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OPTIMIZATION OF PARAMETRIC BALANCED MODULATOR BASED ON FREQUENCY SYMBOLIC METHOD

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Abstract: Application of the frequency symbolic method for analysis of established modes of linear periodically time-variable (LPTV) circuits to solving an optimization task conditioned by the control of their asymptotic stability is considered. The results of optimization of a parametric balanced modulator with respect to the criterion which is based on calculation of the parametric transfer functions approximated by Fourier trigonometric polynomials are presented.

Key words: linear periodically time-variable (LPTV) circuits, frequency symbolic method, asymptotic stability, optimization, objective function, function- characteristic.

1. Introduction

In [1,2,4,5] is shown that the frequency symbolic method (FS-method) is an efficient instrument of analysing the established modes of LPTV circuits in a frequency domain. This method is based on solving differential equations of L. A. Zadeh [3] and approximating the conjugate transfer functions $W(s,t)$ of a LPTV circuit by the Fourier trigonometric polynomial which is easy to present in the complex form

$$\hat{W}(s,t) = W_0(s) + \sum_{r=1}^{k} \begin{bmatrix} W_{+r}(s)\cdot\exp(+j\cdot r\cdot\Omega\cdot t) + \\ +W_{-r}(s)\cdot\exp(-j\cdot r\cdot\Omega\cdot t) \end{bmatrix}, \quad (1)$$

where $s = j\cdot\omega$ is the complex variable of the Laplace transformation, t stands for time, $T = 2\pi/\Omega$ represents the period of changing a parametric element parameter, k denotes the number of members in an approximating polynomial.

Assessment of asymptotic circuit stability can also be conveniently performed on the basis of the FS-method, but by using the normal transfer function $G(s,\xi)$ (ξ is the moment of feeding a delta impulse to a circuit), which is determined by the differential equation [3], similar to the equation of L.A.Zadeh, provided it is approximated by the same Fourier trigonometric polynomial as (1). Due to this, solving the both mentioned differential equations (the equation of L.A.Zadeh and the similar one) is translated into solving a system of linear algebraic equations (SLAE) with time-varying coefficients when s and some or all circuit elements are given in symbolic form. The result of solving the SLAE is the unknown fractional rational expressions $W_0(s)$, $W_{-r}(s)$, $W_{+r}(s)$ of approximable polynomials of the type (1). A certain value of k is chosen so that it can provide the necessary exact matches of the functions $W(s,t)$ and $\hat{W}(s,t)$ as well as $G(s,\xi)$ and $\hat{G}(s,\xi)$ [1, 2].

In this paper, the above mentioned parametric transfer functions are used to determine optimum values of the balanced modulator parameters. Calculations are carried out in the environment of MATLAB using SAPC [5].

2. Formulation of an optimization task

When optimizing characteristics of electric circuits, formation of a optimality criterion (objective function) often occurs through other two functions – the function of goal, which defines desired characteristics of a circuit (goal of optimization), and the function of characteristics, that describes characteristics of the circuit for the selected values of its varied parameters. A solution to the optimization task is considered to be such final values of the varied parameters which provide a minimum (maximum) value of the objective function at specific limitations.

The function of goal is determined in the space of a complex variable s and time t and does not depend on the varied parameters. The function of circuit characteristics is determined in the space of the same independent variables s and t, however, it depends on the varied parameters. The degree of coincidence of these two functions is the objective function. In this case the coincidence is determined for a set of specific values of the independent variables $s_i = j\omega_i$ and t_j which, in the optimization process, are usually fixed, and, therefore, the objective function depends only on the varied parameters.

The area of change of the varied parameters is chosen so that it is situated inside the area of stability of the established circuit mode in the space of the same varied parameters.

In this paper, the objective function F is formed on the basis of the transfer functions of the form (1). A parametric element in the modulator circuit (Fig. 1) is a parametric capacity periodically changing in time t according to the expression:

$$c(t) = c_0(1 + m\cdot\cos(\Omega\cdot t)). \quad (2)$$

The chosen parameters c_0 and m being varied, they should remain in the form of symbols when calculating the transfer functions of a circuit.

By changing the latter, it is nessesary to determine such optimal values of c_0^* and m^* which provide maximum convergence, for example, the module $M_W(c_0, m, \omega, t)$ of the transfer circuit function $W(c_0, m, \omega, t)$ with the module $M_0(\omega, t)$ of the given function in frequency ω_i and time points t_j according to the criterion of a minimum sum of the squared deviations [6]:

$$F(c_0, m) = \sum_{i=1}^{p}\sum_{j=1}^{q}\left(M_W(c_0, m, \omega_i, t_j) - M_0(\omega_i, t_j)\right)^2, \quad (3)$$

and at the same time they provide the circuit stability.

Determination of the area of circuit stability leads to the inequality [4]:

$$m < f(c_0), \quad (4)$$

which has the following meaning: if the condition (4) is met, the circuit is asymptotically stable, if it is not satisfied, the circuit is not stable.

Thus, solving the optimization task is to determine the values c_0^* and m^* which provide a minimum value of the objective function $F(c_0, m) = F_{\min}$, compliance with the conditions of circuit stability $m^* < f(c_0^*)$ and conditions of physical realizability of the parametric element: $c_0^* > 0$, $0 < m^* < 1$.

