# Взаємозв'язок між розташуванням шкіл та вплив викладання у моделі Айткіна-Лонгфорда

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Зміни, які мали місце в Польщі після 1989 року, істотно вплинули на освіту загалом та освіту молоді. Основними організаційними змінами було впровадження трьорівневої навчальної системи, тобто початкової школи, молодшої середньої школи, а потім старшої середньої/вищої школи, а згодом перехід на стадії середньої школи від професійної освіти на користь викладання в загальних вищих школах. Змінам також піддались моделі та стереотипи стосовно місця проживання. Зростання рівня безробіття та розвиток ринку нерухомості викликали зростаючі часті міграції. З одного боку, пошук місця роботи вів у напрямку великих міст, а з іншого – більш заможні люди почали шукати можливості осісти за межами шумних, великих центрів міст, обираючи села та маленькі містечка поблизу промислових та наукових центрів.

Ці процеси міграції вплинули на процес освіти молодого покоління. Переїжджаючи з батьками, діти частіше міняли своє шкільне середовище. Такі зміни можуть мати або позитивний, або ж негативний вплив на викладаня. У даній статті викладена спроба аналізу впливу освіти на викладання на різних маршрутах, що представляють найпоширеніші напрямки міграції. Аналіз було проведено з використанням моделі випадкових ефектів Айткіна та Лонгфорда.

Дані, використані в експериментальній частині стосуються учнів, які склали у 2010 році випускний екзамен після вищої школи - так званий, екзамен на атестат зрілості. Порівняння включає результати тестів шестикласників, екзаменів у молодшій середній школі та на атестат зрілості. З одного боку, результати тестів можна вважати натяком стосовно розташування шкіл, а з іншого – допомогою в прийнятті рішення щодо зміни місця проживання.

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# The relation between location of schools and effects in teaching in the model of Aitkin-Longford

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***The modern socio-economic situation and, in particular, migrations highlight the issue of training quality depending on the location of the school. It happens that students who change their place of stay change the environment in which they learn. These changes may affect the results of training measured by national tests. The content of the article is an analysis of these effects in connection with the location of the school.***

**Keywords** – education, educational added value, education system, migration, panel data, the effectiveness of teaching.

## I. Introduction

The purpose of the article is an analysis of the increase of knowledge of pupils depending on the different types of paths of education. The student who goes to primary school in the village can continue learning at lower secondary level in the countryside or in the city. The next step in education is a upper secondary school, where you can see migrations in the directions of larger or smaller towns.[5] Drawing on surveys carried out by the Carpita, D'Ambra, Vichi and Vittadini (2006), it has been showed how the increase of knowledge looks like in case of different types of locations in which the student completes the school. A student may graduate the primary school in a village, and the later stage of education (lower sevcondary school, high school) continue in the city. The aim will be to show how the increase of knowledge changes depending on the various types of the designated experiences.

The applied model was described by M. Aitkin and N. Longford in the article "Statistical Modeling Issues in School Effectiveness Studies". The article "Statistical Modeling Issues in School Effectiveness Studies" presents several models for testing the effectiveness of education in American schools, using the difference in points between the results of the examination achieved by the pupils graduating the school, and the intelligence quotient as measured before the beginning of the study at the school. In this article the model of random effects is used – described as model no. five by M. Aitkin and N. Longford.

In empirical studies the results of the following exams were used: "sixth grader's test" taken at the end of primary school, two parts of lower secondary school exams (humanist and mathematical-environmental) and final exam after high school from Polish language and mathematics, only for students taking the exam in 2010 in Polish schools. The research is based on two types of

[5] Of course, in part of these cases, migrations between various cities are a necessity arising from the fact that, in the vicinity of the lower secondary school there is not another high school.

models. The first concerns the humanist[6] part and the second model focuses on the exact sciences[7]. An important part of the work is the mathematical description of the model by Aitkin and Longford.

## II. Aitkin-Longoford Model

In the next part of the work the following indications shall be used:

- $x_{ij}$ the number of input points obtained by $i$ the student whose „path of career" (a type of transition) is described by the index $j$,
- $y_{ij}$ the number of points obtained at highschool final examination by $i$ the student whose „path of career" (a type of transition) is described by the index $j$,
- $n_j$ number of students in $j$-type of transition,
- $k$ number of analysed objects – types of transition ($k = 21$),
- $n$ number of all students, i.e. $n = n_1 + \ldots + n_k$,
- $j$ number of types of transitions $j \in \{1, \ldots, 21\}$,
- $\bar{x}_j$, $\bar{y}_j$ average score, respectively input and output at the level of $j$-type of transition.

