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ELECTROMAGNETIC CIRCUIT MODEL OF AN EDDY CURRENT DEFECTOSCOPE

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Abstract: A model of a device for nondestructive control by eddy currents has been proposed. The device and the object under control are represented by an equivalent circuit in the form of an electromagnetic circuit described by the system of differential finite equations in the coordinates of loop magnetic fluxes and currents of windings. The model allows us to take into account three parameters of the defects: depth, width, and conductivity. The spline method of analysis of periodic processes is used to carry out calculation.

Keywords: eddy current, nondestructive control, magnetic flux, excitation coil, sensing coil.

1. Introduction

The eddy current defectoscope is a nondestructive control device consisting of one or more inductive coils which are designed for excitation of eddy currents in a control object. It converts an electromagnetic field, which depends on the parameters of the object, into an information signal of defectoscope. There exist models of the devices for nondestructive control using eddy currents method which are based on Maxwell's equations for monoharmonic processes in terms of magnetic vector potential [3]. These models are complicated for engineering calculations of these devices. In this paper we use the approach proposed in the article [2].

2. Mathematical formulation of the task

This paper deals with a magnetic circular model of the eddy current defectoscope with a ferromagnetic core (Fig. 1).

To build the model we have made the following assumptions:

1. The magnetic field in elementary volumes is considered homogeneous, and therefore these volumes in the schemes are presented as lumped magnetic reluctance.
2. The displacement currents in the windings are neglected.
3. The temperature effect on the electrical conductivity is neglected.
4. The phenomenon of hysteresis (basic magnetization curve) is not considered.
5. The calculated scheme is presented not as a three-dimensional but two-dimensional planar grid taking into account the symmetry of the field.

The domains with defectoscope, the object of control and the environment around are divided into elementary volumes in the form of various-sized cylinders (Fig. 2). The magnetic fluxes transit through these volumes in vertical and horizontal directions. These cylinders represent reluctance R_{mv}, R_{mh} of domain with its vertical and horizontal components of the magnetic flux.

Reluctance R_{mv}, R_{mh} is determined by the following expressions:

$$R_{mv} = h \Big/ \left(\mu\mu_0 \int_{\alpha_1}^{\alpha_2} \int_{r_1}^{r_2} r d\alpha dr \right) = 2h \Big/ \left(\mu\mu_0 \pi \left(r_2^2 - r_1^2 \right) \right); \quad (1)$$

$$R_{mh} = \int_{r_1}^{r_2} dr \big/ \left(\mu\mu_0 h \alpha r \right) = \ln\left(r_2 / r_1 \right) \big/ \left(\mu\mu_0 h \alpha \right); \quad (2)$$

where h, α, r_1, r_2 are the geometrical sizes of elementary volume (Fig. 2); μ is the relative magnetic inductivity of a material of elementary volume; μ_0 stands for the permeability of vacuum.

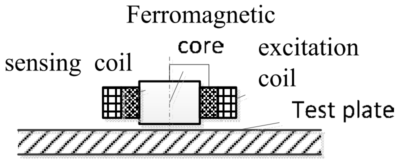

Fig. 1. Eddy current nondestructive testing.

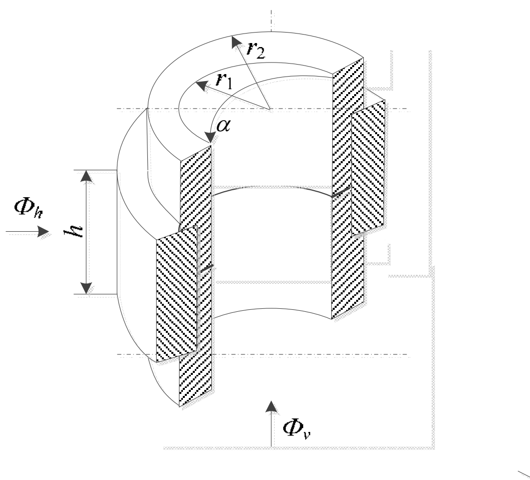

Fig. 2. Elementary volume and its parameters.

