Numerical simulation for the dynamics of the nanoparticles driven by Poisson white noise

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Abstract – In this work we numerically simulated the dynamics of the nanoparticles driven by Poisson white noise in the case of a quadratic potential. In the equation, which described the motion of the nanoparticles, the components of Poisson white noise are represented as a random variables distributed according to the Poisson law. Take it into account, we obtained the mathematical algorithm, based on which, we got the probability density distribution of the nanoparticles that are located in the potential well. The results of numerical simulation are presented on the graph, where they are compared with the previously obtained analytical solution.

Key words – Poisson white noise, probability density distribution, potential well.

I. Introduction

Nowadays, the numerical simulation of the different physical problems, which cannot be described by the analytical methods, is very actual. Using the development of mathematical algorithms we can realize in-depth researches of dynamics of the studied system and get new scientific results. In this work we considered one of this task.

The investigation of the effects of Poisson white noise is of great importance, because it more adequately describe the environmental influence on the various systems than the Gaussian noises. But the Poisson white noise is more complex, therefore we cannot use only analytical methods for the investigation it influence on the particle.

The purposes of our work are to simulate the dynamics of the nanoparticles driven by Poisson white noise using the numerical methods, in the case, when particles are located in the potential well.

II. Numerical simulation

In this paper we consider the simplest model of a single-domain ferromagnetic nanoparticle driven by Poisson white noise in the case of a quadratic potential. It means that the particle is located in the potential well and on its behavior is effected only specific noise – Poisson white noise.

It is well known that this noise is a random sequence of δ-pulses and it defined as [1, 2]:

$$\xi(t) = \sum_{i=1}^{n(t)} z_i \delta(t - t_i). \tag{1}$$

Here $n(t)$ is a Poisson counting process with the probability $P(n(t) = n) = (\lambda t)^n e^{-\lambda t} / n!$ of $n \geq 0$ arrives in the interval $(0,\tau]$, λ is the rate of the process, z_i are independent zero-mean random variables with the some probability density $q(z)$, $\delta(t)$ is the δ function and t_i are the event times. It is assumed also that $\xi(t)=0$ if $n(t)=0$. The noise generating process $\eta(t)$ is a step-wise constant Markov process whose increments $\delta\eta(t) = \int_t^{t+\tau} dt' \xi(t')$ are given by

$$\delta\eta(t) = \begin{cases} 0, \ n(\tau) = 0 \\ \sum_{i=1}^{n(\tau)} z_i, \ n(\tau) \geq 1 \end{cases} \tag{2}$$

Today the random Poisson processes are widely used in modern mathematical simulation of different nature phenomenum. In the physics these processes are generated by the random one-time or successive displacements of atomic and subatomic particles, in biology – by the angular momentums in the nerve fibers, in queuing theory – by the moments of receipt of applications for maintenance. In general, we can say that the random processes, which characterized by a Poisson distribution, are used in many fields of natural sciences, technology and the economy in those cases, where the sequence of events, that occur at specific points in space or at any particular time, are statistically described [3, 4].

In this work we note that the Poisson process is a random point process, which satisfy the three conditions: ordinary, stationary and independence of increments. Nowadays, we have two methods of description of the random point processes, which are closely related. The first of them is based on the consideration thewhole numerical random process, the second is based on the analysis of the random sequence of points in time.

In our work for the creation the program algorithm we used the second approach. The simulation of the random component of the Poisson white noise was conducted in two stages:

1) generation the random number $n(t)$, which characterized by a Poisson distribution;

2) generation the independent random variables z_i, which characterized by the some probability density $q(z)$.

For comparison the analytical and numerical results we assume that independent random variables z_i distributed according to the exponential law and there probability density is given by:

$$q(z) = \frac{r}{2} \cdot e^{-r \cdot |z|}. \tag{3}$$

For obtanening the probability density of a random point process we consider the displacement of points. Suppose that the every particle makes the movements, but at the initial time it located at the point of the origin of coordinates. The initial displasment we can represent in the simple form:

$$X_1(t) = X_0(t) - b \cdot X_0(t) \cdot t + \xi_1(t) \tag{4}$$

Substituting the initial conditions on Eq. (4) we obtained:

$$X_1(t) = \xi_1(t) \tag{5}$$

It means that the first displacement depends only on the random variable distributed according to the Poisson law.