The data used in the article is called unbalanced data or panel data. Panels are unbalanced, when the number of observations for each object (schools) is different i.e. $n_j$. The used model is a random factor model. In economics, this model owes its popularity thanks to the article by Balestr and Nerlove (1966) dealing with the demand for natural gas. This Model is a:

$$y_{ij} = a + b\,x_{ij} + x_j + e_{ij}\,. \qquad (1)$$

In this model it is assumed that $e_{ij}$ is a random variable with the distribution of ($N(0, s^2)$), and $x_j$ a random variable with distribution $N(0, s_I^2)$. Additionally, it is assumed that random components originated from different schools for different students are nor correlated and the individual random component $x_j$ is uncorrelated with the random component $e_{ij}$, i.e. $E(x_j, e_{is}) = 0$. From the assumption it results that

$$\mathrm{var}(y_{ij}) = s_I^2 + s^2,\ \mathrm{cov}(y_{ij}, y_{pj}) = s_I^2,$$

$$r = cor(y_{ij}, y_{pj}) = s_I^2 / (s_I^2 + s^2).$$

The coefficients of the model specified in such a way is estimated using the highest reliability (Atkin, Longford (1986)). Formulas expressing the estimated parameters can be found in the works (Baltagi (2005), Ejsmont (2009)), where the entire algorithm for estimating components of variance $s^2$ and $s_I^2$ also described in detail. In order to verify whether the generated random effects are viable, the test of Breusch-Pagan is used (Hasio (1999), Baltagi (2005)).

Component $x_j$ says how far diverges the averaged result of $j$-type transition from the averaged result of the whole population.[8] To establish the value of component $x_j$ the formula of mean squared error is used (Jakubowski, Sztencel (2004)). Because the components $s^2$ and $s_I^2$ are known before the establishment of the model so that you can use this information as information a priori. Next, the distribution of the conditional random variable $x_j$ is determined, under condition $\bar{y}_j$ (Bayes approach). From the formula (1) the average at the level of $j$-type transition is in the form of $\bar{y}_j = a + b\,\bar{x}_j + x_j + \bar{e}_j$. With $\bar{y}_j$ assumptions made the distribution is normal $N(a + b\,\bar{x}_j, s_I^2 + s^2/n_j)$ (a priori distribution). Besause $x_j$ is a random variable with the distribution of $N(0, s_I^2)$, the conditional distribution $f(x_j / \bar{y}_j)$ will also be a normal distribution. Therefore, after using the general conditional properties of the expected value it is that $f(x_j / \bar{y}_j)$ has the form distribution of $N\left(r n_j^*(\bar{y}_j - a - b\,\bar{x}_j), n_j^*(1-r) s_I^2 / n_j\right)$, where $n_j^* = w_j / (1-r)$ and $w_j = n_j s^2 / (s^2 + n_j s_I^2)$.

Comparison of the analysed groups will be based on a comparison of the average values of the obtained conditional distribution. Hence the effect of training or education value added (EVA) can be defined as:

$$e_j = \hat{r}\,n_j^*(\bar{y}_j - \hat{a} - \hat{b}\,\bar{x}_j)\,. \qquad (2)$$

## III. Data description

In each of the following types of taken examinations the same convention regarding scaling of the data was applied. Both lower secondary and high school final examination points are scaled to level 100, i.e. the students' results from lower secondary and high school final examinations were divided by the maximum number of points possible to acquire and multiplied by 100. This allows easy interp-retation of the results obtained. You can easily see whether a pupil of improved, or worsened its score (%).

The data that was analysed represents three different stages of learning. The first two stages that represent the primary school and lower secondary school were divided into four types of locations i.e.. the village (marked with 1), a city of up to 20 k. inhabitants (2), a city from 20 to 100 k. inhabitants (3), and more than 100 k. residents (4). The third category are high school students, but due to the rarity of high schools in the countryside they were divided into two categories, a town of less than 20 k. inhabitants (marked with 1), and more than 20 k. residents (2). Table 1 shows average results depending on the previously described, different stages of learning. The first digit of

[6] That is, the results of the "six grader's test", lower secondary school examinations – humanist subjects, and the results of the final exam after high school from Polish language at elementary level.

