Higher order spectra analysis for assessing stationarity

Konstantin Stavorovsky, Anton Popov

Biomedical Electronics and Signal Analysis Group,
Physical and Biomedical Electronics Department,
National Technical University of Ukraine «Kyiv Polytechnic Institute», off. 423,Politekhnichna Str., 16, Kyiv, 03056,
E-mail: stavarko@gmail.com

***Abstract* – In this paper the new approach to investigation of signals stationarity based on parameters of bispectrum is proposed. The method was examined using simulated test signals artificially test signals with predefined nonstationarities. As test signals Gaussian noise with varying variance and mean value were proposed. Proposed bispectral characteristics are different for stationary and nonstationary signals.**

Key words – higher order spectra analysis, bispectrum, stationarity, parameters of bispectrum, Gaussian noise.

I. Introduction

Random signals can be categorized into stationary and non-stationary ones. Determination of this property of random signals can provide researchers with a new tool for exploring inherent attributes and defining characteristics of the signal which can tell us about the object under study.

The most well-known and the most widely used approaches of random process stationarity assessment are runs test [1] and reverse arrangements test, based on division of the observed data recording into fragments of equal length and further statistical analysis of obtained sequence. This approach provides only an estimation of the signal nonstationarity in the whole and gives no possibility of detecting signal parts which could be considered as locally stationary. More efficient way to solve this problem is to use more complicated techniques such as correlation, spectral analysis and parametric modeling of signals [2].

In this paper the new approach to stationarity assessment using bispectral characteristics is proposed. The aim of this study is the investigation of difference in the results of bispectrum for stationary and nonstationary test signals.

II. Method of stationarity estimation using bispectrum

Bispectrum $\Phi_3(jw_1, jw_2)$ is the Fourier transform of the third order correlation function $K_3(\tau_1,\tau_2)$. $K_3(\tau_1,\tau_2)$ of signal $x(t)$ is described in [3] as follows:

$$K_3(\tau_1,\tau_2) = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)x(t-\tau_1)x(t-\tau_2)dt \,. \quad (1)$$

Bispectrum can be estimated:

$$\Phi_3(jw_1, jw_2) = (2\Delta f)^2 G_3(jw_1, jw_2)\,. \quad (2)$$

Here, $G_3(jw_1, jw_2)$ denotes the bispectrum density:

$$G_3(jw_1, jw_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_3(\tau_1,\tau_2) e^{[-j(w_1\tau_1 - w_2\tau_2)]} d\tau_1 d\tau_2 \,. \quad (3)$$

The aim of this work is to investigate if there are any bispectrum parameters sensitive to the presence of nonstationary segments in signals under analysis. In this work maximal value (Eq. 4) and mean value (Eq. 5) of bispectrum are proposed as such parameters:

$$\Phi_{\max} = \max(\Phi_3)\,, \quad (4)$$

$$\overline{\Phi} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\Phi_{i,j}}{m\times n}\,. \quad (5)$$

III.Results

To study if nonstationarity of signals can be estimated from the bispectral characteristics, simulated data were used. As reference signal without any nonstationarities, Gaussian noise was employed (Fig. 1).

Several types of nonstationarities were employed: Gaussian noise with linear trend (Fig. 2) and Gaussian noise multiplied by linear function $\gamma = \alpha t$, where $\alpha = 0..1$. (Fig. 3). Duration of each signal was 30 sec, with sampling rate F_s=100 Hz.

