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INVESTIGATION OF THE INFLUENCE OF DATA FINITE PRECISION ON REALIZATION OF DIGITAL CONTROL SYSTEMS

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Abstract. The existence of the problem of impact of the finite precision of hardware in digital control systems which significantly influences on their practical realization was shown. The influence of data finite precision of digital control systems on their behavior using popular discretization methods (the Tustin method and the method of the correspondence of zeros and poles) and forms of discrete transfer functions and different representation of discrete transfer functions has been investigated by the computer model. The existence of the influence of data finite precision on digital control systems practical realization was confirmed and the method of partial reduction of the impact was proposed according to the results of computer simulation.

Keywords: digital control systems, finite precision digital controller, sampled-data systems.

1. Introduction

Digital control systems are the most effective ones, since they provide a high accuracy, universality and possibility of realization to very complicated control algorithms. Their usage is the most perspective from the point of view of operational, energetic and dynamical characteristics that are necessary for the operation of a certain technological system.

The synthesis of a digital control system is as usual based on discretization of continuous prototype. The popular methods of control system discretization include the substitution (for example, a Tustin method), the correspondence of zeros and poles of digital transfer function etc. [1, 2]. The usage of given methods provides the simple effective realizations of digital control systems.

One of the problems in the theory of digital control systems, that is, the problem of impact of finite-precision data on their behavior, is not enough investigated. This fact vastly influences their synthesis and practical realization.

2. Analysis of known solutions

The finite precision of hardware is taken into account in the development of digital filters, for example [3, 4]. As a rule, it means only considering the finite precision of input and output data and discrete disposition of zeros and poles of digital systems on the complex plane.

In the known works [1, 2, 5] the impact of the data finite precision on the behavior of the system is analyzed concerning only two factors:

- the influence of data finite precision on the accuracy of maintaining regulated coordinates, which is, basically, determined by the resolution of an ADC or digital sensors;
- the disposition of zeros and poles of discrete transfer functions only in limited in number discrete points on a complex plane.

That is why that in the majority of cases the analysis of the stability of digital control systems is conducted with classical methods intended for linear systems – exploring the disposition of zeroes and poles on the complex plane, analyzing frequency characteristic etc. [5, 6]. However, conducted research showed some other factors causing the digital control system instability. For example, it may emerge at a certain value of binary resolution of a digital device.

It should be mentioned that the problem of numerical instability in the case of the limited calculation resolution is well-known in applied mathematics. For example, this problem is examined in [7, 8, 9]. However, the results of the investigations conducted by the authors are not popular among the specialists in the digital systems theory so far, especially concerning the impact of the accuracy of discrete transfer function polynomial coefficients on the accuracy of their solutions.

3. Investigations

The most popular methods of the discretization during the syntheses of the digital control systems are the Tustin method and the method of the correspondence of zeros and poles. The discrete transfer functions obtained by those methods could be defined in two ways:

- as a classical transfer function by the ratio of polynomials of a numerator and a denominator;
- in terms of zeros and poles.

The aim of the research was to study the impact of the finite precision of transfer function coefficients on the behavior of a digital system that is realized by the methods listed above.

The quite simple dynamic object with two pairs of complex conjugated poles was chosen as the object of research:

1) $p_{1,2} = 1 \pm j$;
2) $p_{3,4} = 3 \pm 3j$.

Its resulting transfer function has the form:

$$W(s) = \frac{1}{s^4 + 8s^3 + 32s^2 + 48s + 36}.$$

The appropriate discrete transfer functions of digital control systems have been synthesized by above mentioned methods (for example, using Control systems Toolbox of MATLAB software environment). They are obtained in the following forms (on conditions that coefficients depend on the discretization method and step):

1) transfer function obtained by the Tustin method as a ratio of polynomials of the numerator and denominator:

$$W(z) = \frac{b_4 z^4 + b_3 z^3 + b_2 z^2 + b_1 z + b_0}{z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0};$$

2) transfer function obtained by the Tustin method and recorded in the form of zeros and poles:

$$W(z) = \frac{k \cdot (z+1)^4}{(z^2 + p_1 z + q_1)(z^2 + p_2 z + q_2)};$$

3) transfer function obtained by the zero-pole matching method as a ratio of polynomials of the numerator and denominator:

$$W(z) = \frac{b_3^* z^3 + b_2^* z^2 + b_1^* z + b_0^*}{z^4 + a_3^* z^3 + a_2^* z^2 + a_1^* z + a_0^*};$$

4) transfer function obtained by zero-pole matching method recorded in the form of zeros and poles:

$$W(z) = \frac{k^* \cdot (z+1)^4}{(z^2 + p_1^* z + q_1^*)(z^2 + p_2^* z + q_2^*)}.$$

Analysis of the behavior of digital systems obtained was carried out for sampling steps h = 1; 0.3; 0.1; 0.03; 0.01. The modeling software environment Simulink, being the part of the mathematical application MATLAB, was used as a tool for this research.

