Uncertainty Analysis in Acoustic Investigations

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Abstract - The problem of uncertainty assessment in acoustic investigations is presented in the hereby paper. The aspect of the uncertainty asymmetry in processing of data obtained in the measuring test of sound levels, determined in decibels, was sketched. On the basis of the analysis of data obtained in the continuous monitoring of road traffic noise in Krakow typical probability distributions for a day, evening and night were determined. The method of the uncertainty assessment based on the propagation law of the random variables density function was also presented.

Key words - acoustic measurements, statistic analysis of the obtained results, estimation of the distribution, uncertainty

I. Introduction

An important element in the process of controlling environment acoustic hazards is the determination of likelihood of assessment of parameters. This process includes measuring procedures, identification of possible errors in data processing as well as the selection of the proper measures in case of knowing the probability distribution of their occurrence. The most often the normal distribution of measured values is assumed [1-4]. This assumption, in case of sound level measurements expressed in decibels L_{A_i} *; i=1,2,..,n,* was questioned in several papers, which were synthetically discussed in paper [5]. Noticing of this problem resulted in several propositions based on solutions rejecting assumptions of the normal distribution of measuring test results [6-11]. One of the method of the uncertainty determination is identification of the probability distribution of the measured value estimator and giving its uncertainty interval in a form of distribution quantiles. The Guide to the Expression of Uncertainty in Measurements suggests - in such cases - using numerical solutions based on the Monte Carlo algorithm for calculating density functions [12].

The reasons of the uncertainty asymmetry are characterised in the hereby paper. The typical probability distributions of road traffic sound levels, based on the analysis of the results of the continuous monitoring of the road traffic noise in Krakow, are presented. The uncertainty determination method based on the propagation of the density probability function, in which can be used probability density functions described in paper, is presented.

II. Measurements uncertainty

Two main methods of measuring the uncertainty determination are given in references. The first method is the analysis of errors propagation. The second is based on the determination of the output value probability distribution, at the assumption of the probability distribution of input values in the mathematical model of the assessed value.

The analysis of possible errors is performed, as the first procedure, in the errors propagation method. Then, the measurement result is written in a form of the random variable Y dependent on random variables X_i of the measuring process describing directly the measured values, errors of the measuring process, corrections etc.

$$Y = f(X_1, X_2, ..., X_n) \tag{1}$$

Eq. 1 is called the measurement equation. The assessment of Y value, done by substituting random variables in Eq. 1 by their estimators $\hat{x}_i$, is called the measurement result $\hat{y}$. Then, the uncertainty of the assessment $\hat{y}$, is the interval:

$$(\hat{y} - U; \hat{y} + U) \tag{2}$$

Where $U = ku_c$ - is the expanded uncertainty being a product of the expansion factor k and standard uncertainty $u_c(\hat{y})$ determined from the measurement equation by the formula:

$$u_c(\hat{y}) = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial \hat{x}_i}\right)^2 u^2(\hat{x}_i) + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left(\frac{\partial f}{\partial \hat{x}_i}\right)\left(\frac{\partial f}{\partial \hat{x}_j}\right)u(\hat{x}_i, \hat{x}_j)} \tag{3}$$

where $u(\hat{x}_i)$ is the assessment uncertainty $\hat{x}_i$. A determination of the expansion factor k, which is the quantile of the probability distribution of $\hat{y}$, requires knowing the probability distribution of the random variable modelling the measurement result. Since a realisation of this task is often very difficult several simplifications, leading either to an application of approximate methods or to a problem reduction by determining the uncertainty of samples originated from measurements of the normal distribution, were introduced (Type A uncertainty).

In case of the second method the probability distribution of the output value, at assuming the probability distribution of the measured values, is done by the method of distributions propagations. The uncertainty of the assessment constitutes the confidence interval determined by quantiles of its probability distribution. Its length depends on the significance level - with which it is determined - usually 95%. The realisation of such procedure can be found in papers [11],[13-16], concerning the management of the environment protection against noises.