[7] That is, the results of the "sixth grader's test", lower secondary school examinations – mathematics and environment, and the results of the final exam after high school from mathematics at elementary level.

[8] Diagram showing the idea of measuring the increase in the knowledge model Aitkina-Longforda can be seen in the article (Skrondal, Rabe-Hesketh (2008), p. 96).

the column, type of transition, indicates in what type of place the primary school student graduated the school, the second digit, lower secondary school, the third digit, high school. For example, the type of transition 342 must be read in such a way that the student completes primary school (3) in a town from 20 to 100 k. inhabitants, lower secondary school (4) in a city of more than 100 k. inhabitants and high school in a town of more than 20 k. inhabitants. Certain types of transitions were excluded from the analysis due to too small number of observations (e.g. 131). Finally, in each of the group the data concerns at least 500 students.

*Table 1*

**Average results of the various types of examinations provided for students taking "matura" (high school final examination) in 2010 and graduating high schools**

| RP | LU [%] | Average results | | | | |
|---|---|---|---|---|---|---|
| | | Primary school | Lower secondary school | | High school | |
| | | | H-S | M-E | PL | M |
| **111** | **9,26%** | **71,51** | **72,77** | **57,26** | **60,40** | **61,78** |
| **112** | **17,6%** | **75,98** | **76,71** | **63,24** | **63,76** | **67,37** |
| 121 | 2,00% | 73,41 | 71,06 | 55,75 | 60,24 | 62,00 |
| 122 | 1,46% | 77,13 | 76,73 | 63,14 | 63,90 | 67,67 |
| 132 | 1,38% | 77,47 | 73,91 | 60,91 | 63,65 | 67,73 |
| 142 | 0,75% | 81,04 | 76,41 | 66,17 | 66,48 | 71,26 |
| **221** | **9,93%** | **71,92** | **72,49** | **56,91** | **60,96** | **63,59** |
| **222** | **5,26%** | **77,73** | **77,90** | **65,80** | **65,14** | **70,40** |
| 231 | 0,35% | 71,95 | 70,90 | 55,45 | 58,83 | 63,52 |
| 232 | 1,03% | 76,78 | 74,62 | 62,15 | 64,63 | 68,44 |
| 242 | 0,41% | 80,85 | 74,45 | 63,78 | 66,84 | 72,64 |
| 312 | 0,33% | 75,34 | 75,93 | 63,80 | 63,14 | 68,45 |
| 321 | 0,32% | 72,93 | 72,28 | 57,48 | 59,66 | 61,49 |
| 322 | 0,37% | 77,14 | 76,78 | 65,19 | 64,47 | 70,70 |
| 331 | 1,24% | 73,24 | 73,85 | 59,49 | 61,14 | 64,85 |
| **332** | **19,6%** | **76,22** | **75,72** | **62,26** | **64,12** | **68,41** |
| 342 | 0,99% | 78,40 | 74,97 | 62,52 | 65,04 | 68,87 |
| 422 | 0,28% | 77,56 | 78,00 | 64,25 | 64,47 | 70,70 |
| 432 | 0,60% | 79,28 | 76,55 | 64,71 | 66,42 | 70,93 |
| 441 | 0,28% | 72,69 | 71,82 | 54,82 | 61,33 | 60,53 |
| **442** | **26,4%** | **79,13** | **77,81** | **65,82** | **65,22** | **70,23** |
| Σ | 100% | 76,17 | 75,80 | 62,41 | 63,63 | 67,54 |

where:

- H-S - secondary school examinations – humanist subjects,
- M-E - lower secondary school examinations – mathematics and environment,
- P - the final exam after high school from Polish language at elementary level,
- M - the final exam after high school from mathematics at elementary level.

Three most numerous groups of pupils (63,75% of the total) are 442 - students ending all analysed stages of learning in large cities, 332 and 112 graduating primary school and lower secondary school, and then continue in a large city. Bold letters indicate all more numerous groups of students forming approximately 89% of the population. Eventually, considerations were limited to twenty-one of the most common educational paths.

## Conclusion

Figs. 1 and 2 illustrate the effectiveness of teaching in the case of successive stages of the education presented in Table 1. The colour blue is the indicator of increase of knowledge in the lower secondary school, and colour red - increase of knowledge in high school. These two results are summed and divided by two within each analysed category and presented in the chart with colour green. This figure may represent an overall increase of knowledge in the course of study at school.