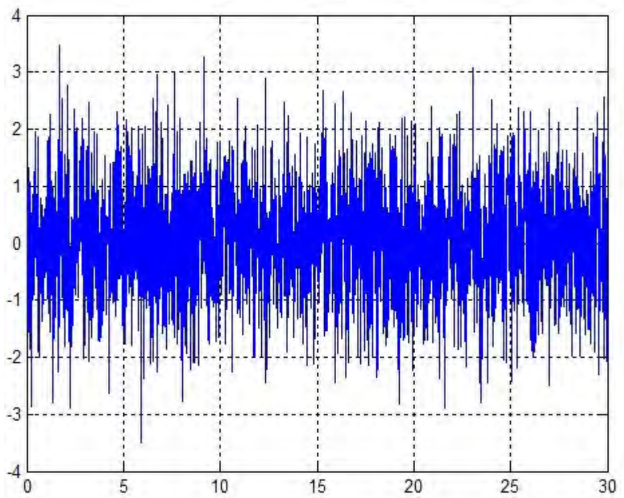

Fig. 1. Gaussian noise

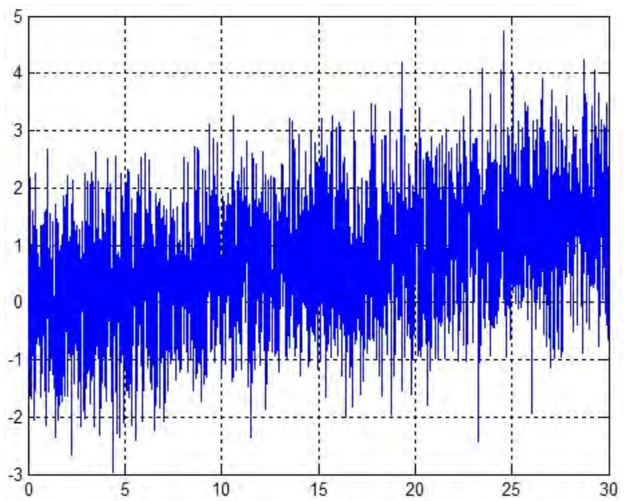

Fig. 2. Gaussian noise with linear trend

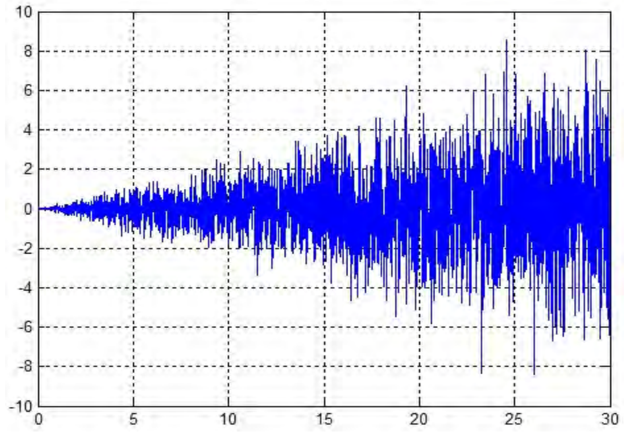

Fig. 3. Gaussian noise with shift dispertion

Reference stationary Gaussian noise has maximum value of bispectrum $\Phi_{\max} = 0.18$ and mean value of bispectrum $\overline{\Phi} = 10^{-3}$.

Dependence of $\Phi_{\max}, \overline{\Phi}$ on α for Gaussian noise with linear trend is given in Fig. 4.

