MATHEMATICAL MODELLING AND COMPUTATIONAL APPROACHES

Modeling of Fluid Filtering N-layer Filters with Multicomponent Pollution

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Abstract – Dimensional mathematical model is formed that takes into account the reverse influence the determinants of the characteristics of the environment in the modeling of multi-cleaning fluid contamination n-layer sorption filters. The algorithm for solving the corresponding model problem described by a system of nonlinear singularly perturbed differential equations "convection-diffusion-mass transfer" is proposed. The results of calculations of the distribution of the impurity concentration and the mass amount of impurities in height porous filter loading for different times and for different number of layers are given.

Key words: filtration, n-layer filter, multicomponent pollution, mathematical model, a computer experiment, the asymptotics.

I. Introduction

Analysis of research results [1, 2] indicates that the complex structure of interdependence of different factors that determine the processes of filtration and filtration through a porous medium, which are not considered in traditional (classical phenomenological) models of such systems. Filtering is an equivalent reduction in the diameter of granules download - one of the methods generally improve the efficiency of filters [1].

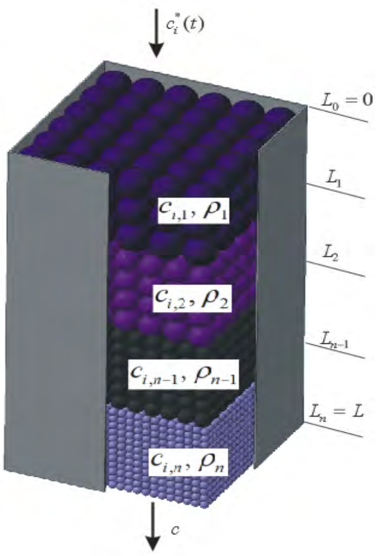

Fig. 1. Schematic representation of the n-layer filter

According to the investigation, we considere and decided the issue of incorporation inverse dependence of the process (the concentration of the liquid and sediment contamination) on the medium characteristics (coefficients porosity, filtration, diffusion, mass transfer, etc.) during the simulation of multi-cleaning fluid from impurities in the n-layer sorption filter.

II. Page Setup

We consider a one-dimensional spatial process of cleaning fluid by filtration in n-layer filter layer of thickness (Fig. 1), which is identified with the segment axis.

Consider [1] that particle of pollution can go from one state to another (processes capture, separation, adsorption, desorption), and the concentration of pollution affects the considered layer.

The concentration of pollution is a multiple, $c=c(x,t)=(c_1,...,c_m)=(c_1(x,t),...,c_m(x,t))$. This process filtration with inverse dependence of the process on the characteristics of the environment (voids ratio, filtration, diffusion, mass transfer, and so on. [1, 2]) is introduced as the following problem:

$$\begin{cases} \frac{\partial(\sigma(\rho)c_i)}{\partial t}+\frac{\partial\rho}{\partial t}+\frac{\partial(vc_i)}{\partial x}=D_i\frac{\partial^2 c_i}{\partial x^2}, \\ i=\overline{1,m},\ (x,t)\in G_k=\{x: L_{k-1}<x<L_k,\ 0<t<\infty\},\ k=\overline{1,l-1}, \\ \frac{\partial\rho}{\partial t}=\beta(\rho)\left(\sum_{i=1}^{m} q_i c_i\right)-\alpha(\rho)\rho+D_*\frac{\partial^2\rho}{\partial x^2}, \end{cases} \tag{1}$$

$$c_i\big|_{x=0}=c_i^*(t),\ \rho\big|_{x=0}=\rho^*(t),\ \frac{\partial c_i}{\partial x}\bigg|_{x=L}=0,\ \frac{\partial\rho}{\partial x}\bigg|_{x=L}=0,$$

$$c_i\big|_{t=0}=c_{*i}^*(x),\ \rho\big|_{t=0}=\rho_*^*(x), \tag{2}$$

$$[c_i]_{x=L_k}=0,\ [\rho]_{x=L_k}=0,\ \left[D_{i,k}\frac{\partial c_i}{dx}+vc_i\right]_{x=L_k}=0,\ \left[D_{*k}\frac{\partial\rho}{dx}\right]_{x=L_k}=0, \tag{3}$$

