Порівняння трьох різних підходів до кінематичного аналізу кривошипного механізму

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Механізми стали частиною нашого життя, вони присутні не лише в технічній сфері, але й у нашому повсякденному житті. Вони допомагають нам виконувати роботу легко і швидко. Механізм – це, як правило, частина машини, де з’єднані дві або більше деталей так, що рух першої деталі призводить до руху інших, який згідно з законом залежить від природи з’єднання. Часто у виробництві потрібно визначити швидкість і прискорення твердого тіла чи якоїсь частини механізму, якщо відомі вхідні параметри привідного механізму.

Найпоширенішим механізмом, який використовується на практиці, вважається кривошипний механізм; він є частиною різноманітних машин, а також практично кожного автомобіля. Цей чотирьох-ланковий механізм має деякі особливі конфігурації, створені шляхом забезпечення нескінченності довжини ланки, і часто використовується для перетворення обертового руху на прямолінійний. Отже кривошипно-повзунний механізм є основним механізмом, який досліджується кінематичним аналізом у навчальному процесі на факультеті виробничих технологій Технічного університету Кошице розташованого в Пряшеві в рамках курсу «Технічна механіка». Студенти опановують застосовування законів кінематики на практиці, вони також можуть порівняти три різні розв’язки задачі. Вони вчаться застосовувати графічний, числовий та комп’ютерний підходи, а також бачать переваги та недоліки окремих методів. Результати всіх трьох розв’язків повинні співпадати, таким чином студент може себе перевірити.

Варто зазначити, що найпоширенішим підходом є комп’ютерний, оскільки використання відповідного програмного забезпечення полегшує роботу, однак стаття наголошує на тому, що знання теоретичних принципів механіки є необхідним для розв’язання задач.

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Comparison of three various kinematic analysis solutions of crank mechanism

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The article deals with the kinematics analysis of crank mechanism realized by three methods followed by comparison of achieved results. The crank mechanism is the most commonly used mechanism in practice; it is the part of the various machines and almost every car. It belongs to the fundamental mechanisms whereon the kinematic characteristics of mechanism are investigated by students on FMT TU Košice with seat in Prešov within the scope of Technical mechanics lessons. Article hints that the utilization of suitable software can saves labour but the knowledge of theoretical mechanics principles for the problem solution are necessary. The paper was written thanks to support provided by the project KEGA num. 270-014TUKE-4/2010 and VEGA 1/0884/10.

Keywords – mechanism, kinematic analysis, methods, solution

I. Introduction

The slider-crank mechanism is the basic mechanism, which kinematic analysis is investigated as first in educational process on FMT TU Kosice with seat in Prešov within the Technical Mechanics subject. This four-bar mechanism has some special configurations created by making link infinite in length and it is often used to transform rotary motion into linear motion. Students learn to use the laws of kinematics in real situation. Figure 1 shows the real slider-crank mechanism of minibike Jawa 90 showed in Figure 2. [1]

Fig. 1. Real slider-crank mechanism of minibike Jawa 90

The aim of the kinematic analysis is to investigate the motion of individual components of mechanism (or their choices points) in dependence on the motion of drivers. To investigate of the motion means to determine the dependency of the position, velocity and acceleration of the examined members and important points on the motion of driven members or on the time. A point moving in space describes a line called its path, which may be rectilinear or curvilinear. The motion of a body is determined by the paths of three of its points not on a straight line. If the motion is in a plane, two points suffice, and if rectilinear, one point suffices to determine the motion. [3].

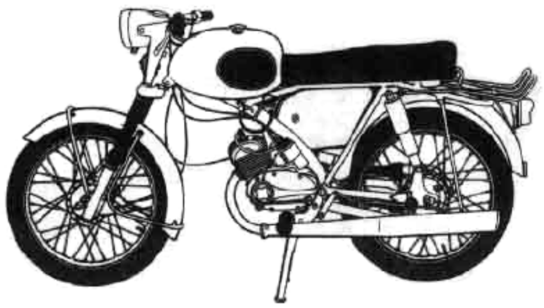

Fig. 2. Minibike Jawa 90 [1]

The input values of this specific mechanism and its position for comparison of values achieved by three various methods are:

- *length* *r = 24 mm;*
- *length* *l=90,25 mm = 0,09025 mm;*
- *angular position of crank 2* $\varphi_{21}=35^o$;
- *angular velocity of crank 2* $\omega_{21}= 31,416$ *rad/s;*

The kinematic analysis can be done by several manners such as analytical, graphical and computer aided solution.