3. Procedure of optimization

The procedure of optimization is implemented by the following method:

1. By the FS-method we form the denominator $\Delta_G(s)$ of the function $G(s, \xi)$ for symbolic values c_0 and m.

2. For each value c_0 from the series of values of the given range, by the roots of the polynomial $\Delta_G(s)$ we determin a limit value $m_{гр}$ at which the circuit stability changes to instability. The found dependence $m_{гр} = f(c_0)$ having been approximated by the exponential polynomial, we determine the area of asymptotic stability of the circuit in the form of inequality (4).

3. The function of goal $M_0(\omega, t)$ as a function of two variables is given by a set of values in the discrete points ω_i, t_j that form a surface in the frequency-time coordinates.

4. The function of circuit characteristics $M_W(c_0, m, \omega, t)$ is determined by frequency symbolic method provided their varied parameters are given by symbols. According to the obtained expression, we calculate the set of values of the function- characteristic in the same discrete points ω_i, t_j as the function of goal, but with unknown (the varied) parameters given in symbolic form.

5. The objective function $F(c_0, m)$ is formed as the sum of squared deviations between the values of the function-characteristic and the function of goal in the selected discrete points ω_i, t_j as the surface in the coordinates of the varied parameters.

6. The minimum value of the objective function (3), determined by one of the selected methods of optimization in case of circuit stability (4) and physical realizability of the parametric circuit element, determines the desired values of the varied parameters.

Note, that by the FS-method arbitrary parameters of the circuit elements as well as their arbitrary numbers can be chosen as the varied parameter. In this case, inequality (4) determines a multi-dimensional area of asymptotic stability of the circuit

The modulator is optimized by means of MATLAB according to the function «patternsearch» [7] and the program SAPC.

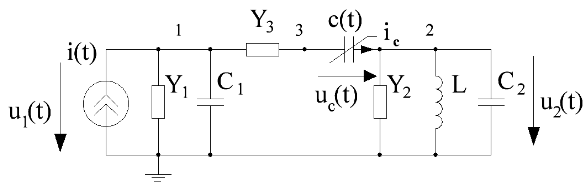

Fig. 1. Modulator.

$Y_1 = 0.001S;\ C_1 = 20\,pF;\ L = 6.53\cdot10^{-8}\,H;$

$Y_2 = 0.01S;\ C_2 = 1.812\,pF;\ Y_3 = 0.0125S;$

$i(t) = 0.01\cos(2\pi\cdot10^7 t);$

$c(t) = c_0(1 + m\cos(\Omega\cdot t));\ \Omega = 2\pi\cdot4.5\cdot10^8\,rad/s.$

4. Optimization of a parametric modulator

In the optimization process, it is necessary to determine the values c_0^* and m^* that provide a minimum of the objective function $F(c_0, m) = F_{\min}$ that should be formed by using the parametric transfer function $Z_2(s,t) = U_2(s,t)/I(s)$ of the modulator shown in Fig. 1 subject to a condition of its asymptotic stability and physical realizability of the parameters c_0^* and m^* of the parametric element $c(t)$ in the frequency range $\omega_i = 1.5\cdot\pi\cdot10^7 - 2.5\cdot\pi\cdot10^7\,rad/s$ and in the time range $t_j = 0 - 2.2222\cdot10^{-9}\,s$. Physically realized parameters c_0^* and m^* are considered as the parameters that are in the limits $0 < c_0 < 1pF$ and $0 < m < 1$, respectively.

The algorithm of the optimization procedure is as follows.

1. Formed by the FS-method the denominator $\Delta_G(s)$ of the functions $G(s,\xi)$ for the circuit presented in Fig.1 when c_0, m and s are in symbolic form has the following form:

$\Delta_G(m, c_0, s) =$ (.25e-93*c0^3*m^2-.51e-93*c0^3)*s^12+(.33e-95*c0^2*m^2-.19e-94*c0^2+.99e-83*c0^3*m^2-.20e-82*c0^3)*s^11+(-.60e-84*c0^2-.24e-96*c0+.15e-72*c0^3*m^2+.99e-85*c0^2*m^2-.29e-72*c0^3)*s^10+(.11e-62*c0^3*m^2-.22e-62*c0^3-.57e-86*c0+.12e-74*c0^2*m^2-.99e-99-.72e-74*c0^2)*s^9+(-.45e-64*c0^2+.75e-65*c0^2*m^2-.54e-76*c0-.10e-52*c0^3-.16e-88+.51e-53*c0^3*m^2)*s^8+(-.33e-66*c0+.33e-55*c0^2*m^2-.14e-78-.21e-54*c0^2+.22e-43*c0^3*m^2-.45e-43*c0^3)*s^7+(.60e-34*c0^3*m^2-.78e-69+.99e-46*c0^2*m^2-.12e-33*c0^3-.69e-45*c0^2-.13e-56*c0)*s^6+(-.18e-35*c0^2-.33e-24*c0^3+.22e-36*c0^2*m^2-.29e-59-.39e-47*c0+.17e-24*c0^3*m^2)*s^5+(.39e-27*c0^2*m^2+.26e-15*c0^3*m^2-.81e-50-.36e-26*c0^2-.84e-38*c0-.51e-15*c0^3)*s^4+(.42e-6*c0^3*m^2+.30e-18*c0^2*m^2-.13e-28*c0-.17e-40-.48e-17*c0^2-.84e-6*c0^3)*s^3+(-.11e-19*c0-.24e-31+.30e3*c0^3*m^2-.60e3*c0^3+.14e-9*c0^2*m^2-.54e-8*c0^2)*s^2+(-.17e-22+.18e-1*c0^2*m^2-.26e-11*c0+.11e12*c0^3*m^2-.23e12*c0^3-4.5*c0^2)*s-.81e-15-.69e-4*c0-.21e9*c0^2.