$$v=\kappa(\rho)\cdot grad\ P, \tag{4}$$

where $\rho(x,t)$ – is concentration of impurities trapped filter filling; $\beta(\rho)$ – is coefficient characterizing the volume of sediment contaminants per unit time, $(\beta(\rho)=\beta_0-\varepsilon\beta_*\rho(x,t))$; $\alpha(\rho)$ – is coefficient characterizing the amount of particles separated by the same time on the grain filling, $(\alpha(\rho)=\alpha_0+\varepsilon\alpha_*\rho(x,t))$; $c_i^*(t)$ – is impurity concentration at the inlet filter; $\sigma(\rho)$ – is porosity filtering attachments, $\sigma(\rho)=\sigma_0-\varepsilon\sigma_*\rho(x,t)$, where σ_0 – is initial porosity attachments, $\kappa(\rho)$ – is coefficient of filtration, $\kappa(\rho)=\kappa_0-\varepsilon\gamma\rho(x,t)$ ($x\in[L_{k-1},L_k]$); $D_i=\begin{bmatrix} D_{i,1}=b_1\varepsilon, \\ ... \\ D_{i,l}=b_l\varepsilon, \end{bmatrix}$, $D_*=\begin{bmatrix} D_{*1}=b_{*1}\varepsilon, \\ ... \\ D_{*l}=b_{*l}\varepsilon, \end{bmatrix}$,

$\beta_0, \beta_*, \alpha_0, \alpha_*, \sigma_*, b_k, b_{*k}, q_i, \kappa_0, \varepsilon$ – are solid parameters characterizing the corresponding coefficients; $\beta(\rho), \alpha(\rho), \sigma(\rho), \kappa(\rho)$ – are soft parameters found experimental method; ε – is small parameter; v – is rate of filtration, $[L_{k-1}, L_k]$ – is k-th layer of filter; P – is pressure in equations (3).

Asymptotic approximation of solution of the model problem (1) - (4) found in the form of asymptotic series [1, 2]:

$$c_{i,k}(x,t) = c_{i,k,0}(x,t) + \sum_{j=1}^{n} \varepsilon^j c_{i,k,j}(x,t) + \sum_{j=0}^{n+1} \varepsilon^j \underset{\sim}{M}_{i,k,j}(\underset{\sim}{\xi},t) + \sum_{j=0}^{n+1} \varepsilon^j \tilde{M}_{i,k,j}(\tilde{\xi},t) + \sum_{j=0}^{m+1} \varepsilon^j A_{i,l,j}(\xi,t) + R_{ci,k}(x,t,\varepsilon),$$
$$\rho_k(x,t) = \rho_{k,0}(x,t) + \sum_{j=1}^{n} \varepsilon^j \rho_{k,j}(x,t) + \sum_{j=0}^{2n+1} \varepsilon^{j/2} \underset{\sim}{P}_{k,j}(\underset{\sim}{\mu},t) + \sum_{j=0}^{2n+1} \varepsilon^{j/2} \tilde{P}_{k,j}(\tilde{\mu},t) + \sum_{j=0}^{m+1} \varepsilon^{j/2} B_{l,j}(\mu,t) + R_{\rho,k}(x,t,\varepsilon), \quad (5)$$

$R_{cj,k}, R_{\rho,k}$ – are remaining members; $c_{i,k,j}(x,t)$, $\rho_{k,j}(x,t)$, ($i=\overline{1,m}$; $j=\overline{0,n}$; $k=\overline{0,l}$) - are members of the regular units asymptotics; $\underset{\sim}{M}_{i,k,j}(\underset{\sim}{\xi},t)$, $\tilde{M}_{i,k,j}(\tilde{\xi},t)$, ($i=\overline{1,m}$, $j=\overline{0,n+1}$), $\underset{\sim}{P}_{k,j}(\underset{\sim}{\mu},t)$, $\tilde{P}_{k,j}(\tilde{\mu},t)$, ($j=\overline{0,2n+1}$, $k=\overline{0,l-1}$) – are features such as boundary layer in the neighborhood of $x = L_k$ (adjustment for the transition seepage from one of the k-th layer the next filter), $A_{i,l,j}(\xi,t)$, $B_{l,j}(\mu,t)$ ($j=\overline{0,m+1}$) – are features such as boundary layer in the neighborhood of $x = L$ (amendment output seepage flow), $\tilde{\xi} = x\cdot\varepsilon^{-1}$, $\tilde{\mu} = x\cdot\varepsilon^{-1/2}$, $\tilde{\xi} = (L-x)\cdot\varepsilon^{-1}$, $\tilde{\mu} = (L-x)\cdot\varepsilon^{-1/2}$, $\xi = (L-x)\cdot\varepsilon^{-1}$, $\mu = (L-x)\cdot\varepsilon^{-1/2}$ – are appropriate transformations which govern.