II. Graphical solution

Graphical solution is suitable only for the solving of planar mechanism and come out from the kinematic scheme of mechanism sketched in the selected scale with the scaled input parameters in vector form.

The example of graphical solution of wing mechanism is shown in the Fig.2.

This solution consists in investigation of velocity and acceleration field of important mechanism points; in this case it is point B, so it was necessary to determine velocity v_B and acceleration a_B in immediate mechanism position (angular position of crank 2 is given by angle $\varphi_{21}=35^o$).

Graphical solution is suitable for investigation of planar mechanism. This method use vectors and the principles of operating with them, it is based on application of simultaneous motion theory and they prop oneself upon kinematics knowledge. Input parameters are drawn in needed scale, of course it influences the output values which have to be changed after solution according to scale. Students can use the colour lines for the higher limpidity of solution. For length parameters is used scale of length m_d and for velocity parameters is used scale of velocity m_v. After graphical solution it is needed to measure the obtained vectors of kinematic parameters and consequently to calculate the real obtained values of kinematic analysis. From the Fig.3 are visible measured values, but actual achieved values of velocity and acceleration after calculations are:

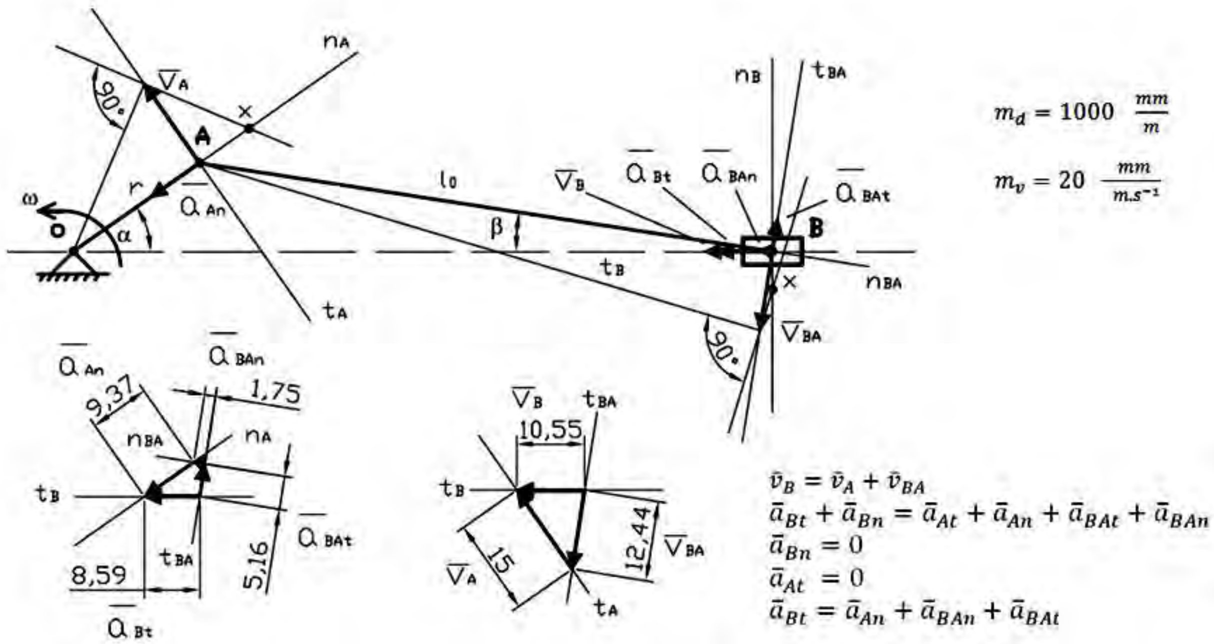

Fig. 3. Graphical solution in selected position of mechanism

$$v_B = \frac{v_{B\text{-measured}}}{m_v} = \frac{10,55}{20} = 0,53 \qquad \left[m.s^{-1}\right]$$

$$a_B = \frac{a_{B\text{-measured}}}{m_a} = \frac{8,6}{0,4} = 21,45 \qquad \left[m.s^{-2}\right]$$

III. Analytical solution

There are several types of analytical solution that is usually concerned on the task of the position. Most often analytical method uses the trigonometric rules and mathematical definitions as are functions, differentiation, equations, etc.