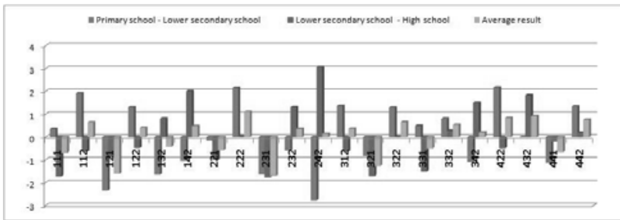


Fig. 1. Education Value Added calculated for subsequent stages of the education from table 1 for the humanistic part

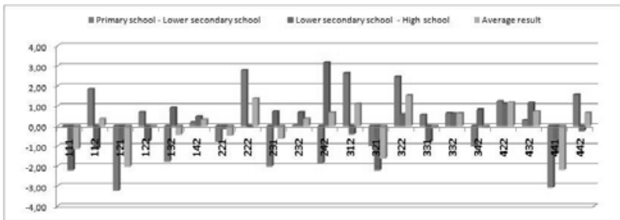


Fig. 2. Edacation Value Added calculated for subsequent stages of the education from table 1 for science part

The first element, which can be noted is an intuitive conclusion that the good result of pupils leaving lower secondary school is not always associated with a high increase of knowledge in high school. Studies show that in all large groups (with more than 10 k. pupils), achieving a high increase in knowledge in lower secondary school led to a small increase at the stage of high school education. This results, of course, from the fact that it is more difficult to achieve any growth from an already high level. In this context, result obtained on the educational path marked 242 is not surprising. There, in turn, starting at a low level it is easy to get a good result in the next stage of education. Any background weakens, However, the fact that the number of students included in this category is so small, less than half a per cent of the population, weakens all generalizations.

Interpretation of the results shown in Fig. 3 is obvious. The location of the lower secondary school in rural areas does not affect negatively the quality of education. No matter where the students recruited from, their growth of knowledge was greater than the one obtained in other centres. This probably results from a smaller environmental impact on younger students, as well as broader background of staff prepared to engage in teaching activities at this level.

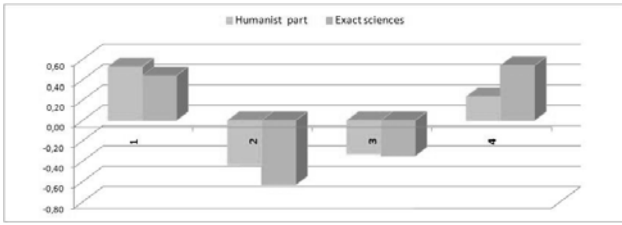


Fig. 3. The growth of knowledge in lower secondary school for graduating students (lower secondary school):

- in a village – 1
- in a small town up to 20 k. – 2
- in a town from 20 to 100 k. – 3
- in a big city – 4

Fundamentally different collusion is drawn from the analysis of the comparisons presented in Figs. 4 and 5. No matter where candidates for high schools located in small towns or villages recruit from, the development of their knowledge in this environment is considered in a negative sense. It seems that because of parents who decide to change their place of residence and move to a small town, their children in high school age are bound to have educational problems. This situation stems directly from qualifications of teachers who have negative effect on the children's education. These findings can also be used as an argument in the discussion about the viability of university off-campus teaching centres.

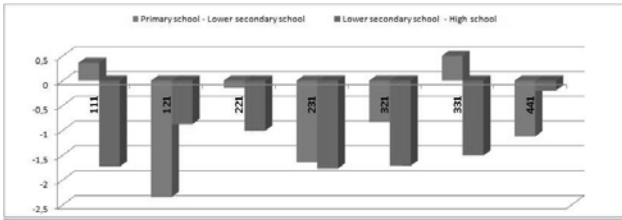


Fig. 4. The growth of knowledge for students graduating high school in a small town – humanistic part

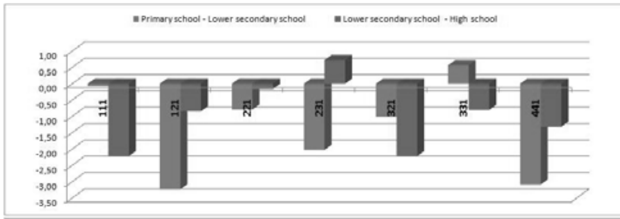


Fig. 5. The growth of knowledge for students graduating high school in a small town – science part

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