2. The results of the modulator stability assessment are the following. The value of the real parts of the roots of a polynomial $\Delta_G(m, c_0, s)$, for all admissible by the condition of optimization values of c_0 and m are negative. This suggests that the circuit is asymptotically stable in all allowable range of values of c_0 and m. Therefore, henceforth, the inequality (4) is accepted in the form:

$$m < 1.$$

Before performing the next step of the procedure of optimization, we present the results of calculating the appropriate transfer functions of the modulator.

By the FS-method, using the method of independent additional source of signal [5,8], we develop a symbolic frequency model of the modulator depicted in Fig.2 from which we define the parametric transfer function $Z_2(s,t)$. For this, in the time domain, we form a differential equation that relates the input current $i(t)$ to one of the variables of a parametric element. However, the preliminary analysis showed [5] that the differential equation, composed with respect to the variables $i(t)$ and $u_c(t)$ has a considerably simpler form compared to the differential equation, composed with respect to the variables $i(t)$ and $i_c(t)$. In this connection we choose a differential equation which connects the input current $i(t)$ of the circuit in Fig.1 with the voltage $u_c(t)$ on the parametric capacitor, and it is as follows:

$$\begin{aligned}
&[(Y_1+Y_3)\frac{1}{L}c'(t)+(Y_1Y_2+Y_1Y_3+Y_2Y_3+C_1/L)c''(t)+\\
&+((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)c'''(t)+C_1C_2c''''(t)+\\
&+Y_1Y_3\cdot(1/L)+]u_c(t)+[(Y_1Y_2+C_1/L)Y_3+(Y_1+Y_3)+\\
&+(1/L)\cdot c(t)+3((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)c''(t)+\\
&\quad +2(Y_1Y_2+Y_1Y_3+Y_2Y_3+C_1/L)c'(t)+\\
&+4C_1C_2c'''(t)]u_c'(t)+[(Y_1C_2+Y_2C_1)Y_3+\\
&\quad +(Y_1Y_2+Y_1Y_3+Y_2Y_3+C_1/L)c(t)+3((Y_1+Y_3)C_2+\\
&\quad +(Y_2+Y_3)C_1)c'(t)+6C_1C_2c''(t)]u_c''(t)+\\
&\quad +[Y_3C_1C_2+((Y_1+Y_3)C_2+(Y_2+Y_3)C_1c(t)+\\
&\quad +4C_1C_2c'(t))]u_c'''(t)+C_1C_2c(t)u_c''''(t)=\\
&\quad =(1/L)\cdot Y_3\cdot i(t)+Y_3Y_2\cdot i'(t)+Y_3C_2\cdot i''(t).
\end{aligned} \tag{5}$$

The equation of L.A.Zadeh obtained from the equation (5) for the transfer function $Z(s,t) = U_c(s,t)/I(s)$ has the form:

$$\begin{aligned}
&(C_1C_2c(t)24)Z''''(s,t)\frac{1}{24}+(C_1C_2c(t)24\cdot s\\
&+[Y_3C_1C_2+((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)c(t)+\\
&+4C_1C_2c'(t)]6)Z'''(s,t)\frac{1}{6}+(C_1C_2c(t)12\cdot s^2+\\
&+[Y_3C_1C_2+((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)c(t)+\\
&+4C_1C_2c'(t)]6\cdot s+[(Y_1C_2+Y_2C_1)Y_3+(Y_1Y_2+\\
&+(C_1/L)+Y_1Y_3+Y_2Y_3)c(t)+3((Y_1+Y_3)C_2+\\
&+(Y_2+Y_3)C_1)c'(t)+6C_1C_2c''(t)]2)Z''(s,t)\frac{1}{2}+\\
&+(C_1C_2c(t)4\cdot s^3+[Y_3C_1C_2+((Y_1+Y_3)C_2+\\
&+(Y_2+Y_3)C_1)c(t)+4C_1C_2c'(t)]3s^2+[(Y_1C_2+\\
&+Y_2C_1)Y_3+(Y_1Y_2+Y_1Y_3+Y_2Y_3+C_1/L)+\\
&+c(t)+3((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)c'(t)+\\
&+6C_1C_2c''(t)]2\cdot s+[(Y_1Y_2+C_1/L)y_3+\\
&+(Y_1+Y_3)\cdot(1/L)\cdot c(t)+2(Y_1Y_2+Y_1Y_3+(C_1/L)+\\
&+Y_2Y_3)c'(t)+3((Y_1+Y_3)C_2+(Y_2+Y_3)C_1)+\\
&+c''(t)+4C_1C_2c'''(t)])\cdot Z'(s,t)+([Y_1Y_3\cdot(1/L)+
\end{aligned}$$

