Mathematical Model of Steady-State Visual Evoked Potential in Problems of Ophthalmological Information Technologies

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***Abstract* – The mathematical model of steady-state visual evoked potential as a linear periodical random process is justified. The exciting models of the VEP signal are described. The estimation of mathematical expectation and variance are found by φ- series method.**

Key words – characteristic function, impulse, linear random process, mathematical expectation, mathematical model, periodical process, steady-state visual evoked potential.

I. Introduction

Percentage of the visual system pathology and disease at the structure of human morbidity has tendency to increase for every year. The main reasons of this fact are significant effect of the computer work owing to the rapid scientific and technical progress, reduction of human provision level, deterioration of the environmental conditions.

At present time, in the every-day practice ophthalmologists try to use the new information technologies (ITs), for example: DX-NT, OKULYAR, Neuron-Spectr, Sierra, Neurofax and etc. One of the methods that use such technologies for further diagnosis is registration of visual evoked potentials (VEP – electrical response of human visual system to external light stimuli). Depending on the frequency of stimulation EPs are divided into transient (1-4 Hz) and steady-state (5-30 Hz). Most of the present ophthalmic ITs use transient potentials, but steady-state EPs can allow to estimate the complex state of the visual system and the ability to long-term action [1].

Fig. 1 illustrates examples of steady-state VEP realization at a photo stimulation frequency 10 Hz. Its realization is considerably noisy by background electrical activity of the brain and that's why periodicity (due to periodic stimulation) can't be seen without signal pre-processing.

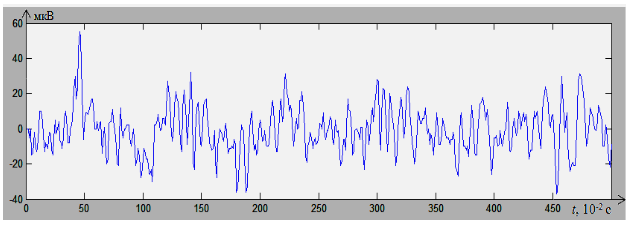

Fig.1 Realization steady-state VEP
(The stimulation frequency is 10 Hz)

The field of VEP application is not limited by ofthtalmodiagnosis. This type of signal is also used in the brain-computer interface, in the clinical neurology (diagnosis of epilepsy, schizophrenia) [2].

Taking into account the above-mentioned considerations, exploration of the steady-state VEPs is actual scientific problem. However, the main task of such signal usage in the IT is to build an adequate mathematical model that represent the physical generating process of the VEP and allow to make a qualitative diagnosis.

II. Exciting models of the VEP signal

The additive model is the widespread mathematic model. It is a sum of centered weakly stationary random process (background electrical activity) and determined function (VEP) [1]. Main disadvantage of this model is case that it takes into account only the first moment function and it cannot descript signal in details.

We find VEPs description with a help of component model in the works of many researchers [4]. It comprises processes which are caused by separate electrical brain activity sources. The independent components analysis (ICA) and the principal components analysis (PCA) are used to find separate components. Herewith we have to descript every component by the corresponding mathematic models separately.

Whereas steady-state VEP is generated under the influence of the high frequency stimulation, so we observe cycle rhythm during its realization. Such feature is used in the signal description by the random periodical process, probabilistic characteristics of which are the periodical functions of time. The disadvantage of such model is the impossibility of the description of the physical generation process by neurons.

The artificial neural network model is used for VEP analyzing and classification in work [4], but it does not take into account random nature of the electrical brain activity by separate neurons. This fact is very important for medical diagnostic. Also the learning process can be so long to find the optimal weight coefficient.

The mathematic model of VEP in a form of the linear random process is used in work [5]. Such model takes into account biophysical process of the electrical brain activity and allows to descript signal using cumulant and moment functions of high order. Authors descript the resulting signals as sum of the linear piecewise stationary random stochastic process and determined function. The authors [5] investigate the received signal realization only on stationary intervals.

The designing of the adequate mathematic model which allows not only to estimate diagnostic parameters by corresponding methods but also could give their biophysical explanations is the main task of oftalmodiagnosis using the steady-state VEPS.

III. Mathematical model of steady-state VEP signal

After the human visual system stimulation neurons generate during the random sequential time points $\{..., \tau_{-2}, \tau_{-1}, \tau_0, \tau_1, \tau_2, ..., \tau_n, ...\} = \{\tau_n, n \in \mathbf{Z}\}$ action potential (AP), excitatory or inhibitory postsynaptic potentials (EPSP and IPSP) [4]. These responses are called impulses end they are different by the frequency and amplitude. The values of time interval between impulse appearances are independent random variables and exponentially distributed with parameter $\lambda(\tau)$, $\tau \in (-\infty, \infty)$, that characterizes the intensity of impulses occurrence. Basing on the above-mentioned considerations, the sequence of time moments τ_n, $n \in \mathbf{Z}$ is the non-uniform Poisson flow with parameter $\lambda(\tau)$.