III. Non-linearity of acoustic effects

Assumptions of a normal distribution of a random test of sound level measurement results are applied in several analysis. For samples with this characteristic the Type A uncertainty analysis is applied, which significantly simplifies the uncertainty assessment process. The value of the assessed parameter is replacing by the test arithmetic mean $\overline{x}$ and its uncertainty Eq. 4 is determined by using the standard deviation s.

$$u(\overline{x}) = \frac{s}{\sqrt{n}} \tag{4}$$

However, such approach does not take into account the character of the measuring test of values measured in decibels. In the results of measurements of sound levels occurs asymmetry, which result directly from the decibel construction Eq. 5

$$L = 10\log\frac{p^2}{p_0^2}, \qquad (5)$$

where p_0 is the reference level (the most often $p_0 = 2x10^{-5}\,Pa$ - the smallest pressure change which human ear can record).

When operations on decibel values are to be performed the Weber-Fechner Law Eq. 6, which informs how to add sound levels expressed in decibels, should be applied.

When two sound levels L_A, L_B expressed in decibels are added, their total level equals:

$$L_A \overset{dB}{+} L_B = 10\log\left(10^{0,1L_A} + 10^{0,1L_B}\right) \qquad (6)$$

Other operations on decibel numbers are defined in a similar fashion Eq. 7,8.

Subtraction:

$$L_A \overset{dB}{-} L_B = 10\log\left(10^{0,1L_A} - 10^{0,1L_B}\right) \qquad (7)$$

Averaging:

$$\overline{L}^{dB} = 10\log\left(\frac{1}{n}\sum_{i=1}^{n} 10^{0,1L_{A_i}}\right) \qquad (8)$$

Non-linearity disturbances of sound pressure transformations and nonlinearity operations on decibel values causes an asymmetry of probability density functions of decibel values. This affects the asymmetry of the uncertainty interval of parameter estimates and indicators used in acoustics and in the processes related to the environment protection against noises.

IV. Transformations of acoustic pressure disturbances and pressure square distributions

When determining the noise estimation likelihood it is necessary to take into account the asymmetry occurring in the random test of sound level measurement results. One of the possibilities for the correct consideration of this effect is using the method of the probability distribution propagation. The probability distributions useful in the determination of asymmetric uncertainty intervals were presented in the paper [15]. They are formed by means of transformations of the acoustic pressure disturbances and pressure square, for which the normal probability distributions can be assumed. The first one is the transformed normal probability distribution of the acoustic pressure. For variable P , which is representing the acoustic pressure values, based on an experimental practice the normal distribution trimmed off at the interval $[0,+\infty)$ can be assigned. When considering pressure disturbances (limited to changes noticeable by a human ear) the normal distribution Eq. 9 trimmed off at the interval $[p_0,+\infty)$.

$$f_P(x) = \begin{cases} \dfrac{1}{1-F\left(p_0\right)}\dfrac{1}{\sqrt{2\pi}s}e^{-\frac{(x-m)^2}{2s^2}}, & x \ge p_o \\ 0 & x < p_o \end{cases} \qquad (9)$$

For the problem focused in the above presented way, in accordance with the procedure described in paper [15], the sound level probability distribution, expressed in decibels, will be described by Eq. 9

$$f_Y(y) = \frac{\ln 10}{20}\frac{1}{1-F\left(p_0\right)}\frac{\sqrt{10^{0,1y}}}{\sqrt{2\pi}\,\dfrac{s}{p_0}}e^{-\frac{\left(\sqrt{10^{0,1y}}-\frac{m}{p_0}\right)^2}{2\left(\frac{s}{p_0}\right)^2}} \qquad (10)$$

$y \in [0,\infty)$

The probability distribution described by equation Eq. 10 can be assigned to sample measurement of decibel noise value. It is characterised by the left-sided asymmetry. The Mode and Median are shifted to the right-side with respect to the average value. This asymmetry is characteristic for acoustic investigations, however sometimes the measurement results are of the right-sided asymmetry [16]. The example of the density probability distribution function described by equation Eq. 10 is presented in the diagram Fig. 1.. The distribution obtained by the transformation of acoustic pressure disturbances is presented in the diagram together with the normal probability distribution applied at the uncertainty determination by means of the A method.