$$+\left(Y_1+Y_3\right)\frac{1}{L}c'(t)+\left(Y_1Y_2+Y_1Y_3+Y_2Y_3+\left(C_1/L\right)\right)+$$
$$+c''(t)+\left(\left(Y_1+Y_3\right)C_2+(Y_2+Y_3)C_1\right)c'''(t)+$$
$$+C_1C_2c''''(t)]+[\left(Y_1Y_2+\left(C_1/L\right)\right)Y_3+\left(Y_1+Y_3\right)\cdot$$
$$\cdot\left(1/L\right)\cdot c(t)+2\left(Y_1Y_2+Y_1Y_3+Y_2Y_3+\left(C_1/L\right)\right)c'(t)+$$
$$+3\left(\left(Y_1+Y_3\right)C_2+(Y_2+Y_3)C_1\right)c''(t)+$$
$$+4C_1C_2c'''(t)]s+[(Y_1C_2+Y_2C_1)Y_3+\left(Y_1Y_2+\right.$$
$$\left.+\left(C_1/L\right)+Y_1Y_3+Y_2Y_3\right)c(t)+3\cdot\left(\left(Y_1+Y_3\right)C_2+\right.$$
$$\left.+(Y_2+Y_3)C_1\right)c'(t)+6C_1C_2c''(t)]s^2+$$
$$+[Y_3C_1C_2+\left(\left(Y_1+Y_3\right)C_2+(Y_2+Y_3)C_1\right)c(t)+ \qquad (6)$$
$$4C_1C_2c'(t)]s^3+C_1C_2c(t)s^4)\cdot Z(s,t)=$$
$$=\left(1/L\right)\cdot Y_3+Y_3Y_2s+Y_3C_2s^2.$$

The solution to the equation of L.A. Zadeh (6) by the FS-method under approximation $Z(s,t)$ by the trigonometric complex polynomial (1) when k = 1 and symbolic parameters c_0 and m is as follows:

$$Z(m,c_0,s,t)=Z_0(m,c_0,s)+$$
$$+Z_{-1}(m,c_0,s)\cdot\exp(-j\cdot 2\pi\cdot 4.5\cdot 10^8\cdot t)+, \qquad (7)$$
$$+Z_{+1}(m,c_0,s)\cdot\exp(j\cdot 2\pi\cdot 4.5\cdot 10^8\cdot t),$$

where $Z_0(m,c_0,s)=\dfrac{z_0(m,c_0,s)}{\Delta_z(m,c_0,s)}$,

$$Z_{-1}(m,c_0,s)=\frac{z_{-1}(m,c_0,s)}{\Delta_z(m,c_0,s)},Z_{+1}(m,c_0,s)=\frac{z_{+1}(m,c_0,s)}{\Delta_z(m,c_0,s)},$$

$z_0(m,c_0,s)=$ 52e123*c0^2*m*s^10+((.66e133-.57e133*i)*c0^2*m+.44e85*c0*m*(.15e37+.13e49*i*c0+.15e49*c0)+.29e133*c0^2*m)*s^9+((.18e143-.88e143*i)*c0^2*m+(.55e95-.47e95*i)*c0*m*(.15e37 + .13e49*i*c0+.15e49*c0)+.44e85*c0*m*(-.41e58*c0 +.13e59*i*c0+.81e46+.12e47*i)+.24e95*c0*m*(.15e37+.13e49*i*c0+.15e49*c0)+.44e142*c0^2*m)*s^8+((-.20e153-.35e153*i)*c0^2*m+(.15e105-.74e105*i)*c0* m*(.15e37+.13e49*i*c0+.15e49*c0)+(.55e95-.47e95*i) *c0*m*(-.41e58*c0+.13e59*i*c0+.81e46+.12e47*i)+ .44e85*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0)+.24e95*c0*m*(-.41e58*c0+.13e59*i*c0 +.81e46+.12e47*i)+.37e104*c0*m*(.15e37+.13e49*i*c0+.15e49*c0))*s^7+((-.10e163-.28e162*i)*c0^2*m-(.17e115+.29e115*i)*c0*m*(.15e37+.13e49*i*c0+.15e49*c0)+(.15e105-.74e105*i)*c0*m*(-.41e58*c0+.13e59* i*c0+.81e46+.12e47*i)+(.55e95-.47e95*i)*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0)+.44e85*c0 *m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0) +.24e95*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0)+.37e104*c0*m*(-.41e58*c0+.13e59*i*c0 +.81e46+.12e47*i))*s^6+((-.87e124-.23e124*i)*c0* m*(.15e37+.13e49*i*c0+.15e49*c0)-(.17e115+ .29e115*i)*c0*m*(-.41e58*c0+.13e59*i*c0+ .81e46+.12e47*i)+(.15e105-.74e105*i)*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0)+(.55e95-.47e95*i)*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0)+(-.14e172+.88e171*i)*c0^2*m + .24e95*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0)+.37e104*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0))*s^5+((-.87e124-.23e124*i)*c0*m*(-.41e58*c0+.13e59*i*c0+.81e46+. 12e47*i)-(.17e115+.29e115*i)*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0)+(.15e105-.74e105*i) *c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0)+(-.12e134+.74e133*i)*c0*m*(.15e37+.13e49*i*c0+.15e49*c0)+(-.15e180+.12e181*i)*c0^2*m+.37e104*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0))*s^4+((-.87e124-.23e124*i)*c0*m*(-.23e67*i*c0-.22e56 +.46e56*i-.36e68*c0)-(.17e115+.29e115*i)*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0)+(-.12e134+.74e133*i)*c0*m*(-.41e58*c0+.13e59*i*c0+ .81e46+.12e47*i)+(-.13e142+.10e143*i) *c0*m*(.15e37+.13e49*i*c0+.15e49*c0))*s^3+((-.87e124-.23e124*i)*c0*m*(-.64e65+.30e64*i-.42e76 *c0-.33e77*i*c0)+(-.12e134+.74e133*i)*c0*m*(-.23e67 *i*c0-.22e56+.46e56*i-.36e68*c0)+(-.13e142+.10e143* i)*c0*m*(-.41e58*c0+.13e59*i*c0+.81e46 +.12e47*i))* s^2+((-.12e134+.74e133*i)*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0)+(-.13e142+.10e143*i)*c0*m*(-.23e67*i*c0-.22e56+.46e56*i-.36e68*c0))*s+(-.13e142 +.10e143*i)*c0*m*(-.64e65+.30e64*i-.42e76*c0-.33e77*i*c0);