The simple representation of real mechanism that serves as the basis for next processing is the kinematic scheme. The individual components of mechanism in this scheme are numbered due to the numerical solution limpidity. The frame has number 1, the driver is the crank with number *2* that rotates with angular speed ω_{21} and the goal is to define the motion of the slider (component 4), which all points describe line path. The kinematic scheme of crank-slider mechanism was prepared on the basis of real slider-crank mechanism and it is shown in the Fig. 4.

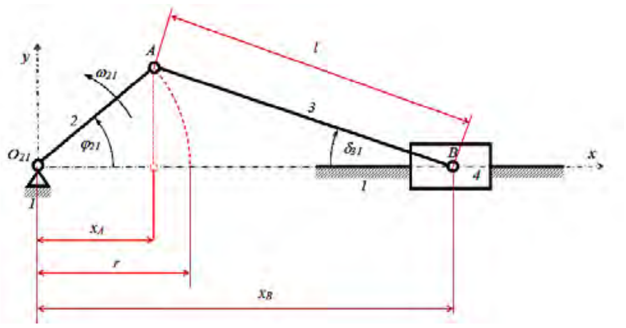

Fig. 4. Kinematic scheme of mechanism

The coordinate system of mechanism was for numerical solution located into the point O_{21}, position of the important points is described by *x* a *y* coordinates. The dependency of these coordinates on the angle $\varphi_{21(t)}$ is defined by trigonometric method. It can be said, that the kinematic values of component 4 is determined by kinematic values of point B, because slider 4 executes the linear motion and all its points make the same move.

Angle position of the crank *2* is determined by angle φ_{21} that is changed with time. The position of slider 4 is changed with time, too, because its motion depends on the motion of crank 2. Kinematic dependency of angle φ_{21} on time, if angular speed ω_{21} is constant can be expressed:

$$\omega_{21} = \frac{d\varphi}{dt}$$

$$\int_0^{\varphi_{21}} d\varphi = \omega_{21} \int_0^t dt$$

$$\varphi_{21} = \omega_{21} . t \qquad (1)$$

For point B holds

$$v_{Bx} = -r\omega_{21} \sin \omega_{21} t - \frac{r^2 \omega_{21} \sin 2\omega_{21} t}{2.\sqrt{\left(l^2 - r^2 \sin^2 \omega_{21} t\right)}} \qquad (2)$$

$$v_{By} = \dot{y}_B = 0 \qquad (3)$$

Due to the linear motion, the total velocity of the point B is the same as its horizontal velocity, what can be expressed:

$$v_B = \left|\bar{v}_B\right| = \sqrt{\dot{x}_B^2 + \dot{y}_B^2} = \left|\dot{x}_B\right| \quad \left[m.s^{-1}\right] \qquad (4)$$

Particular and total acceleration of point B are given by relations:

$$a_{Bx} = -r\omega_{21}^2 \cos \omega_{21} t - \frac{r^2 \omega_{21}^2}{4.\left(l^2 - r^2 \sin^2 \omega_{21} t\right)^{3/2}} .c$$

$$c = \left[4.\left(l^2 - r^2 \sin^2 \omega_{21} t\right).\cos(2\omega_{21} t) + r^2 .\sin^2(2\omega_{21} t)\right]$$

$$a_{By} = \dot{v}_{By} = 0 \qquad (6)$$

$$a_B = \sqrt{a_{Bx}^2 + a_{By}^2} = \sqrt{a_{Bx}^2 + 0} = a_{Bx} \quad \left[m.s^{-2}\right] \qquad (7)$$

After institution of concrete values *r, l,* φ_{21} and ω_{21} into the equations (2), (3), (4) , (5), (6) and (7), the values of velocity and acceleration are:

$$v_{Bx} = -0,528644964 \quad m.s^{-1}$$
$$a_{Bx} = -21,37024049 \ m.s^{-2}$$
$$v_{By} = 0 m.s^{-1}$$
$$a_{By} = 0 m.s^{-2}$$
$$v_B = -0,528644964 \quad m.s^{-1}$$
$$a_B = -21,37024049 \quad m.s^{-2}$$

IV. Computer aided solution

Computer aided solution use the special software dedicated for it. Today there are very interactive and user friendly 3D software in the market, which can simulate not only the motion of the mechanism, but they can define the position, velocity, acceleration, forces, moments and other parameters in every moments of time in graph or vector version, for example. [6] Inside computer application primarily it is necessary to create the 3D models of individual components of mechanism, secondary to join them by kinematic linkage which removes needed number of the degree of freedom. The virtual model of slider-crank mechanism was created in software Pro/Engineer and it is showed on the Fig.5.