In order to estimate the parameters of the probability distribution obtained by the transformation method the recalculation of decibel values $L_i, i = 1, 2, ..., n$ into pressure values is done according to Eq. 10:

$$p = p_0\sqrt{10^{0.1L_i}} \qquad (11)$$

Then, the standard deviation and arithmetic mean are calculated and the parameters are put into Eq. 10. This procedure called the Method of Moments, lies in substituting moments of the random variable by the estimators obtained from the sample. More information concerning various methods of the sound level measurements estimation can be found in paper [16].

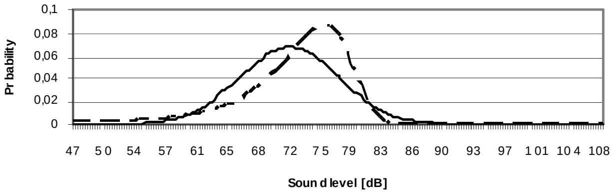

Fig 1. Diagrams of the probability of the transformed normal distribution of pressures and the normal distribution for decibels

Parameters of distributions from Fig. 1. are presented in Table 1

TABLE 1

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG 1

Normal distribution		Transform of the normal distribution	
EN	71.80dB	EY	72.40dB
σN	5.90dB	σY	7.10dB
m_e^N	71.80dB	m_e^Y	74.00dB
m_o^N	71.80dB	m_o^Y	76.00dB
$q_{0.025}$	60.17dB	$q_{0.025}$	56.47dB
$q_{0.05}$	60.04dB	$q_{0.05}$	60.42dB
$q_{0.95}$	81.51dB	$q_{0.95}$	80.37dB
$q_{0.975}$	83.37dB	$q_{0.975}$	81.42dB

The data presented in Table 1 and the shape of diagrams indicate the left-sided asymmetry of the transformed normal distribution of pressures. Both the Mode and Median are shifted to the right in relation to the average value. There is a significant difference between the standard deviations of these two types of distributions. The uncertainty interval for the normal distribution is narrower and shifted to the right with respect to the interval of the transformed distribution of pressure. For the normal distribution it is equal (60.04dB;83.37dB), while for the transformed normal distribution (56.47dB;81.42dB). This has an essential influence on the determination of uncertainty intervals.

The second important distribution, from the point of view of the uncertainty determination, is the logarithmic transformation of the normal distribution (described in paper [15]). This distribution occurs in case of determining the uncertainty for a large measuring test . Using the Central Limit Theorem, it can be assumed that the probability distribution of the arithmetic mean of the acoustic pressure square values or energy levels is the normal distribution trimmed off at the interval [18]. The transformation of the logarithmic mean of energy levels is marked by Z Eq. 12

$$Z = 10\log\left(\frac{1}{n}\sum_{i=1}^{n}10^{L_i}\right) = 10\log\bar{X} \qquad (12)$$

The probability distribution for the random variable Z at the assumption that will given by Eq. 13.

$$\varphi_Z(z) = \frac{\ln 10}{10}\varphi(10^{0,1z})10^{0,1z} = $$

$$= \frac{\ln 10}{10}\frac{1}{(1-\psi(1))\sqrt{2\pi}\sigma}e^{-\frac{\left(10^{0,1z}-\mu\right)^2}{2\sigma^2}}10^{0,1z} \qquad (13)$$

The example of the probability distribution for variable Z is presented in the diagram in Fig. 2

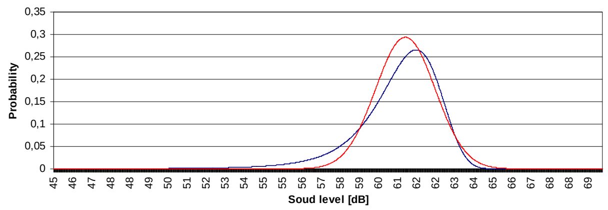

Fig 2. Probability distributions of the normal distribution and of the logarithm of the normal distribution given by Eq. 13

Parameters of distributions presented on Fig. 2 are given in Table 2.

