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BELLMAN DYNAMIC PROGRAMMING OF THE TIME-VARYING ELECTRIC DRIVE

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Abstract: The modern electric drives with variable parameters such a winding mechanism require an updated approach to construction of an optimal control system. The expression of an interpolation polynomial for tuning a speed regulator of a winding mechanism based on dynamic programming by Bellman is given in this paper. As a result, the proposed third-order interpolation polynomial can be used to construct an adaptive control system of a drive system such as a winding drive.

Key words: Thyristor electrical drive, winding mechanisms, dynamics.

1. Introduction

In the modern paper and metallurgy industry on the coilers for sheets of the paper and metal, thyristor electrical drives are used. It is known that processes of winding the continuous growth of the moment of inertia takes place. In connection with this, the electric drive of the considered aggregates presents drives with variable parameters. During the transition regime, the change of the moment of inertia is not essential and we consider it to be constant. However, during the cycle of the mechanism operation, it changes essentially. For regulation of the system of such electric drives, the usage of regulators with constant coeffitients is inadmissible. It is clear that if we adjust regulators orienting on initial values of object parameters, after some period adjustment will be nonoptimal and make the system unstable.

On the basis of our investigation [1], we consider it sufficient to divide the period of operation cycle of the windable mechanism into several time intervals and to abjust regulators (with the help of additional devices) in accordance with initial parameter values for each interval. For optimal control by rooling drive (or coiler), we use the method of dynamic programming by Bellman [1].

Let us write equations of dynamics of electrical drives for winding mechanisms without taking into consideration disturbance:

$$\frac{d\vec{x}}{dt} = \mathbf{A}(t) + \vec{B}u \,, \qquad (1)$$

where

$$\mathbf{A}(t) = \begin{bmatrix} 0 & J_2^{-1}(t) & 0 & 0 \\ -C_{12} & 0 & C_{12} & 0 \\ 0 & -J_1^{-1} & 0 & C_e J_1^{-1} \\ 0 & 0 & -C_e L^{-1} & -rL^{-1} \end{bmatrix}$$

$$\vec{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_0 L^{-1} \end{bmatrix}$$

$\vec{x}^T = [x_1 \quad x_2 \quad x_3 \quad x_4]$; x_1, x_2, x_3 and x_4 correspondingly are angular speed of mechanism, drive elastic moment, angular speed and current of the engine; u is signal of the drive control; C_e, J_1, r, L are the electromechanical parameters of the drive; K_0 stands for the amplification factor of thyristor converter feeding anchor of electric drive; $J_2(t)$ are the moments of inertia of rolling (with a roll) defined by formula [2]:

$$J_2(t) = \frac{(a+bt)t}{c+dt}, \; t \in [0;2000] \qquad (2)$$

where a, b, c and d are the factors depend on geometrical size of a winding roll, working speed of mechanism and production material .

For right introduction of a technological process of the given mechanism, the electric drive must support as a minimum following functional [3]:

$$J = \int_0^\infty \left(\vec{x}^T \mathbf{Q} \vec{x} + \vec{u}^T \mathbf{R} \vec{u} \right) dt \to \min, \qquad (3)$$

where $\vec{x}^T \mathbf{Q} \vec{x}$ and $\vec{u}^T \mathbf{R} \vec{u}$ are the positively defined square matrices; $\mathbf{Q}$ and $\mathbf{R}$ represent the given diagonal matrices defined as follows:

$$\mathbf{Q} = \begin{bmatrix} \tilde{x}_1^{-2}(t) & 0 & 0 & 0 \\ 0 & \tilde{x}_2^{-2}(t) & 0 & 0 \\ 0 & 0 & \tilde{x}_3^{-2}(t) & 0 \\ 0 & 0 & 0 & \tilde{x}_4^{-2}(t) \end{bmatrix};$$

$$\mathbf{R} = \tilde{u}^{-2}(t), t \in [0;3], \qquad (4)$$

where $\tilde{x}_1^{-2}(t), \tilde{x}_2^{-2}(t), \tilde{x}_3^{-2}(t), \tilde{x}_4^{-2}(t)$ and $\tilde{u}$ are the maximum admissible values of deviation from desirable values of variable state and controlling action of the system.

For realization (3), i.e. optimal control by rolling drive, we set up an equation of dynamic programming according to Bellman:

$$\min[\frac{\partial\psi_i}{\partial t} + J_2^{-1}(t_1)x_2\frac{\partial\psi_i}{\partial x_1} + C_{12}(x_3 - x_2)\frac{\partial\psi_i}{\partial x_2} +$$

$$+J_1^{-1}(C_e x_4 - x_2)\frac{\partial\psi_i}{\partial x_3} + L^{-1}(K_0 U - C_e x_3 - \tag{5}$$

$$-rx_4)\frac{\partial\psi_i}{\partial x_4} + (1{,}15\omega_B)^{-2}(x_1^2 + x_3^2) +$$

$$+(2I_{NOM})^2(x_2^2 + x_4^2)(0{,}1U_B)^{-2}U^2] = 0.$$

The values with index "B" accord to basis regime of the system.