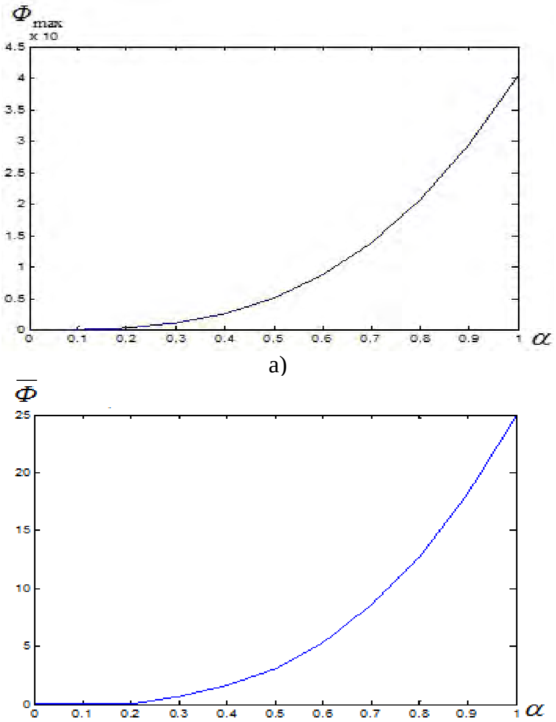

Fig. 4. Dependence of $\Phi_{\max}$ (a) and $\overline{\Phi}$ (b) on α for Gaussian noise with trend

Both parameters of bisectrum have nonlinear increase with α growth, and dependences of $\Phi_{\max}, \overline{\Phi}$ have similar shape. A constant component of linear trend has minor effect on dependence.

For next test Gaussian noise multiplied on γ was used.

Changing tendency of bispectrum parameters for this type of nonstationary signal is the same as in the previously described case (Figs. 5 and 6). $\Phi_{\max}, \overline{\Phi}$ increase with the increasing of α.

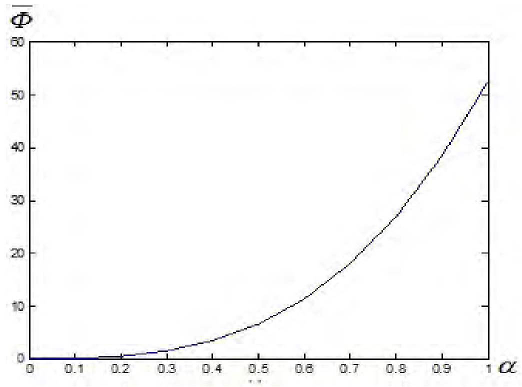

Fig. 5. Dependece $\overline{\Phi}$ from α for Gaussian noise with shift dispertion

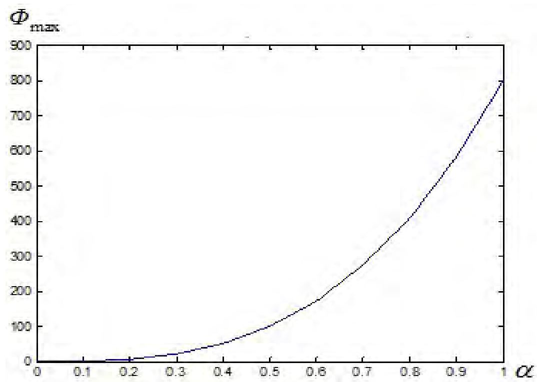

Fig. 6. Dependece $\Phi_{\max}$ from α for Gaussian noise with varying variance.

Experimental results show that proposed parameters are different for the signals with different nonstationarities, and therefore these parameters can be used on to measure nonstationarity of the signals.

Conclusion

In this paper the results of bispectral analysis for assessing stationarity were presented. As the parameters to be analyzed as nonstationarity measures maximum and mean value of bispectrum were proposed.

The results show that bispectra can be used for investigation of stationary parts of signals. This approach would be useful in identification of local stationary segments of signals algorithms.

References

[1] J. S.Bendat, Al. G.Piersol, Random Data: Analysis and Measurement Procedures, 3nd ed. Wiley-Interscience, 2000.

[2] A.N. Kalinichenko, M.I. Nilicheva, S.V. Khasheva, O.D. Yurieva, "Signal stationarity assessment for the heart ratevariability spectral analysis", Computers in Cardiology, vol. 35, pp. 965-968, 2010 [Online]. Available: IEEE Xplore, http://www.ieee.org [Accessed: 14 Sept. 2013].

[3] V. F. Kravchenko, Cifrovaja obrabotka signalov i izobrazhenij v radiofizicheskih prilozhenijah [Digital signal and image processing in radiophysical applications]. Moscow: FIZMATLIT, 2007.