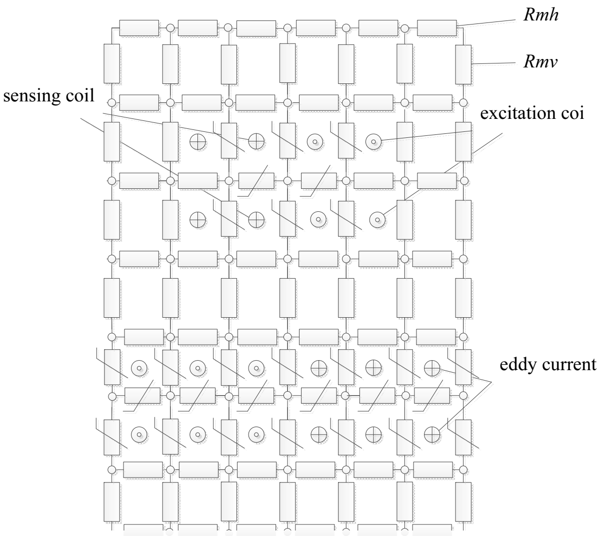

Fig. 3. Magnetic circuit diagram.

The studied objects can be made of ferromagnetic and non ferromagnetic conductive materials, i.e. the electromagnetic circuit is nonlinear (Fig. 3). Defects in these objects are approximated by the resistance change of the respective domain of broken area. This approach allows you to explore the parameters of defects, namely their depth, width and conductivity. The depth of the defect is given by the reluctances R_{mv} of the respective cylinders, which represent the object and its width – reluctances R_{mh}.

The mathematical model of nondestructive testing instrument is formed in the coordinates of loop magnetic fluxes and currents of windings as a system of differential finite equations:

$$\mathbf{\Gamma}_m \vec{U}_m(\Phi) - \mathbf{W}_t \vec{i} = 0 \, ; \qquad (3)$$

$$\mathbf{W}\, d\vec{\Phi}_\kappa / dt + \mathbf{R}\vec{i} - \vec{u} = 0 \, ; \qquad (4)$$

where: $\mathbf{\Gamma}_m$ is $(q \times s) \times p_m$-dimensional second incidence matrix of the magnetic circuit (here $q \times s$ is the contours of the magnetic circuit; p_m stands for the branches of the magnetic circuit); $\bar{U}_m(\vec{\Phi}) - p_m$ represents the dimensional column vector of the magnetic branches voltages; $\bar{\Phi}_\kappa$ is the column vector of the loop magnetic fluxes; $\mathbf{W} - n \times (q \times s)$ stands for the dimensional matrix of the elementary loops turns of the magnetic circuit; $\vec{i} = (i_1, i_2, \ldots i_n)_t$ represents the column vector of the defectoscope coils currents and eddy currents; $\vec{u} = (u_1, 0, \ldots 0)_t$ is the column vector of the voltages of the solenoid and equivalent windings; $\mathbf{R} = \mathrm{diag}(R_1, R_2, \ldots R_n)$ is the resistance diagonal matrix.

The model of nondestructive testing instrument in the coordinates of the current branches is created by differentiating equation (1) with respect to magnetic flux Φ

$$\mathbf{\Gamma}_m \, d\vec{U}_m / d\vec{\Phi} - \mathbf{W}_t \, d\vec{i} / d\vec{\Phi} = 0 \, . \qquad (5)$$

From equation (5) the following is received

$$\partial\vec{\Phi}_k / \partial\vec{i} = \left(\mathbf{\Gamma}_m \partial\vec{U}_m / \partial\vec{\Phi}\, \mathbf{\Gamma}_{mt}\right)^{-1} \mathbf{W}_t = \mathbf{R}_{mk}^{-1} \mathbf{W}_t \qquad (6)$$

where $\vec{\Phi} = \mathbf{\Gamma}_{mt} \vec{\Phi}_k$.

After some transformations we obtain the following equation (4)

$$\mathbf{M}\, d\vec{i} / dt + \mathbf{R}\vec{i} - \vec{u} = 0 \, , \qquad (7)$$

where $\mathbf{M} = \mathbf{W}\, \partial\vec{\Phi}_k / \partial\vec{i} = \mathbf{W}\mathbf{R}_{mk}^{-1}\mathbf{W}_t$; $\mathbf{R}_{mk}^{-1}$ is the circuit matrix of differential magnetic reluctance.