TABLE 2

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG. 2

Normal distribution		Transform of the normal distribution	
EN	61.02dB	EZ	60.44dB
σN	1.35dB	σZ	4.28dB
m_e^N	61.02dB	m_e^Z	61.03dB
m_e^N	61.02dB	m_o^Z	61.52dB
$q_{0.025}$	58.36dB	$q_{0.025}$	55.84dB
$q_{0.05}$	58.79dB	$q_{0.05}$	57.15dB
$q_{0.95}$	63.26dB	$q_{0.95}$	63.10dB
$q_{0.975}$	63.69dB	$q_{0.975}$	63.42dB

The data presented in Table 2 and the shape of the probability function allow to notice the left-sided asymmetry of transformed normal distribution of acoustic pressure. Apart from that, the nearly threefold difference in the standard deviation (for the normal distribution) and shifting of uncertainty intervals, can be noticed. 95% of uncertainty interval for the variable Z distribution equals (55.84dB;63.42dB), while for the normal distribution equals (58.36dB;63.69dB). The left end of the interval is shifted by 2.52 dB to the left, while the right end by 0.27dB. The uncertainty interval span equals 7.58dB and 5.33dB respectively, which constitutes an essential difference in the interval length and has a significant influence on the uncertainty determination. It means, that it has also an influence on the uncertainty of the expanded assessment of logarithmic mean, on the basis of which the majority of parameters - used in the environment noise protection - is constructed.

The proper identification of the probability density function, the estimator of the random variable, is the key problem at the determination of the uncertainty of parameters estimations, which are assessed on the basis of the measuring test. Its improper identification can cause erroneous decisions in the analysis of the measurement data. In case of determination of indices in the environment noise protection it can cause erroneously imposed fines for exceeding the permissible noise standards.

V. Characteristics of the probability distribution of sound levels in road traffic - on the basis of the monitoring of Krakow

The uncertainty determination by means of the A method can be realized, when the probability distribution of measurement values is determined by the normal distribution. Thus, an essential aspect constitutes the verification, which distribution type obtain the measured values of sound levels. During the analysis of data related to road traffic, air, railway noises or industrial noises usually a random test of sound levels, in which essential deviations from normal distributions, is obtained.

Analysis from the continuous monitoring of road traffic noise in Krakow [19] confirm lack of normal character of sound level density probability functions Based on the data it can be specified typical distribution functions, which can be assigned to the samples measurements.

Frequency bar charts of the measured sound levels as well as the estimated for it density are presented in Fig. 8-11. The recorded sound levels are determined from the equation:

$$L_{A_i} = 10\log\left(\int_{t_i}^{t_{i+1}} \frac{p^2(t)}{p_0}dt\right) \qquad (14)$$

where integration is done after the time interval of one second length. Twenty-four hours can be divided - in the acoustic monitoring of road traffic - into three time periods. Day - from 6.00 to 18.00, evening - from 18.00 to 22.00 and night - from 22.00 to 6.00. Typical probability distributions, normal distributions (MM) and frequencies bar chart, obtained from the monitoring, are presented in Fig. 8-11. The typical probability distribution for the day is shown in Fig. 3

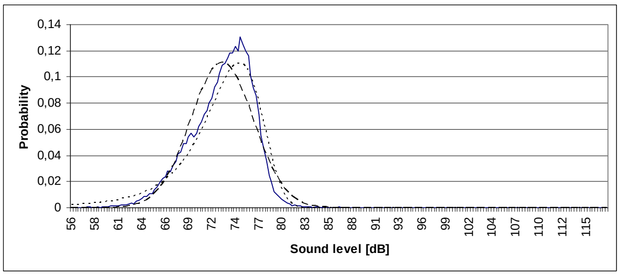

Fig 3. Density probability diagram for the day 08.01.2010

Solid line - bar chart of empirical data, dotted line - probability distribution obtained from Eq. 10, dashed line - normal distribution estimated by the method of moments.