$z_{-1}(m,c_0,s)=$ -.10e124*c0^2*s^10+(-.32e134*c0^2-.25e122*c0)*s^9+(-.60e132*c0-.15e120-.38e144*c0^2)*s^8+(-.26e130-.57e142*c0-.24e154*c0^2)*s^7+(-.22e140-.12e164*c0^2-.34e152*c0)*s^6+(-.14e162*c0-.12e150-.46e173*c0^2)*s^5+(-.45e159-.41e171*c0-.13e183*c0^2)*s^4+(-.12e169-.32e192*c0^2-.90e180*c0)*s^3+(-.56e201*c0^2-.14e190*c0-.25e178)*s^2+(-.72e210*c0^2-.12e199*c0-.35e187)*s-.67e219*c0^2-.21e207*c0-.25e196;

$z_{+1}(m,c_0,s)=$ .52e123*c0^2*m*s^10+((.66e133+.57e133*i)*c0^2*m+.44e85*c0*m*(-.13e49*i*c0+.15e49*c0 +.15e37)+.29e133*c0^2*m)*s^9+((.18e143+.88e143*i)*c0^2*m+(.55e95+.47e95*i)*c0*m*(-.13e49*i*c0+.15e49*c0+.15e37)+.44e85*c0*m*(-.41e58*c0+.81e46-.12e47*i-.13e59*i*c0)+.24e95*c0*m*(-.13e49*i*c0 +.15e49*c0+.15e37)+.44e142*c0^2*m)*s^8+((-.20e153+.35e153*i)*c0^2*m+(.15e105+.74e105*i)*c0*m*(-.13e49*i*c0+.15e49*c0+.15e37)+(.55e95+.47e95*i)*c0*m*(-.41e58*c0+.81e46-.12e47*i-.13e59 *i*c0)+.44e85*c0*m*(-.36e68*c0+.23e67*i*c0-.22e56-.46e56*i)+.24e95*c0*m*(-.41e58*c0+.81e46-.12e47*i-.13e59*i*c0)+.37e104*c0*m*(-.13e49*i*c0+.15e49*c0

+.15e37))*s^7+((-.10e163+.28e162*i)*c0^2*m+(-
.17e115+.29e115*i)*c0*m*(-.13e49*i*c0+.15e49*c0+
.15e37)+(.15e105+.74e105*i)*c0*m*(-.41e58*c0+
.81e46-.12e47*i-.13e59*i*c0)+(.55e95+.47e95*i)
*c0*m*(-.36e68*c0+.23e67*i*c0-.22e56-.46e56*i)
+.44e85*c0*m*(.33e77*i*c0-.42e76*c0-.64e65-
.30e64*i)+.24e95*c0*m*(-.36e68*c0+.23e67*i*c0-
.22e56-.46e56*i)+.37e104*c0*m*(-.41e58*c0+.81e46-
.12e47*i-.13e59*i*c0))*s^6+((-.87e124+.23e124*i)*
c0*m*(-.13e49*i*c0+.15e49*c0+.15e37)+(-.17e115
+.29e115*i)*c0*m*(-.41e58*c0+.81e46-.12e47*i-
.13e59*i*c0)+(.15e105+.74e105*i)*c0*m*(-.36e68
*c0+.23e67*i*c0-.22e56-.46e56*i)+(.55e95+.47e95*i)
*c0*m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i)-
(.14e172+.88e171*i)*c0^2*m+.24e95*c0*m*(.33e77*i*
c0-.42e76*c0-.64e65-.30e64*i)+.37e104*c0*m*(-.36e68
*c0+.23e67*i*c0-.22e56-.46e56*i))*s^5+((-.87e124+
.23e124*i)*c0*m*(-.41e58*c0+.81e46-.12e47*i-.13e59
*i*c0)+(-.17e115+.29e115*i)*c0*m*(-.36e68*c0+
.23e67*i*c0-.22e56-.46e56*i)+(.15e105+.74e105*i)*
c0*m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i)-
(.12e134+.74e133*i)*c0*m*(-.13e49*i*c0+.15e49*c0+
.15e37)-(.15e180+.12e181*i)*c0^2*m+.37e104*
c0*m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i))*s^4
+((-.87e124+.23e124*i)*c0*m*(-.36e68*c0+.23e67
*i*c0-.22e56-.46e56*i)+(-.17e115+.29e115*i)*c0*
m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i)-(.12e134
+.74e133*i)*c0*m*(-.41e58*c0+.81e46-.12e47*i-
.13e59*i*c0)-(.13e142+.10e143*i)*c0*m*(-.13e49*i*c0
+.15e49*c0+.15e37))*s^3+((-.87e124+.23e124*i)*
c0*m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i)-
(.12e134+.74e133*i)*c0*m*(-.36e68*c0+.23e67*i*c0-
.22e56-.46e56*i)-(.13e142+.10e143*i)*c0*m*(-.41e58
*c0+.81e46-.12e47*i-.13e59*i*c0))*s^2+((-.12e134-
.74e133*i)*c0*m*(.33e77*i*c0-.42e76*c0-.64e65-
.30e64*i)-(.13e142+.10e143*i)*c0*m*(-.36e68*c0+
.23e67*i*c0-.22e56-.46e56*i))*s-(.13e142+.10e143*i)*
c0*m*(.33e77*i*c0-.42e76*c0-.64e65-.30e64*i)

