

# Mathematical model and software to recover blurry graphic images

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*Abstract – The analysis of methods for removing blur from graphic images was accomplished. A suitable mathematical software was developed. The algorithm of restoring images corrupted by blur and noise was developed.*

Key words – deconvolution, Wiener filter.

## I. Relevance of the Problem

Blurred images can be obtained from all types of visualization starting from microscopy to astronomy. Numerous different things can cause blur and hide the details on the image - camera movement, camera defocusing, light absence. Some of these images can be obtained only once, therefore it is important to have the possibility to remove the blur and restore the image without new artifacts or noise.

Recovering corrupted images is one of the most interesting and important problems in image processing - both from theoretical and practical points of view.

## II. Analysis of the Known Results

The task of blur removing algorithm is to restore as many as possible details lost or distorted.

Lucy-Richardson method is one of the most popular and implemented in some programs, such as Matlab Processing Toolbox. It requires the known blurring function. But even with well-selected blur feature, this method can cause noise and edge effects.

Another popular algorithm is the Wiener filter. It is useful when the distortion includes noise, that is why it is suitable for real images obtained, for example from the camera.

## III. Task set

The main objective of the project is to develop software based on one of the deconvolution algorithms. The program should allow the image restoring distorted due to blur, smearing or defocusing.

Application operation of the distorting function to another function (the image in this case) is called the convolution, i.e. a certain output image area minimizes to one pixel of the distorted image. [1] Mathematically, an  $f$  image  $M \times N$  of size and  $h$  distorting function  $m \times n$  of size is written as:

$$g(x, y) = h(x, y) * f(x, y) = \sum_{i=-a}^a \sum_{j=-b}^b h(i, j) f(x+i, y+j), \quad (1)$$

where  $a = (m-1) / 2$ ,  $b = (n-1) / 2$ .

Reverse to curtailing operation is called deconvolution and solution of this task is quite nontrivial.

The curtailing operation in the spatial area is equivalent to usual multiplication in the frequency area (with elementwise multiplication, not matrix). [1]

Correspondingly, the operation inverse to convolution is equivalent to the division in the frequency area, this can be written as:

$$h(x, y) * f(x, y) \Leftrightarrow H(u, v) F(u, v), \quad (2)$$

where  $H(U, V)$ ,  $F(U, V)$  - Fourier figures of the corresponding functions. So the process of the formula distortion can be rewritten in the frequency area as:

$$G(U, V) = F(U, V) \cdot H(U, V) + N(U, V)$$

where  $F$  is the Fourier transformation of the "ideal" version of the image,  $H$  is the blur function,  $N$  - noise.

As already mentioned, the best way to solve the smearing problem is to use the Wiener blur filter. This tool solves  $F$  estimation according to the following formula:

$$F(U, V) = |H(U, V)|^2 \cdot G(U, V) / (|H(U, V)|^2 \cdot H(U, V) + K(U, V)), \quad (3)$$

$K$  is an experimentally optimized constant.

## IV. Software Designing

To implement the algorithm the diagram of classes has been developed. Diagram is shown in Figure 1.

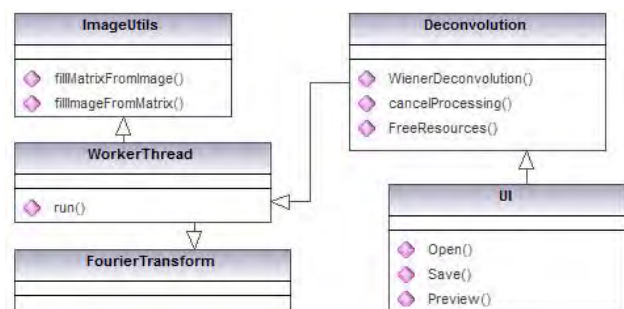


Fig. 1. Class diagram

The developed diagram shows only basic classes and methods.

Class UI is the main and responsible for the user's graphical interface. The process of deconvolution is started and controlled in it through the Deconvolution class.

Worker Thread class implements work flow, in which computation will be fulfilled. ImageUtils class contains helper functions necessary for working with

images. Fourier Transform class implements the Fourier transformations.

### Conclusion

This article describes a mathematical model and algorithm of development software to restore images corrupted by blur and noise.

### References

- [1] Gonzales R.C., Woods R.E. "Digital Image Processing" Prentice Hall; 2nd edition (January 15, 2002)
- [2] Shan Q., Jia J., Agarwala A. "High-quality motion deblurring from a single image" ACM Transactions on Graphics, 2008