Here is the same equation written in the Cauchy form

$$\mathbf{M}\, d\vec{i} / dt = \vec{u} - \mathbf{R}\vec{i} \, , \qquad (8)$$

where $\vec{u}$ is the periodic function.

Since we are interested in a stationary mode in this circuit, it is better to apply the method of calculation which allows doing this without calculating a transient.

We use the spline method for calculating the discussed periodic processes.

A solution to equation (8) is a periodic dependence $\vec{i}(t) = \vec{i}(t+T)$.

Let us draw a n+1 nodes grid at the period T and obtain n domains. The coordinates of vector $\vec{i}$ at each of these domains are approximated by the cubic spline

$$\vec{i}(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3 , \qquad (9)$$

where a_i, b_i, c_i, d_i are the spline coefficients; $i = (1, n)$ is the number of a domain.

We are using a regular nodes grid with the step $h = t_{i+1} - t_i$.

From the structure of cubic spline we can see

$$i(t_i) = i_i = a_i \, , \qquad (10)$$

$$(di/dt)_{t_i} = b_i \, . \qquad (11)$$

Taking into account the conditions of periodicity

$$a_{i+1} = a_i \, ; \; b_{i+1} = b_i \qquad (12)$$

an algebraic analogue of the system of differential equations (6) is obtained as a system of n non-linear algebraic equations:

$$\begin{Vmatrix} \mathbf{M}\cdot\mathbf{B}_{11} & \mathbf{M}\cdot\mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{Vmatrix} \begin{Vmatrix} \vec{a} \\ \vec{c} \end{Vmatrix} = \vec{u} - \mathbf{R}\vec{a} \, ;$$

$$\vec{c} = -\mathbf{B}_{22}^{-1}\mathbf{B}_{21}\vec{a} \, ;$$

$$\mathbf{M}\left(\mathbf{B}_{11} - \mathbf{B}_{12}\mathbf{B}_{22}^{-1}\mathbf{B}_{21}\right)\vec{a} = \vec{u} - \mathbf{R}\vec{a} \, , \qquad (13)$$

or

$$\mathbf{Q}\vec{a} = \vec{u} \tag{14}$$

where $\vec{a} = (a_1, \ldots, a_m)_t$ is the column-vector of all the currents values in the grid nodes; $\mathbf{B}_{i,j}$ are the square $(m \times m)$ matrixes; $\mathbf{Q} = \mathbf{M}\left(\mathbf{B}_{11} - \mathbf{B}_{12}\mathbf{B}_{22}^{-1}\mathbf{B}_{21}\right) - \mathbf{R}$ is the square $(m \times m)$ matrix; $m = k \cdot n$; k is the number of currents in the circuit .

$$\mathbf{B}_{11} = \left\| \begin{matrix} 1/h & -1/h & 0 & \ldots & 0 \\ 0 & 1/h & -1/h & \ldots & 0 \\ 0 & 0 & \ldots & \ldots & 0 \\ 0 & \ldots & 0 & 1/h & -1/h \\ 0 & -1/h & 0 & \ldots & 1/h \end{matrix} \right\|; \tag{15}$$

$$\mathbf{B}_{12} = \left\| \begin{matrix} -2h/3 & -h/3 & 0 & \ldots & 0 \\ 0 & -2h/3 & -h/3 & \ldots & 0 \\ 0 & 0 & \ldots & \ldots & 0 \\ 0 & \ldots & 0 & -2h/3 & -h/3 \\ 0 & -h/3 & 0 & \ldots & -2h/3 \end{matrix} \right\|; \tag{16}$$

$$\mathbf{B}_{21} = \left\| \begin{matrix} 3/h & -6/h & 3/h & \ldots & 0 \\ 0 & 3/h & -6/h & 3/h & 0 \\ 0 & 0 & \ldots & \ldots & 0 \\ 0 & 3/h & \ldots & 3/h & -6/h \\ 0 & -6/h & 3/h & \ldots & 3/h \end{matrix} \right\|; \tag{17}$$

$$\mathbf{B}_{22} = \left\| \begin{matrix} -h & -4h & -h & \ldots & 0 \\ 0 & -h & -4h & -h & 0 \\ 0 & 0 & \ldots & \ldots & 0 \\ 0 & -h & \ldots & -h & -4h \\ 0 & -4h & -h & \ldots & -h \end{matrix} \right\|. \tag{18}$$

The system of nonlinear algebraic equations (14) has been solved by Newton's method and the method of continuation with respect to a parameter

$$\mathbf{Q}\Delta\vec{a}^{(k)} = \vec{F}\left(\vec{a}^{(k)}\right), \tag{19}$$

where $\vec{F}\left(\vec{a}^{(k)}\right) = \mathbf{Q}\vec{a}^{(k)} - \varepsilon\vec{u}$ stands for the vector of residuals of the system (14).