$\Delta_z(m,c_0,s)=$(.79e114*c0^3*m^2-.16e115*c0^3)*
s^12+(.62e125*c0^3+.99e112*c0^2*m^2+.31e125*
c0^3*m^2-.59e113*c0^2)*s^11+(-.92e135*c0^3+
.46e135*c0^3*m^2-.19e124*c0^2-.74e111*c0+
.31e123*c0^2*m^2)*s^10+(.37e133*c0^2*m^2-
.22e134*c0^2-.18e122*c0+.34e145*c0^3*m^2-
.68e145*c0^3-.31e109)*s^9+(-.52e119-.14e144*
c0^2+.23e143*c0^2*m^2-.33e155*c0^3-.17e132*
c0+.16e155*c0^3*m^2)*s^8+(.70e164*c0^3*m^2-
.14e165*c0^3+.10e153*c0^2*m^2-.44e129-.65e153*
c0^2-.10e142*c0)*s^7+(.31e162*c0^2*m^2+.19e174*
c0^3*m^2-.22e163*c0^2-.24e139-.41e151*c0-
.38e174*c0^3)*s^6+(.70e171*c0^2*m^2-.91e148-
.10e184*c0^3-.55e172*c0^2+.52e183*c0^3*m^2-
.12e161*c0)*s^5+(.81e192*c0^3*m^2-.11e182*c0^2-
.16e193*c0^3-.27e170*c0+.12e181*c0^2*m^2-
.25e158)*s^4+(-.15e191*c0^2+.97e189*c0^2*m^2-
.26e202*c0^3+.13e202*c0^3*m^2-.52e167-.42e179*
c0)*s^3+(.94e210*c0^3*m^2-.19e211*c0^3-.34e188*
c0+.42e198*c0^2*m^2-.74e176-.16e200*c0^2)*s^2+
(-.82e196*c0+.36e219*c0^3*m^2-.72e219*c0^3-
.54e185+.57e206*c0^2*m^2-.14e209*c0^2)*s+
.52e214*c0^3*m^2+.83e201*c0^2*m^2-.21e204*c0-
.66e216*c0^2-.25e193.

Voltage on the parametric modulator capacity in the complex form $U_c(s,t)$ we form by using (7) in the following form

$$U_c(m,c_0,s,t)=Z(m,c_0,s,t)\cdot I(s)\,. \tag{8}$$

In the given circuit due to the defined voltage $U_c(m,c_0,s,t)$ we have the opportunity to replace the parametric capacity with a corresponding source of the voltage $E_c=-U_c$ [5,8]. Such a change generates a frequency model of the given parametric circuit, which is shown in Fig.2.

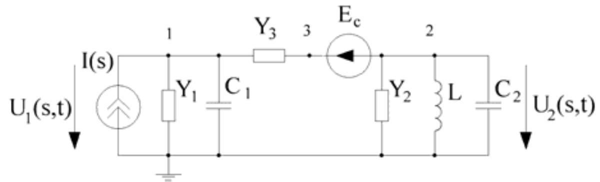

Fig. 2. An equivalent principle diagram of the circuit depicted in Fig.1 with the parametric capacity being replaced with the voltage source $E_c(m,c_0,s,t)$

Since the method of nodal voltages has been chosen as the basis for further calculations in a frequency domain, then, the source of voltage $E_c(m,c_0,s,t)$ in a frequency model of the circuit (Fig.2) is transformed into an equivalent source of current $J_c=-U_c\cdot Y_3=-Z\cdot Y_3\cdot I$, which leads to the construction of a frequency model of the circuit suitable to be analysed by this method (Fig. 3).

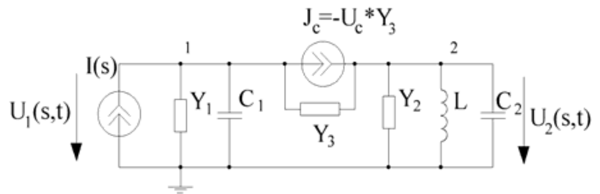

Fig. 3. An equivalent principle diagram of the circuit depicted in Fig.1 with the parametric capacity being replaced with the current source $J_c(m,c_0,s,t)$

Using the diagram shown in Fig.3, and by means of the method of nodal voltages, we can develop a mathematical frequency model of the circuit with two independent sources of current $I(s)$ and $J_c(m,c_0,s,t)$ in the form

$$\begin{bmatrix} \begin{pmatrix} Y_1+Y_3+ \\ +s_rC_1 \end{pmatrix} & -Y_3 \\ -Y_3 & \begin{pmatrix} Y_3+Y_2+ \\ +s_rC_2+\dfrac{1}{s_rL} \end{pmatrix} \end{bmatrix} \cdot \begin{bmatrix} U_1(s,t) \\ U_2(s,t) \end{bmatrix} =$$

$$= \begin{bmatrix} \begin{pmatrix} I(s)- \\ -J_c(m,c_0,s,t) \end{pmatrix} \\ +J_c(m,c_0,s,t) \end{bmatrix}, \qquad (9)$$

which results in an expression for the transfer function of circuit $Z_2(m,c_0,s,t)$:

$$Z_2(m,c_0,s,t) = U_2/I = \qquad (10)$$

$$= \frac{Y_3 - Z(m,c_0,s,t)Y_3Y_1 - Z(m,c_0,s,t)Y_3s_rC_1}{(s_rC_1+Y_1)\cdot\left(Y_3+Y_2+s_rC_2+\dfrac{1}{s_rL}\right)+Y_3\left(Y_2+s_rC_2+\dfrac{1}{s_rL}\right)}.$$

Appearance of the variable s_r in (9) and (10) indicates that the source of current J_c is not harmonic and contains harmonic components of a signal of the series $s_r = j(\omega \pm r\cdot\Omega)$. Mathematical operations with such harmonic components are described in the works [5, 8] in sufficient detail.