TABLE3

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG 3

	Estimated distribution	Empirical data	Normal distribution
Logarithmic mean	-	73.68dB	-
Expected value	72.67dB	72.97dB	72.97dB
Mode	73.8dB	75.3dB	72.97dB
Mediane	75.1dB	73.5dB	72.97dB
Skewness	-2.34	-0.46	0
Kurtosis	14.21	4.10	3
Standard deviation	5.30dB	3.60dB	3.60dB
$q_{0.025}$	59.25dB	65.10dB	65.92dB
$q_{0.05}$	63.18dB	66.40dB	67.05dB
$q_{0.95}$	78.59dB	77.80dB	78.88dB
$q_{0.975}$	79.27dB	78.50dB	80.02dB

The typical probability distribution for the evening is presented in Fig. 4

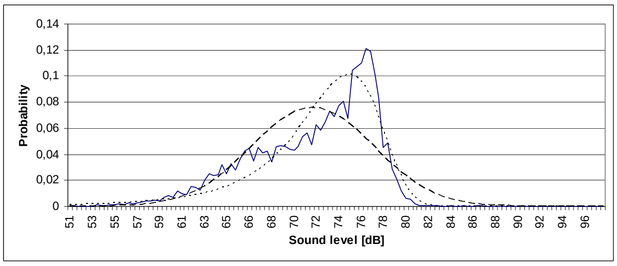

Fig 4. Density probability diagram for the evening 16.01.2010. Solid line - bar chart of empirical data, dotted line - probability distribution obtained from Eq. 10, dashed line - normal distribution estimated by the method of moments

Parameters of probability distributions are given in Table 4

TABLE4

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG 4

	Estimated distribution	Empirical data	Normal distribution
Logarithmic mean	-	74.37dB	-
Expected value	71.98dB	71.96dB	71.96dB
Mode	75.05dB	76.67dB	71.96dB
Mediane	73.54dB	73.27dB	71.96dB
Skewness	-0.37	-0.75	0
Kurtosis	4.47	3.14	3
Standard deviation	7.7dB	5.27dB	5.27dB
$q_{0.025}$	58.42dB	60.10dB	61.63dB
$q_{0.05}$	62.11dB	62.10dB	63.28dB
$q_{0.95}$	78.88dB	78.30dB	80.63dB
$q_{0.975}$	79.72dB	78.90dB	82.29dB

Two types of distribution are observed for the night, triangular distribution Fig. 5 and bimodal distribution Fig. 6.

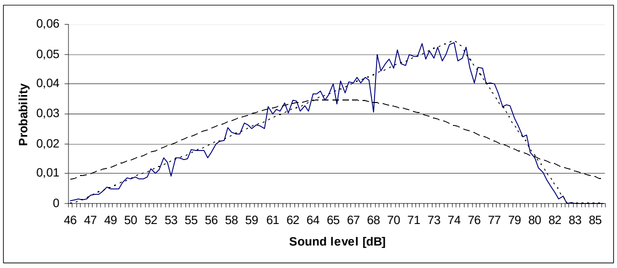

Figure 5 Probability distribution of the data for the night 10.01.2010. Solid line - bar chart of empirical data, dotted line - estimated probability distribution dashed line - normal distribution estimated by the method of moments

TABLE 5

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG 5

	Estimated distribution	Empirical data	Normal distribution
Logarithmic mean	-	72.8dB	-
Expected value	67.74dB	67.8dB	67.8dB
Mode	74.76dB	74.13dB	67.8dB
Mediane	68.65dB	68.61dB	67.8dB
Skewness	-0.43	-0.44	0
Kurtosis	2.39	2.42	3
Standard deviation	7.79dB	7.77dB	7.77dB
$q_{0.025}$	46.74dB	51.40dB	52.58dB
$q_{0.05}$	46.95dB	53.50dB	55.57dB
$q_{0.95}$	82.27dB	78.70dB	80.57dB
$q_{0.975}$	82.37dB	97.80dB	83.02dB