Equation (5) is nonlinear with frequent derivatives and constant coefficients, its solution is given in the following form:

$$\Psi_i(t) = \sum_{k=1}^{4}\sum_{j=1}^{4}\gamma_{kj}(t_1)x_k x_j,\ \gamma_{kj}(t_i) = \gamma_{jk}(t_i),\ k \neq j \tag{6}$$

It should be noted that coefficients in (6) can be found easily when we substitute (6) into (5) and use a software for their computation. Optimal control for the given drive based on dynamic programming has a form:

$$U_i = -\frac{K_0}{2rL}\cdot\frac{\partial\Psi_i}{\partial x_4} \tag{7}$$

In (7), the factor with partial derivative is with the help of (6), i.e.

$$\frac{\partial\Psi_i}{\partial x_4} = 2\left[\gamma_{14}(t_i)x_1 + \gamma_{24}(t_i)x_2 + \gamma_{34}(t_i)x_3 + \gamma_{44}(t_i)x_4\right] \tag{8}$$

Putting (8) into (7) we obtain:

$$U_i = \frac{K_0}{rL}\left[\gamma_{14}(t_i), \gamma_{24}(t_i), \gamma_{34}(t_i), \gamma_{44}(t_i)\right]x = \Omega_i x \tag{9}$$

Expression (9) allows defining optimal factors of the system feedback for any preliminary shosen t_i-time moment. To choose a polynomial function of n order for regulators (feedback), we use the method of interpolation from the theory of numerical analysis.

Let the unknown interpolation polynom has a form:

$$P_n(t) = \sum_{k=0}^{n} C_k t^{n-k};\ P_n(t_i) = \Omega_i, \tag{10}$$

Then the following equation is just:

$$\begin{bmatrix} t_0^n & t_0^{n-1} & \dots & t_0 & 1 \\ t_1^n & t_1^{n-1} & \dots & t_1 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ t_n^n & t_n^{n-1} & \dots & t_n & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \dots \\ C_n \end{bmatrix} = \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \dots \\ \Omega_n \end{bmatrix} \tag{11}$$

From (11) one can find:

$$\begin{bmatrix} C_0 \\ C_1 \\ \dots \\ C_n \end{bmatrix} = \begin{bmatrix} t_0^n & t_0^{n-1} & \dots & t_0 & 1 \\ t_1^n & t_1^{n-1} & \dots & t_1 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ t_n^n & t_n^{n-1} & \dots & t_n & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \dots \\ \Omega_n \end{bmatrix} \tag{12}$$

Thus on the basis of (9), (10) and (12) factors of drive regulators for fixed time moment $(t_i, i = 0, \overline{n})$

If $n \to \infty$ must be defined, then (10) will ensure a continuous functional change of regulator factors. It means that the optimal drive system of winding mechanisms must be solve-adjusting.

In private case at n=3 expression (3) has a form:

$$P_3(t) = C_0 t^2 + C_1 t^2 + C_2 \tag{13}$$

where

$$C_0 = \left[(t_1 - t_2)\Omega_0 - (t_0 - t_2)\Omega_1 + (t_0 - t_1)\Omega_2\right]\Delta^{-1};$$

$$C_1 = \left[(t_2^2 - t_1^2)\Omega_0 + (t_0^2 - t_2^2)\Omega_1 - (t_0^2 - t_1^2)\Omega_2\right]\Delta^{-1};$$

$$C_2 = [t_1 t_2(t_1 - t_2)\Omega_0 + t_0 t_2(t_0 - t_2)\Omega_1 +$$

$$+t_0 t_1(t_0 - t_1)\Omega_2]\Delta^{-1};$$

$$\Delta = t_1 t_0{}^2 + t_0 t_2{}^2 + t_1{}^2 t_2 - t_1 t_2{}^2 - t_0{}^2 t_2 - t_0 t_1{}^2.$$

If $n \to \infty$, the expression defines functional variation (tuning) of the drive regulator coefficients. Therefore, for optimal control of the drive system of wind mechanisms, it is contingent to build a system with adaptive principles.

References

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ДИНАМІЧНА ОПТИМІЗАЦІЯ БЕЛЛМАНА ДЛЯ ЗМІННОГО В ЧАСІ ЕЛЕКТРОПРИВОДУ

Джумбер Дочвірі, Олег Хачапурідзе

Сучасні електричні приводи зі змінними параметрами, такі як обертові механізми, вимагають нових підходів у конструюванні оптимальних систем керування. У даній статті ми пропонуємо вираз поліному інтерполяції для налаштування регулятора швидкості обертового механізму, що грунтується на математичному методі оптимізації, названому динамічним програмуванням Беллмана. Як наслідок, використовуючи запропонований інтерполяційний поліном третього порядку, можна побудувати адаптивну систему керування приводом, виконаним, наприклад у вигляді обертового механізму.

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