3. Let the function of goal, in this particular case, be chosen in such a way so that the modulator could perform it. Therefore, we calculate the function using the expression (10), by arbitrarily choosing appropriate values c_0 and m within the limits of physical realizability of the parametric element. These very values must be found as c_0^* and m^* as a result of optimization that will also indicate the correctness of the optimization process.

The function of goal $M_0(\omega_i,t_j)$ is given as a module of the transfer function

$$Z_2(\omega_i,t_j) = \frac{0.0125-}{\left(s_r\cdot 20\cdot 10^{-12}+0.001\right)\cdot}$$

$$\frac{-Z(\omega_i,t_j)\cdot 0.0125\cdot 0.001-}{\cdot\left(0.0125+s_r\cdot 1.812\cdot 10^{-12}+0.01+\dfrac{1}{s_r\cdot 65.3\cdot 10^{-9}}\right)+}$$

$$\frac{-Z(\omega_i,t_j)\cdot 0.0125\cdot s_r\cdot 20\cdot 10^{-12}}{+0.0125\cdot\left(0.01+s_r\cdot 1.812\cdot 10^{-12}+\dfrac{1}{s_r\cdot 65.3\cdot 10^{-9}}\right)}$$

of the circuit when $c_0 = 0.9n\Phi$, $m = 0.1$ and $\omega = \omega_i$, $t = t_j$, $s_r = j(\omega \pm r\cdot\Omega)$, $k = 2$. Figure 4 shows a graphic view of the module $M_0(\omega_i,t_j)$ of the function of goal at the given in optimization conditions values ω_i and t_j: $1.5\cdot\pi\cdot 10^7 - 2.5\cdot\pi\cdot 10^7\, rad/s$ in $0.1\cdot\pi\cdot 10^7\, rad/s$ interval, and $0 - 2.2222\cdot 10^{-9}s$ in $0.02\cdot 10^{-9}s$ interval, respectively.

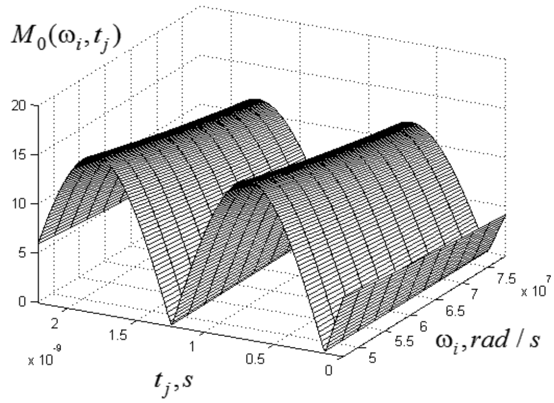

Fig. 4 Module $M_0(\omega_i,t_j)$ of the function of goal of a modulator.

4. The function-characteristic of the modulator we choose as the module $M_Z(c_0,m,\omega_i,t_j)$ of the transfer function

$$Z_2(m,c_0,\omega_i,t_j) = \frac{0.0125-}{\left(s_r\cdot 20\cdot 10^{-12}+0.001\right)\cdot}$$

$$\frac{-Z(m,c_0,\omega_i,t_j)\cdot 0.0125\cdot 0.001-}{\cdot\left(0.0125+s_r\cdot 1.812\cdot 10^{-12}+0.01+\dfrac{1}{s_r\cdot 65.3\cdot 10^{-9}}\right)+}$$

$$\frac{-Z(m,c_0,\omega_i,t_j)\cdot 0.0125\cdot s_r\cdot 20\cdot 10^{-12}}{+0.0125\left(0.01+s_r\cdot 1.812\cdot 10^{-12}+\dfrac{1}{s_r\cdot 65.3\cdot 10^{-9}}\right)}$$

of the circuit when $s_r = j(\omega \pm r\cdot\Omega)$, $k = 2$ and $\omega = \omega_i$, $t = t_j$, that are chosen by analogy with the function of goal: $1.5\cdot\pi\cdot 10^7 - 2.5\cdot\pi\cdot 10^7\, rad/s$ in $0.1\cdot\pi\cdot 10^7$ rad/s interval, and $0 - 2.2222\cdot 10^{-9}s$ in $0.02\cdot 10^{-9}s$ interval, respectively.