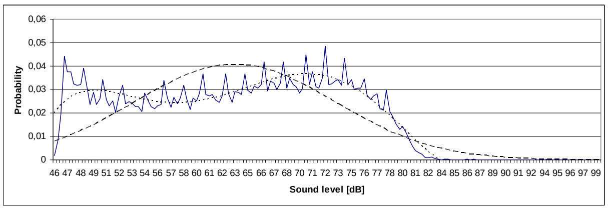

Fig 6. Probability distribution of data from the night 25.01.2010. Solid line - bar chart of empirical data, dotted line - estimated probability distribution dashed line - normal distribution estimated by the method of moments

TABLE 6

PARAMETERS OF DISTRIBUTIONS PRESENTED IN FIG 6

	Estimated distribution	Emirical data	Normal distribution
Logarithmic mean	-	71.35dB	-
Expected value	63.30dB	63.29dB	63.29dB
Mode	-	-	63.29dB
Mediane	64.33dB	64.02dB	63.29dB
Skewness	-0,12	-0.11	0
Kurtosis	1.79	1.83	3
Standard deviation	10.03dB	9.80dB	9.80dB
$q_{0.025}$	46.69dB	46.80dB	44.08dB
$q_{0.05}$	47.64dB	47.40dB	47.17dB
$q_{0.95}$	78.19dB	77.90dB	79.41dB
$q_{0.975}$	79.69dB	79.10dB	82.49dB

In the carried out analysis of allows to notice the repeatability of results. Probability distributions obtained for the day and night are similar to the transformed normal distribution of pressure. Two types of distributions are obtained for the night, the triangular distribution and bimodal distribution, which can be approximated e.g. by a multinomial of the fifth or sixth degree. All distributions obtained from analyses significantly differ from normal distributions, which can be observed when analysing the skewness and kurtosis coefficients as well as 95% confidence intervals for logarithmic means.

Knowing the form of the probability distribution of the results of sound levels measurements, it is possible to find - for small size tests - the probability distribution of the logarithmic mean described by Eq. 8. On the basis of this distribution the majority of coefficients and factors used in the noise environment protection are constructed. Convolutions of probability distributions can be applied for this aim. The determination procedure of such probability distribution is given in papers: [11] and [14].
For large size tests the probability distribution is determined from Eq. 13 The expected value μ is substituted by the test arithmetic mean $\mu_{\overline{X}} = \frac{1}{n}\sum_{i=1}^{n} 10^{0,1L_i}$ and the standard deviation σ by the standard deviation from the test $\sigma_{\overline{X}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(10^{0,1L_i} - \mu_{\overline{X}}\right)^2}$ energy levels . In such case the uncertainty interval for the logarithmic mean of sound levels is expressed by the following equation:

$$\left(a_l < 10\log\left(\overline{X}\right) < a_r\right) \qquad (15)$$

$$a_l = 10\log\left(\mu_{\overline{X}} - u(1-\alpha/2)\sigma_{\overline{X}}/\sqrt{n}\right)$$

$$a_r 10\log\left(\mu_{\overline{X}} + u(1-\alpha/2)\sigma_{\overline{X}}/\sqrt{n}\right)$$

Conclusion

The model of the uncertainty determination based on the method of the propagation of the density probability function is presented in the hereby paper. An aspect of an asymmetry of the measuring test of sound levels expressed in decibels was shown. On the bases of the data obtained from the continuous noise monitoring in Krakow typical probability distributions, which can be attributed to the results of sound levels measurements in road traffic for the day, evening and night, were characterised. The distribution obtained by means of the transformation of the normal distribution of acoustic pressures for the day and evening as well as two types of distribution, triangular and bimodal approximated by a multinomial of the fifth and sixth degree - for the night, were presented.

The analysis of probability distributions, obtained from monitoring, indicates their left sided asymmetry and multiple deviations from the normal distribution, which using classical methods of the uncertainty assessment renders impossible. Applying procedures described in this paper it is possible to calculate, for the obtained types of probability distributions, asymmetric uncertainty intervals of coefficients, based on a logarithmic mean of sound levels, used in the monitoring of the environment acoustic states.

The method of the uncertainty determination for models based on large and small measuring tests is given. In case of a large measuring sample asymptotic properties of the average value of energy levels are used, while in case of a small one the logarithmic mean probability distributions are determined by means of the convolution calculus.