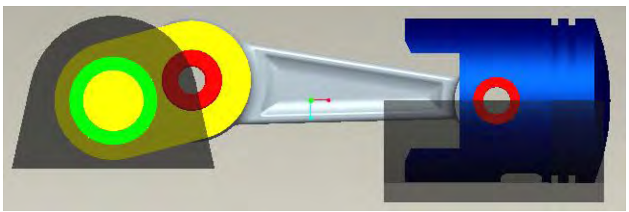

Fig. 5. Virtual model of real slider-crank mechanism

The simulation tools of Pro/Engineer belong in the software, which are suitable for the analysis and the control rationalization of complicated processes. It provides students to perform the kinematic motion simulation and behavioural insight into the assembly through the easy definition and animation of connections. Once assembled, students can observe how their mechanism designs will behave geometrically through interactive part dragging and user-defined motion simulations. Predefined motion simulations, using drivers to simulate motors or actuators, also provide animation.

After the modelling mechanism, joints and input parameter definition, it was possible to provide the kinematic analysis. Output data could be designed direct in software Pro/Eas values or as graphs or it can be sent to other software for the next processing. Fig.6 shows the velocity [mm/s] and the acceleration [mm/s^2] profile of the slider 4 after analysis has been executed depending up the angle position crank 2 measured in Deg [º].

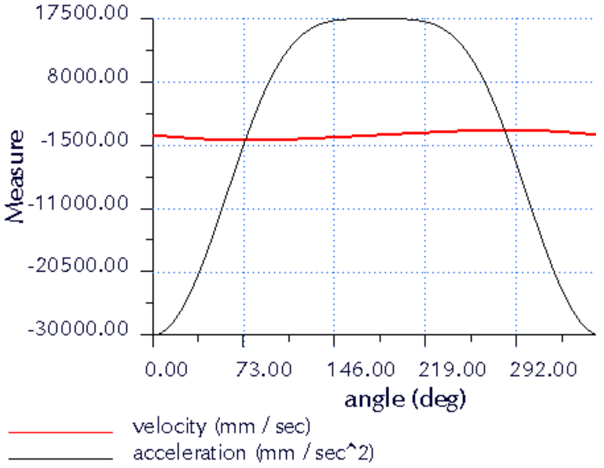

Fig. 6. Output data of kinematic analysis

Conclusion

The values obtained by all approaches to kinematic analysis are presented in Tab.1. It can be said that the results are the same and students can compare the achieved values. If one of them is different, they have to look for the mistake.

That is, kinematics deals with the functional relationships between the parts are interconnected, and how those parts move relative to each other. Only after choices have been made regarding those three factors can matters such as strengths, materials, fabrication techniques, and costs be seriously addressed. Failure to devote the proper attention to kinematics "up front" can, and often does, result in the design of a system with substandard or nonoptimum performance and/or with unsatisfactory reliability. [5]

Fortunately, today, the ready availability of very powerful personal computers and the associated software allows kinematic synthesis and analysis, which were formerly laborious, to be performed quickly and cheaply. There is no longer an excuse for avoiding doing careful kinematic design up front. [4] Because of the availability of these computer aids and the consequent incentives to apply kinematic principles in design, it is becoming increasingly important for the practicing engineer to have a good understanding of those kinematic principles. [2]

Table 1

Kinematic analysis results

Kinematic analysis – point B *Values for φ_{21}=35°*		**Methods**		
		Graphical solution	***Analytical***	***Computer aided***
Speed	v_B [m.s^{-1}]	*- 0,53*	*-0,5286*	*- 0,52864496*
Acceleration	a_B [m.s^{-2}]	*- 21,45*	*-21,3702*	*- 21,37024049*

Whereas all three solutions use the same principles, students have to understand them, because the design of a machine or mechanism or any moving mechanical system always starts with a consideration of kinematics because kinematics is the study of the geometry of motion. Actually before engineers can start to use a computer for synthesis or analysis of a machine, they must develop some initial concept of how the machine will operate.

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