frac{d\dot{z}}{\sqrt{g} + k\dot{z}} \right) = dt,$$

$$\frac{1}{2k\sqrt{g}} \ln \frac{\sqrt{g} + k\dot{z}}{\sqrt{g} - k\dot{z}} = t + \frac{\ln c_1}{2k\sqrt{g}}, \quad \frac{\sqrt{g} + k\dot{z}}{\sqrt{g} - k\dot{z}} = C_1 e^{2k\sqrt{g}t}.$$

The constant magnitude C_1 is determined by application of the initial condition $\dot{z}(0)=0$, then:

$$\frac{\sqrt{g} + k \cdot 0}{\sqrt{g} - k \cdot 0} = C_1^{2k\sqrt{g} \cdot 0}, \quad \frac{\sqrt{g}}{\sqrt{g}} = C_1; \quad C_1 = 1.$$

Therefore, the velocity of material particle motion about the axis Oz is equal to:

$$\sqrt{g} + k\dot{z} = \left(\sqrt{g} - k\dot{z} \right) e^{2k\sqrt{gt}} : k\dot{z}\left(1 + e^{2k\sqrt{gt}} \right) = \sqrt{g}\left(e^{2k\sqrt{gt}} - 1 \right),$$

$$\dot{z}(t)=\frac{\sqrt{g}\left(e^{2k\sqrt{g}t}-1\right)}{k\left(e^{2k\sqrt{g}t}+1\right)}:\dot{z}(t)=\sqrt{\frac{mg}{C_zF_zP}}\cdot\frac{e^{2\sqrt{\frac{CzFz\rho g}{m}}t}-1}{e^{2\sqrt{\frac{CzFz\rho g}{m}}t}+1}. \qquad (15)$$

Let us consider, that material particles are in the form of the ball, then:

$$m=\frac{4}{3}\pi r_m{}^3\cdot\rho_m:F_r=F_\varphi=F_z=\pi r_m{}^2,$$

where: r_m – the ball radius; ρ_M – the material particle specific mass. The differential equation system (12), (13) acquires the form [19]:

$$\frac{d^2r(t)}{dt^2}-r(t)\left(\frac{d\varphi(t)}{dt}\right)^2=-\frac{3C_r\rho}{4r\rho_m}\left(\frac{dr(t)}{dt}\right)^2, \qquad (16)$$

$$r(t)\frac{d^2\varphi(t)}{dt^2}+2\frac{d\varphi(t)}{dt}\cdot\frac{dr(t)}{dt}=$$

$$=\frac{3C_r\rho}{4r_m\rho_m}\left(\frac{V_0}{r_0}-\frac{d\varphi(t)}{dt}\right)^2r^2sign\left(\omega_0-\dot{\varphi}\right). \qquad (17)$$

Setting certain values for the magnitudes:

$0{,}5\le C_r\le 1$; $0{,}5\le C_\varphi\le 1$, $\rho=1,\dots$, $r_m=0{,}05$ *м*,

$\rho_m=2$, $\frac{кг}{м^3}$, $V_0=4$, $м/c$, $r_0=0{,}8$ *м*,

we can determine the instant time of material particle contact with the apparatus casing walls with provision that:

$$r(t_k)=r_a,$$

where: r_a – the radius of apparatus casing.

The velocity of material particle motion about the axis *Oz* is depicted by the equation:

$$\dot{z}(t)=\sqrt{\frac{4r_m\rho_m g}{3C_z\rho}}\cdot\frac{e^{\sqrt{\frac{3C_z\rho g}{r_m\rho_m}}t}-1}{e^{\sqrt{\frac{3C_z\rho g}{r_m\rho_m}}}+1}. \qquad (18)$$

Solving this differential equation, in view of the initial condition that z(0)=0, we shall obtain the coordinate, on which the material particle contacts the casing wall, to accomplish this we use the condition that h=z(t_k) [20].

CONCLUSIONS

Thus, the performed theoretical analysis and suggested by us model allows [21]:

1 – to reveal the physical essence of the motion of the air, being cleansed, in the projected apparatus, to determine the influence of the forces, which affect the particle in radial direction on its motion nature;

2 – to reduce dramatically the number of experimental researches on the study of apparatus parameters influence on air cleansing efficacy and conduct them purposefully;

3 – to create fundamentally new designs of centrifugal inertia dust-catchers.

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