III. Page Setup

As a result of experiment using a computer algorithm solution for the problem (1) - (4) with $c_1^*(t) = 170$ mg/l, $c_2^*(t) = 35$ mg/l, $L = 0.8$ m; $v = 1/360$ m/s; $\beta_0 = 0.3$ s^{-1}; $q_1 = q_2 = 1$; $\alpha_0 = 0.0056$ s^{-1}; $\sigma_0 = 0.5$; $\alpha_* = 1$; $\beta_* = 1$; $\sigma_* = 1$; $b = b_* = 1$; $\varepsilon = 0.001$, the portraits of pollution concentration distribution at the inlet filter at time *t* (Fig. 2) and graphs of pollution concentration distribution at the output of the filter at time *t* (Fig. 3). The resulting field experiments (corresponding index "*exp*") is according to the classical model of Mintz (corresponding index "*M*") [13] and calculated by (5) (corresponding to the index "*p*"). We see that the calculations by formulas (5) is more accurate than the classical model Mintz. From Figure 3 we see that when *k = 3* and *k = 4* and a protective effect does not change, so three layers are sufficient to ensure the maximum effectiveness of treatment on these criteria, which used a 3-layer filter in practice.

These results make it possible to calculate the dynamics of the concentration of pollution and sedimentation pollution along the filter.

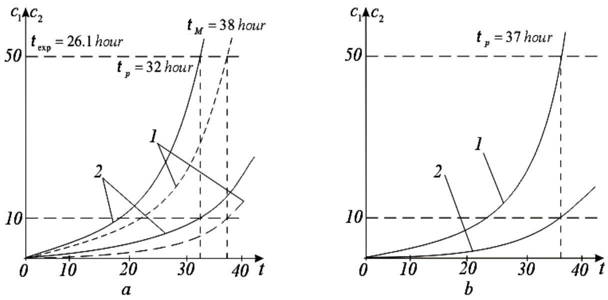

Fig. 2. Graphs of pollution concentration distribution at the output of the filter at time *t*: 1 – of Mintz; 2 – by (5) for $k = 1$ – (*a*) and the formula (5) for $k = 2$ – (*b*)

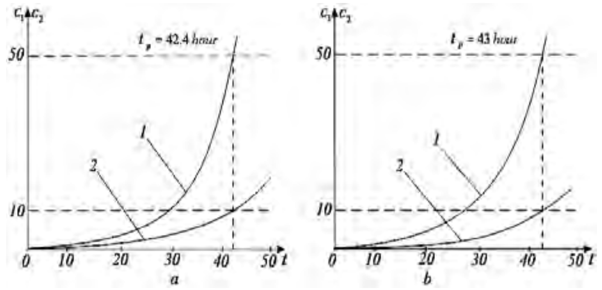

Fig. 3. Graphs of pollution concentration distribution at the output of the filter at time *t* by (5) for $k = 3$ – (*a*) and $k = 4$ – (*b*)

Conclusion

A spatial mathematical model that takes into account the reverse effect determining factors (concentration of fluid and sediment contamination) on the medium characteristics (porosity ratios, filtering, diffusion, mass transfer, etc..). During the modeling of multicomponent fluids clean dirt *n*-layer sorption filters.

The algorithm for solving the corresponding model problem is given. The results of calculations of separation of impurity concentration and mass amount of impurities in height porous filter loading for different times and for different number of layers. The comparative characterization data obtained by experimental, calculated on the basis of the classical model of Mintz and received by us as a result of calculations (Fig. 2).

References

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