The method presented in the paper allows for the correct determination of uncertainties of coefficients, of the environment noise protection, constructed on the basis of the sound levels logarithmic mean. It takes into account the skewness of the measuring test of the sound level results, which has an essential influence on the uncertainty interval asymmetry. This method can be applied either for the uncertainty assessment or for the validation of results obtained by other methods.

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References

[1] Guide to the Expression of Uncertainty Measurement (1995), International Organization for Standardization, ISBN 92-67-10188-9.

[2] D. Senczyk, Podstawy Teorii Pomiarów Wydawnictwo Politechniki Poznańskiej, Poznań, 2003

[3] Measurement Uncertainty Analysis Principles and Methods. NASA Measurement Quality Assurance Handbook, ANNEX 3, July, 2010.

[4] J. Kłysz, S. Lisiecki, Szacowanie Niepewności Pomiaru, Research Works Of Air Force Institute of Technology, No 22, p. 47-79, 2007

[5] W. Batko, Nowe rozwiązania metrologiczne dla monitoringu stanu zagrożeń akustycznych środowiska realizowane w Katedrze Mechaniki i Wibroakustyki. Problemy Inżynierii Mechanicznej i Robotyki , nr 60,

monografia: Badania statutowe na Wydziale Inżynierii Mechanicznej i Robotyki, p. 95-130, Wyd. AGH. WIMiR, Kraków, 2012.
[6] W. Batko, P. Pawlik, *Uncertainty Evaluation in Modelling of Acoustic Phenomena With Uncertain Parameters Using Interval Arithmetic*, Acta Physica Polonica A, vol. 121 No. 1-A, p. A-152-155, Warszawa, 2012.
[7] W. Batko, P. Pawlik, "*New approach to uncertainty assessment of acoustic effects in the environment*", Archives of Acoustics, vol. 37 no. 1 p. 57-61, 2012.
[8] W. Batko, B. Stępień, Application of the bootstrap estimator for uncertainty analysis of the long-term noise Acta Physica Polonica A vol. 118, p. 11–16 2010
[9] W. Batko, B. Stępień, Application of the Bayesian inference for estimation of the long-term noise indicators and their uncertainty Acta Physica Polonica A vol. 119 no. 6-A s. 916–920, 2011.
[10] W. Batko, R. Bal, Controlling of violations of the basic assumptions of the recurrent assessment model of long-term noise indicators, Archives of Acoustic, 35, 361–369, 2010.
[11] W. Batko, B. Przysucha Determination of the Probability Distribution of the Mean Sound Level, Archives of Acoustic Vol 35, No 4, p 543-550, 2010.
[12] Evaluation of measurement data _ Supplement 1 to the _Guide to the expression of uncertainty in measurement_ _ Propagation of distributions using a Monte Carlo method (GUM), BIPM, JCGM 101:2008.
[13] W. Batko, B. Przysucha, Random distribution of long-term indicators of variable emission conditions Acta Physica Polonica A Warszawa ; ISSN 0587-4246 vol. 119 6-A, p. 1086–1090, 2011
[14] W. Batko B. Przysucha, Statistical Analysis of the equivalent noise Level 59 OSA 10-14.092012 Boszkowo p 37-41 2012
[15] W. Batko B. Przysucha, Cascade Method Of The Determination Of The Sound Level Probability Distribution Function Złożono do Czasopisma Acta Physica Polonica A.
[16] W. Batko, B. Przysucha, Identification Of The Probability Distribution Form For The Results Of Sound Level Measurements, Mechanics and Control Vol. 32 No. 1 p. 6-12 2013
[17] W. Batko, B. Przysucha, Assessment of the accuracy of the noise sources identification at the determined background level Postępy Akustyki Rzeszów 2013 p 249-255
[18] W. Batko, B. Przysucha, P. Pawlik, Comparison of uncertainty determination methods of environmental noise threats assessment Postępy Akustyki Rzeszów 2013, p 255-267.
[19] K. Turzański, W. Batko, Stan zagrożenia hałasem na terenie miasta Krakowa, Oficyna Wydawnicza Text – Krakow, 1998.