5. According to the expression (3) we form the objective function in the frequency range ω_i and time range t_j as**:**

$$F(c_0,m) = \sum_{i=1}^{11}\sum_{j=1}^{112}\left(M_Z(c_0,m,\omega_i,t_j) - M_0(\omega_i,t_j)\right)^2 . (11)$$

Fig. 5 shows a graphic view of the objective function $F(c_0, m)$ when values c_0, m are within the limits $0.8 \cdot 10^{-12} - 1.2 \cdot 10^{-12} F$ in $0.01 \cdot 10^{-12} F$ interval, and $0.05 - 0.15$ in 0.001 interval, respectively, for the same values t_j and ω_i common both to the function of goal and function-characteristic of the circuit**.**

6. The function of optimization «patternsearch», at the given constraints $0 < c_0 < 1pF$, $0 < m < 1$ and at randomly selected initial values $c_0 = 0.95 \cdot 10^{-12} F$, $m = 0.05$ in 1606 iterations, determines a minimum of $F_{\min}$ at $c_0^* = 0.9 \cdot 10^{-12} F$ and $m^* = 0.1$, which, in Fig. 5, is marked by symbol ▣ .

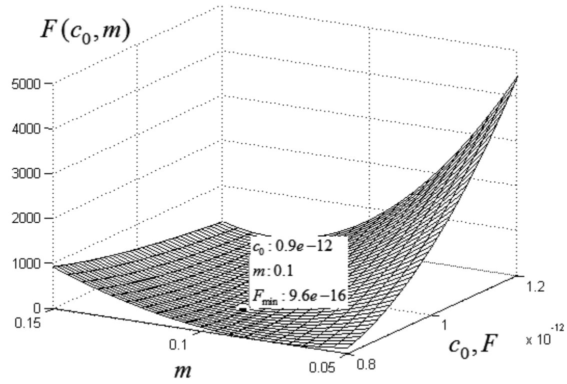

Fig. 5. The objective function $F(c_0, m)$ in the coordinates c_0 and m .

These values coincides with the above selected ones when forming the function of goal.

5. Conclusions

1. The paper shows peculiarities of applying the frequency symbolic method to the solution of an optimization task concerning the design of a parametric balanced modulator in the frequency domain provided its asymptotic stability is under control.

2. Since the analysis of modulator circuit stability has shown its asymptotical stability in all permissible range of values c_0 and m, the condition for the stability takes the form $m < 1$.

3. Selection of variables in differential equations of the form (5), on the basis of which transfer functions of the circuit under investigation are defined, significantly affects the bulkiness of these equations. In particular, when choosing $i(t)$ and $i_c(t)$ as variables, the equation (5) becomes dozens times cumbersome. Such an unsuccessful choice of the variables without affecting the accuracy of the final result requires more considerable amount of computer resources of memory and time that can even cause an interruption of the calculation process.

4. The used «patternsearch» optimization function of MATLAB 7.6.0 has correctly identified a minimum of the objective function in 1606 iterations.

5. Maximum values of the variables i and j of the objective function have been dictated by practical possibilities of MATLAB 7.6.0 on the computer with a 2.30 GHz AMD TurionX2 Dual Core Mobile RM-76 processor and 3.00 GB random access memory.

6. The described approach to the optimization of LPTV circuits does not restrict the choice of varied parameters, because this is not limited by the used frequency symbolic method.

References

1. Y. Shapovalov, The Peculiarities of Analysis of Linear Parametric Circuit Performed by Frequency-Symbolic Method // Przeglad Elektrotechniczny. – Vol. 86, № 1. – 2010. – P.158–160.

2. Y. Shapovalov, B. Mandziy,. Symbolic Analysisi of Linear Parametric Circuits: Current Approach and Application Directions // Teoretychna elektrotekhika. – Lviv, Ukraine: Publishing house of Ivan Franko National University of Lviv. – № 59. – 2007. – P. 3-9.

3. A. Solodov, F. Pietrov, Linear Automatic Systems with Varying Parameters. – Moscow, Ruaais: Nauka, 1971. – 620 p. (Russian)

4. Y. Shapovalov, B. Mandziy, D. Smal, Frequency Symbolic Method for Estimation of Asysmptotic Stability of Linear Parametric Circuits. // International Journal of Computing. – Ternopil, Ukraine: Publishing house of Ternopil National Economic University. – Vol. 11, № 2. – 2012. – P. 156–165.

5. Y. Shapovalov, Devepopment of Theory of Symbolic Analysis of Linear Parametric Circuits in the Frequency Domain // Dissertation for the Degree of Doctor of Sciences in Engineering. – Lviv, Ukraine: Publishing house of Lviv Polytechnic National University. – 2012. – 413 p. (Ukrainian)

6. Automation of Schematic Design [Edited by V. Ilin] . – Moscow, Russia: Radio i sviaz. – 1987. – 368 p. (Russian)

7. Y. Tryus, V. Tryus, Optymization of Multi-extremum Functions by Applying Hybrid Methods in MATLAB R2007A. // Prykladna matematyka. Informatyka». Cherkasy, Ukraine: Publishing house of Cherkasy University. – № 172. – 2010. – P. 104-122. (Ukrainian)

8. Y. Shapovalov, B. Mandziy, D. Smal, J. Lipinski, Frequency Symbolic Models of Linear Parametric Circuits // Przeglad Elektrotechniczny. - Vol. 88, № 3a. – 2012. – P. 52–54.

ОПТИМІЗАЦІЯ ПАРАМЕТРИЧНОГО БАЛАНСНОГО МОДУЛЯТОРА НА ОСНОВІ ЧАСТОТНОГО СИМВОЛЬНОГО МЕТОДУ

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Розглянуто застосування частотного символьного методу аналізу усталених режимів лінійних параметричних кіл до розв’язування оптимізаційної задачі за умови контролю їх асимптотичної стійкості. Наведено результати оптимізації параметричного балансного модулятора.

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