Whereas the correlation coefficient between the potentials of different neurons is in the range 0,006-0,01, so we assumed that the impulses are statistically independent. Random function that describes the electrical potential changing of separate neuron can be written in the next form:

$$V_n(\tau_n, t) = \alpha_n \varphi(\tau_n, t), \qquad (1)$$

where τ_n – is the random occurrence moment time of n-th impulse, t – is the moment of observation;

α_n, $n \in \mathbf{Z}$ – is the sequence of independent random variables which characterize impulse amplitude with distribution function $F_\alpha(x;\tau)$, $x \in \mathbf{R}$ and the finite variance;

$\varphi(\tau, t)$ – is the nonrandom function, which can be presented in the next form:

$$\varphi(\tau, t) = e^{-\beta(\tau)(t-\tau)} \sin(\omega(\tau)(t-\tau)) U(t-\tau) \qquad (2)$$

$U(s)$ – is the Heaviside function;

$\beta(\tau) > 0$ – is the nonrandom function that describes the coefficient of impulse damping;

$\omega(\tau) > 0$ – is the nonrandom function that describes the impulse frequency.

Whereas, the VEP is the total response to stimulation of all active neurons, so the resulting signal can be written in the form of random process:

$$\xi(t) = \sum_{n=-\infty}^{\infty} \alpha_n \varphi(\tau_n, t) \quad . \qquad (3)$$

Let us to introduce $\pi_1(\tau)$, $\tau \in (-\infty, \infty)$, $\mathbf{P}\{\pi_1(0) = 0\} = 1$ is the non-uniform generalized Poisson process, jumps of which happen at the time moments τ_n, $n \in \mathbf{Z}$ and the value of jump is equal to the random variables α_n, $n \in \mathbf{Z}$. Using Eqs. (1) and (2) the process Eq.(3) can be represented in the form of a linear random process (LRP) [6]:

$$\xi(t) = \int_{-\infty}^{\infty} \varphi(\tau, t) d\pi_1(\tau) \qquad (4)$$

It can be shown that due to T-periodic photo stimulation, the process $\pi_1(\tau)$, $\tau \in (-\infty, \infty)$ is the process with T-periodic independent increments and the kernel of LRP expressed by Eq.(2) is periodical. According to the definitions given in the papers [7], random process (4) can be named as linear cyclostationary random process and the every m-dimensional $(m = 1, 2, 3, \ldots)$ characteristic function is T-periodical of the time arguments totality. Consequently, the mathematical expectation and correlation function is also T-periodical. As the result, process expressed by Eq. (4) is the periodical correlated random one.

Periodicity of probabilistic characteristics of the random process $\xi(t), t \in (-\infty, \infty)$ can allowed to use well-known methods of time series analysis. Figs. 2 and 3 illustrates realization graphics of mathematical expectation and variance estimations of steady-state VEP (photo stimulation frequency is 10 Hz) obtained using the φ - series method.

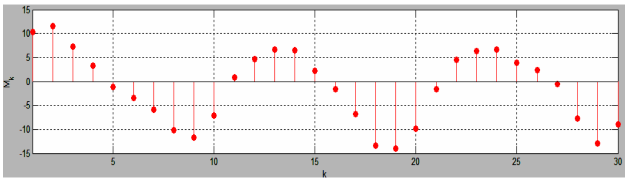

Fig. 2. Realization of the estimation of mathematical expectation

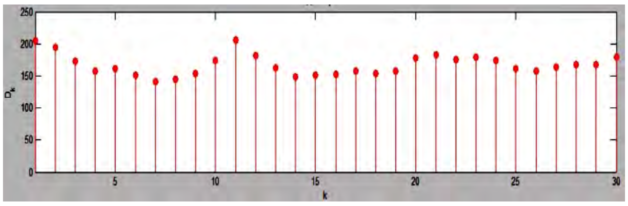

Fig. 2. Realization of the variance estimation

The obtained results show the validity of mathematical model for ophthalmic IT in the form of a linear stochastic process.

Conclusion

Based on and requirements for ophthalmic ITs the mathematical model of steady-state visual evoked potential in a form of a linear periodical random process is justified. This model allows not only identifying information and diagnostic parameters, but also gives them a biophysical explanation. The estimation of mathematical expectation and variance are showed, which prove the correctness of the chosen model.

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