The computer model of research, made in MATLAB environment, is shown in Fig. 2. All coefficients of the numerator and denominator of the discrete transfer functions and the coefficients in zero-pole form are defined using only four decimal digits as the realization of their finite precision.

It is commonly known that in the case of the decrease of the sampling step the system theoretically approaches its behavior to a continuous model. However, finite-precision notation of transfer function coefficients in the system leads to some unexpected consequences – decreasing of sampling step leads to the non-typical behavior of the obtained digital system deviating from continuity.

It has been determined that the digital control system represented by the classic discrete transfer function (as a ratio of polynomials of the numerator and denominator) becomes unstable in case of sampling step less than 0.03 s (Fig. 3). However, in the case of the zero-pole presentation of the discrete transfer function the digital control system remains stable (Fig. 4).

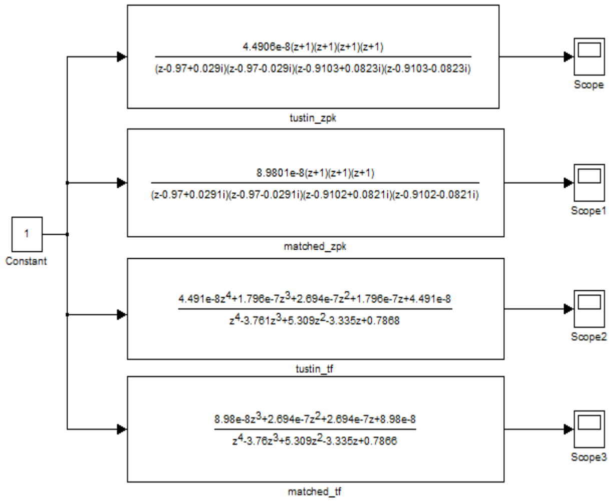

Fig. 2. Computer model of the digital system.

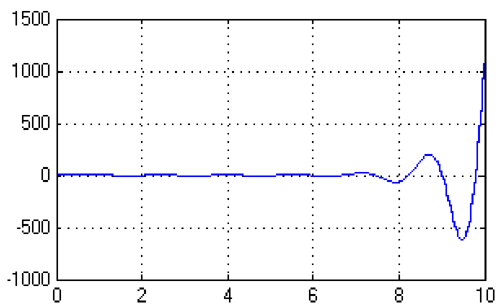

Fig. 3. The influence of finite-precision data on digital control system with a step h = 0.03 s.

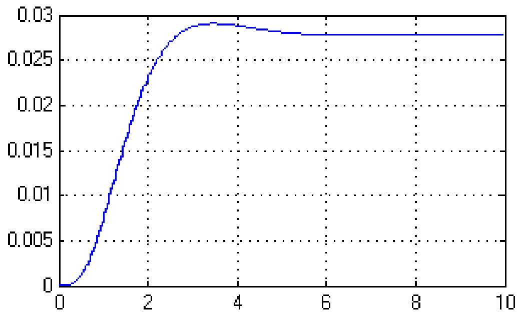

Fig. 4. Transfer characteristic of a system recorded in zero-pole form with a step h = 0.03 s.

The explanation of this phenomenon can be found in publications on applied mathematics, such as [8, 9]. It is shown there that polynomials with multiple or close roots are very sensitive to accuracy of the transfer function coefficients. Decreasing of sampling step leads to the displacement of all zeros and poles of the discrete transfer function to a unity. So, all solutions of the polynomials in the numerator and denominator become very close. Consequently, polynomials become singular and, as a result, very sensitive to accuracy of the coefficient resolution.

According to the results of research done the representation of the discrete transfer function in the form of zeros and poles is more resistant to this kind of a drawback.

4. Conclusion

Data finite precision in digital control systems significantly influences on their practical realization. The appearance of this problem can be explained by the sensitivity of coefficients of the discrete transfer function and resulted values of its zeros and poles to small errors occurred while presenting the data in mathematical notation.

For decreasing the influence of the data finite precision in digital control systems it is necessary to use the presentation of the discrete transfer functions in the form of zeros and poles.

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ДОСЛІДЖЕННЯ ВПЛИВУ ОБМЕЖЕНОЇ РОЗРЯДНОСТІ НА РЕАЛІЗАЦІЮ ЦИФРОВИХ СИСТЕМ

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Показано, що у цифрових системах керування існує проблема обмеженої розрядності апаратної частини, яка суттєво впливає на їх синтез та практичну реалізацію. На основі комп'ютерної моделі досліджено вплив обмеженої розрядності задавання параметрів цифрових систем керування на їх поведінку у випадку використання популярних методів дискретизації (підстановки Тастіна, методу відповідності нулів та полюсів) та на форму запису дискретних передатних функцій. Відповідно до результатів комп'ютерного моделювання, підтвер-джено існування впливу обмеженої розрядності на практичну реалізацію цифрових систем керування та запропоновано спосіб часткового зменшення даного впливу.

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