All next displacements we find as

$$X_i(t) = X_{i-1}(t) - b \cdot X_{i-1}(t) + \xi_i(t) \tag{6}$$

Take into account, that for each movement is calculated new random component of noise, we derive the probability density distribution particles after the some displacements. This is the first step in investigation of the effects of Poisson white noise in behavior of the particles.

In this work we make a calculation for the N=1000000 particles, each of which make a i=1000 movement with the rate of the process λ=100 and the time interval t=0,01.

III. Analysis of results

Previously, for determining the probability density of the nanoparticles driven by Poisson white noise in the case of a quadratic potential ($U(x,t) = U(x) = bx^2/2$ $(b > 0)$) was obtained the following generalized Fokker-Planck equation [5]:

$$\frac{\partial}{\partial t} P_k(t) + bk \frac{\partial}{\partial k} P_k(t) = P_k(t)\phi_k. \quad (7)$$

From the solution of the Eq. (7) the stationary probability density of the particle driven by Poisson white noise takes the form:

$$P_{st}(x) = \sqrt{\frac{2}{\pi}} \frac{r\left(r|x|\right)^{s-1/2}}{2^s \Gamma(s)} K_{s-1/2}\left(r|x|\right), \quad (8)$$

here $\Gamma(s)$ is the gamma functions, $K_{s-1/2}\left(r|x|\right)$ is the modified Bessel function of the third kind (or Macdonald function) [6], $s = \lambda/(2b), b, r$ are the parametrs, λ is the rate of the noise.

In this analytical solution the dynamics of the nanoparticles driven by Poisson white noise, which are located in the potential well, is considered only as a special case. Using this solution, we can determine the presence of Poisson white noise-induced transitions.

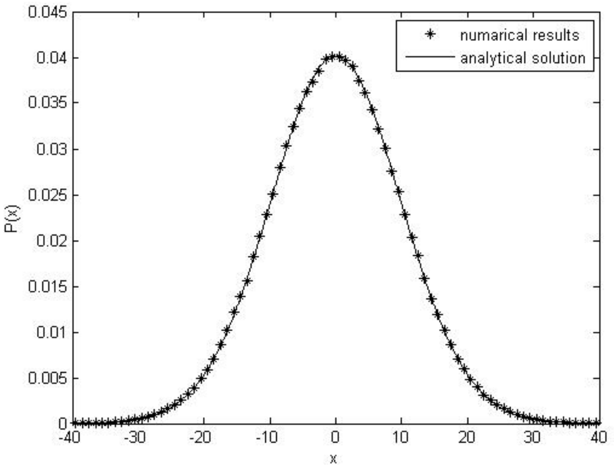

Fig.1 The probability density of the particles driven by Poisson white noise in the case of a quadratic potential.

For another cases, a similar analys cannot be obtained from the analytical expressions. Our numerical algorithm can be used for the investigation of behavior of the particles driven by Poisson white noise with different input parameters.

For analysis our numerical results we compared it with the probability density distribution, which is obtained by analytical method.

The fig. 1 is shown that the numerical values of the probability density of the particles on preset intervals and analytical solution based on Eq (7) are similar. Hence, our developed numerical algorithm fully describes behavior of the particle driven by Poisson white noise which is located in a potential well. This gives us an opportunity to research the dynamics of the nanoparticles and to determinate the presence of Poisson white noise-induced transitions.

Conclusion

We have reviewed the dynamics of the nanoparticles driven by Poisson white noise, which are located in the potential well. The developed algorithm fully describes the considered physical problem. This is substantiated by the comparative analysis of numerical and analytical results. Thus, we can use our numerical algorithm for the investigation the behavior of the nanoparticles driven by Poisson white noise with different input parameters.

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