Multi-stage problem of concentration plant location

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***Abstract* – The problem of placing mines and processing plants in a given area is discussed. Mathematical model of combined formulation is offered. Aggregate cost of product delivery is chosen as a criterion for optimal location problem.**

Key words – partition problem, mathematical model, concentration plant, multi-stage problem, location-allocation problem, combined formulation, problem of optimal partitioning set.

I. Introduction

Logistics is vital to sustaining many industrial, commercial, and administrative activities. It is often composed of the logistics service providers and the customers being serviced. The goal of service providers is to maximize revenues by servicing customers efficiently within their preferred timelines. To achieve this goal, they are often involved in activities of location-allocation planning, that is, which logistics facilities should be opened, where they should be opened, and how customer allocations should be performed to ensure timely service to customers at least delivery costs to logistics operators.

II. Multi-stage problem

Facility location problems have proven to be a fertile ground for operations researchers interested in modeling, algorithm development, and complexity theory. When dealing with a wide range of issues in a variety of practice areas there often arises the problem of optimal location of production in a given area. The importance of this issue is proved by the large number of publications devoted to the development of mathematical models of effective methods and algorithms solving various problems of allocation. Applications of location modelling include locating emergency medical service (EMS) bases[1], fire stations, schools, hospitals, reserves for endangered species, airline hubs, waste disposal sites, and warehouses [2] to list only a small subset of the numerous areas in which location models have been applied. Location models have also found applications in nontraditional areas, including medical diagnosis, vehicle routing, alignment of candidates and parties along a political spectrum, and the analysis of archeological sites [3].

In the Multi-Stage Facility Location Problem we are given a set of facilities and a set of customers. Each customer must be serviced by a sequence of different facilities. These sequences are defined by hierarchy of production and distribution system and can be presented as facility paths. The set of admissible facility paths is given. Each facility has fixed cost. Each customer has transportation costs for servicing by the facility paths. The problem is to select facilities in order to service the customers with minimal total cost.

Fig. 1. presents an example of hierarchical multi-stage production system.

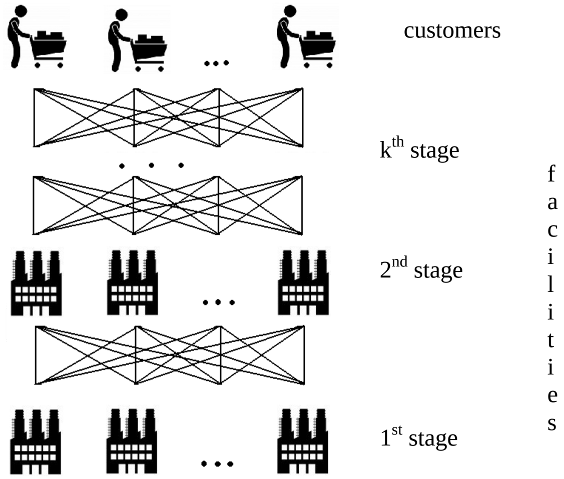

Fig. 1/ Feasible solution of the problem

The problem of optimal partitioning set for concentration plant locating is one of the examples of a multi-stage problem. In this paper, mathematical model for the problem of concentration plant locating is given.

A characteristic feature of this production process is the presence of two phases implemented at enterprises of various types. The organizational structure of production is shown schematically in Fig. 2.

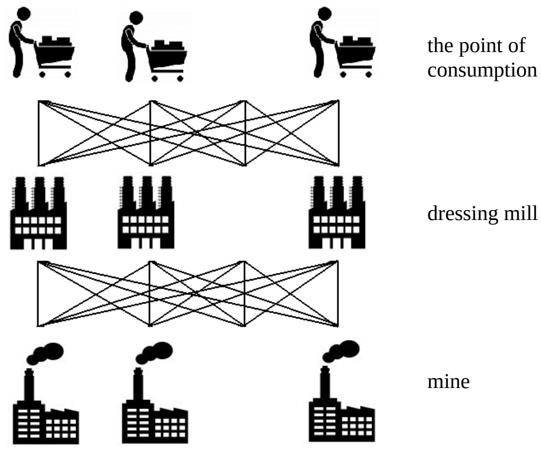

Fig. 2/ The standard scheme of concentrating production

Ore is mined at a mine and it is then sent for concentration via hydraulic transport. The concentration plant directs the resulting commercial concentrate to the consumption point (e.g. warehouses, or other businesses). Thus, the manufacturing process involves several stages requiring a variety of production resources. Furthermore, when locating companies of the first stage it is necessary to consider the distribution of raw materials in the given area based on geological exploration. Location criteria may be different: one of the most common is to minimize the transportation costs for the delivery of raw materials and finished products.