3. An application example

The approbation of the model has been performed for the eddy current nondestructive testing device and the object under control with such parameters.

Fig. 4 shows the result of calculation of the current in the excitation coil.

Table 2 displays changes of the excitation coil impedance and voltage of the coil under investigation depending on the location of the defect.

Table 1

The model parameters

Parameter	Value	Unit of measure
U_m	24	V
N_{ex}	600	Number of windings
R_{ex}	52,3	Ω
d_c	0,003	m
h_c	0,006	m

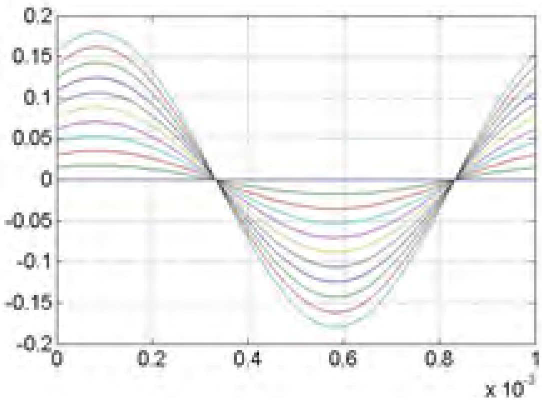

Fig. 4. Current of the excitation coil.

Table2

Results of calculation

	1 kHz		**10 kHz**	
	ΔZ, Ω	**Δ*U*, mV**	**ΔZ, Ω**	**Δ*U*, mV**
Defect in an upper layer	0.22	694.4	90.42	299.5
Defect in a lower layer	0.2176	109.3	2.2	3.6
Defects in all layers	0.5657	1407.3	207.5	659.0

4. Conclusion

The proposed model allows us to optimize the parameters of a nondestructive testing instrument (its design, voltage of an excitation coil as a function of time) for the gain in sensitivity to defects.

References

[1] V. Muzhytskyy, A. Byzyulev, R. Zahydulyn, „Eddy current defectoscope VD-12NFP and methods of processing the measured signal from the defect“, *Defectoscopiya*, no.5, pp. 85-91, 2004. (Russian)

[2] V. Perkhach, T. Shelepeten, V. Horyachko, “Digital quasi etalon model electromagnetic devices// Mathematical modeling of electrical engineering, electronics and power”, in *Proc. 3rd International Conference on Matematychne modeliuvannia v elektrotekhnitsi, elektronitsi ta energetytsi,* pp. 208-209, Lviv, Ukraine: Lviv Polytechnic National University, 1999. (Ukrainian)

[3] V. S. Cecco, G.van Drunen, F. L. Sharp, *Manual on eddy current method,* Chalk River, ON: Canada, 1981.

[4] V. Malyar, "The main provisions of the spline method of calculating periodic modes of operation of electrical circuits", *Elektronika i svyaz*, no. 5, pp. 11-14, 1998. (Russian)

МАГНІТНО-КОЛОВА МОДЕЛЬ ВИХРОСТРУМОВОГО ДЕФЕКТОСКОПА

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Всеволод Горячко

Запропоновано модель пристрою неруйнівного контролю за методом вихрових струмів. Пристрій і досліджуваний об'єкт представлені заступною схемою у вигляді електромагнітного кола, описаних системою диференційно-скінчених рівнянь в координатах контурних магнітних потоків та струмів обвиток. Модель дозволяє врахувати три параметри дефектів: глибину, ширину й провідність. Для розрахунку використовується сплайн-метод аналізу періодичних режимів.

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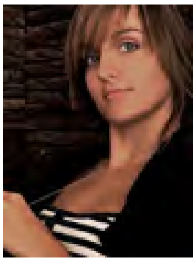

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