III. Mathematical model of combined formulation

Informal statement of location problem for a multi-stage processing industry can be summarized as follows: it is necessary to place concentration production, which includes mines and processing plants in the region Ω, so that the cost of shipping raw materials and products was minimal. It is assumed that mines and factories can be placed anywhere in the area Ω, the location of a finite number of users is known in advance.

To construct a mathematical model, we introduce the following notation :

Ω - given domain;

Ωi – fixed area of i-th mine;

N – the number of mines;

M - the number of plants to be placed in the area.

b_{ir} - the yield of i-th mine ;

$c^p_{ij}=c^p(\tau_i, \tau_j)$ - cost of shipping a unit of raw material from i-th mine to j-th plant;

K - finite number of consumers;

$c^u_{jk}=c^u(\tau_j, \tau_k)$ - the cost of delivery of products from j- th factory to k-th consumers;

b^p_j - maximum production capacity of the plant ;

b^u_k - the demand of k-th consumer;

$\rho^r(x)$ – thickness of a seam at the x point of Ω;

τ^r - coordinates of i-th mine; $\tau^r_i=(\tau^r_{1i}, \tau^r_{2i})$;

τ^p - coordinates of j-th plant; $\tau^p_j=(\tau^p_{1j}, \tau^p_{2j})$;

τ^u_k - coordinates of the consumer, $\tau^u_k=(\tau^u_{1k}, \tau^u_{2k})$;

v^p_{ij} - the volume of products delivered from i-th of the mine to j-th plant;

v^u_{jk} - the volume of products delivered from j-th factory to k-th consumers.

Then the above problem can be described by the following model:

$$\sum_{i=1}^{N}\int_{\Omega_i} c_i^r(x,\tau_i^r)\rho^r(x)dx+\sum_{i=1}^{N}\sum_{j=1}^{M}c_j^p(\tau_i^p,\tau_j^p)v_{ij}\lambda_j^p + \\ +\sum_{k=1}^{K}\sum_{j=1}^{M}c_k^u(\tau_j^u,\tau_k^u)v_{jk}\lambda_j^p \to \min_{\tau^r,\tau^p} \quad (1)$$

$$\tau^r = (\tau_1^r,\tau_2^r,...\tau_N^r)\in\Omega^N \quad (2)$$

$$\tau^p = (\tau_1^p,\tau_2^p,...\tau_M^p)\in\Omega^M \quad (3)$$

$$\bigcup_{i=1}^{N}\Omega_i=\Omega \quad (4)$$

$$\Omega_i\bigcap\Omega_j=0, i\neq j; i,j=1,n \quad (5)$$

$$\sum_{i=1}^{N}v_{ij}=b_j^p,\ j=1,M \quad (6)$$

$$\sum_{j=1}^{M}v_{jk}=b_k^u, k=1,K \quad (7)$$

where the constraints (4, 5) mean a partition of Ω into the service areas for mines, that is, to cover the whole area Ω (4), and to serve each point of the area by only one mine (5); (6,7) - mean that corporate demand of plants of the second stage and consumers must be satisfied.

The approach to solution of this type of problem is based on the sequential solution of the problem of optimal partitioning set (OPS) [5] and discrete multi-stage location problem [4]. The method of solution, the algorithm and the results of OPS are collected and systematized in [4]. It also described a unified approach to solving such problems. It lies in the conversion of the initial problems into infinite-dimensional mathematical programming problems by means of the characteristic functions, and then into the finite optimization problem using Lagrangian functional. In addition, for the majority of the tasks necessary and sufficient conditions of optimality are provided. The case with a finite number of possible locations for factories is proposed in [6, 7].

As a final note, we suggest that the relevance of the continuous location-allocation problem in the real world is increasing and will continue to do so. One of the main reasons is that much larger problem instances are being dealt with in practice. It should be noted that transport costs account for over 50% of the cost of the extracted ore and overburden. Therefore, even a slight decrease in this index results in significant economic effect.

Conclusion

Facility location problems have been a fertile ground for the development of new modeling techniques, innovative solution algorithms and exciting applications. Location modeling remains an exciting area of research for operations researchers and of application for practitioners. This paper considers a new formulation for the Multi-stage problem of concentration plant location. Aggregate cost of product delivery is chosen as a criterion for optimal location problem.

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