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THERMOSTRESSED CONDITION AND LOAD-CARRYING CAPACITY OF NON-FERROMAGNETIC CONDUCTIVE PLATES UNDER THE ACTION OF ELECTROMAGNETIC PULSES

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Abstract: The paper describes a mathematical model of elastic deformation of non-ferromagnetic conductive bodies under the action of pulsed electromagnetic fields (EMF), which takes into account the adiabatic character of processes of heating and deformation of conductive bodies by pulsed electromagnetic fields. Within this model, the dynamic problem of thermo-mechanics has been formulated for a non-ferromagnetic conductive planar layer under the action of pulsed electromagnetic fields. A technique of approximate solution to the formulated problem, that applies a cubic polynomial approximation of all key functions' distributions with respect to a thickness coordinate, has been proposed. The solution to the problem under consideration in case of homogeneous electromagnetic pulse has been found and discussed. The thermo-mechanical behavior and load-carrying capacity of non-ferromagnetic conductive plates under the action of electromagnetic pulses (EMP) has been investigated.

Key words: non-ferromagnetic plates, electromagnetic pulse, thermo-mechanical behavior, load-carrying capacity, critical EMP parameters.

1. Introduction

Structural elements of modern engineering work under the impact of multivariate loads that dictate their respective thermo-mechanical behavior. One kind of those loads is frequently represented by an electromagnetic field, particularly by a pulsed one [1, 10]. The action of such load is principal for work of various electrical apparatus, and it is also used in modern technologies of structural elements processing in many industries [1, 5, 9]. Some of the above mentioned devices, namely those for removing undesirable extraneous formations from surfaces of functional elements of various engineering products (by creating mechanical vibrations removing the formations) and reusable pulse induction systems, particularly those for pulse processing of machine components, mechanisms and devices, have become widely employed. These applications basically require that the load-carrying capacity of the relevant structural elements be ensured.

To address the mentioned engineering and technology problems, the theory of thermo-mechanics of non-ferromagnetic conductive bodies under the action of pulsed electromagnetic fields was developed [2, 3, 7, 8], that takes into account the special features of these fields impact on the material continuum. The theory is applied for the rational design and development of devices for removing extraneous formations (including icing) using pulsed electromagnetic fields, of reusable pulse induction systems and their operation modes as well as the modes of structural elements magnetic pulse treatments, for securing the load-carrying capacity of both the elements and products as a whole.

Based on the developed theory, this paper presents a description of the mathematical model of elastic deformation of conductive bodies by pulsed electromagnetic fields and formulates the dynamic problem of thermo-mechanics for non-ferromagnetic conductive planar layer being under the action of an electromagnetic pulse. Using the technique of numerical solution of corresponding initial boundary value problems of electrodynamics and thermo-elasticity, a solution to the formulated dynamic problem has been obtained; a numerical analysis of dependency of thermo-mechanical behavior and load-carrying capacity of the examined layer on the EMP parameters has been performed.

2. Mathematical model of conductive body elastic deformation under the action of pulsed EMF and estimation of its load-carrying capacity in such condition

Let us set forth basic physical and mathematical conditions and write down the system of basic equations of the theory of thermo-mechanics of non-ferromagnetic conductive bodies under the action of pulsed electromagnetic fields that describe the relationship between electromagnetic, thermal and mechanical processes by taking into account the special features of EMF impact on a conductive body.

We consider a conductive isotropic body that takes a domain $V \in R^3$ with the boundary surface S exposed to the action of pulsed electromagnetic fields which can be described by the magnetic field intensity vector $\vec{H}\left(\vec{r},t\right)$ on the surface of the conductive body in the form

$\vec{H}(\vec{r}_0,t)=H_*(t)\vec{H}_0^*(\vec{r}_0)$. Here $\vec{H}_0^*(\vec{r}_0)$ is a function that describes the distribution of the magnetic field intensity vector $\vec{H}(\vec{r},t)$ on the surface S of the conductive body; the surface is described by the equation $\vec{r}=\vec{r}_0$; $H_*(t)$ is a pulse function that describes time-dependency of the magnetic field intensity vector $\vec{H}(\vec{r},t)$ on the surface S of the conductive body and satisfies the conditions:

$$H_*(t)\le 1 \quad t\in[0,t_i],\ H_*(0)=0,\ H_*(t_i)=0;$$

where t_i – electromagnetic pulse duration; $\vec{r}$ and $\vec{r}_0$ – radius-vectors of points inside the conductive body and on its surface.

We assume that the parameters $\vec{H}_0^*(\vec{r}_0)$, $H_*(t)$, t_i describing the acting EMF are such that the EMF belongs to the class of "non-destructive" pulsed EMF [5, 9], i.e. its pulse duration is considerably less than one tenth of the second (t <0,01c) and its maximum magnetic induction value is less than 50 T ($B_{\max}\le 50$ T). We consider such EMP whose action doesn't lead to the emergence of a shock front ($H_{\max}\le 10^7$ A/m, where $H_{\max}$ – the maximum value of magnetic field intensity on the body surface).

The corresponding problem of mathematical physics describing the thermal and mechanical processes in the case of such electromagnetic action may be formulated as follows.

In the case of the above assumed parameters of the electromagnetic pulse the stress and deformations as well as their velocity in the body are so small that linear elasticity theory can be applied and the influence of medium movements on the EMF characteristics is negligible [2,3,5,7-10].

It was experimentally proven that fluidity limit σ_m for different materials increases along with the increase of deformation $\dot{\varepsilon}$ velocity at normal temperature, both in the process of stretching and compression. That is why in the case of dynamic power load the stress σ_i – deformation ε_i diagrams for the majority of metals and their alloys are different from the static ones [4,5].

Therefore, we can assume that the process of conductive body deformation under the action of pulsed EMF is of dynamic character and is featured by all the mechanic behavior peculiarities of deformable bodies exposed to dynamic and pulsed power and thermal loads (the value of elastic deformation dynamic limit σ_d may increase by a factor of $2\div 3$ in comparison with elastic deformation static limit σ_s; it was determined experimentally for different materials and depends on deformation velocity).

We consider widespread homogeneous isotropic non-dielectric non-ferromagnetic bodies [3, 7, 8, 10], for which electromechanical and thermoelectric effects are negligible, induction vectors $\vec{D}$ and $\vec{B}$ are collinear to electric $\vec{E}$ and magnetic $\vec{H}$ field intensity, conduction current density $\vec{j}$ is collinear to $\vec{E}$. In that case we may describe EMF by means of medium-related electrodynamics equations [3, 8, 10]: $\vec{D}=\varepsilon\vec{E}$, $\vec{B}=\mu\vec{H}$, $\vec{j}=\sigma_0\vec{E}$, where $\varepsilon=\varepsilon_0\varepsilon_*$, $\mu=\mu_0\mu_*$; ε_*, μ_* – relative dielectric permittivity and magnetic permeability; σ_0 – conductivity, ε_0, μ_0 – electric and magnetic constant.

According to the accepted assumptions the influence of a pulsed EMF on processes of thermal and elastic deformation in a conductive body may be taken into account by Joule heat $Q=\sigma_0\vec{E}\cdot\vec{E}$ and ponderomotive force $\vec{F}=\sigma_0\mu\vec{E}\times\vec{H}$ [5, 9, 10]. These factors give rise to unsteady thermal and mechanical fields.

On the basis of such approximating assumptions and given material characteristics (they are assumed to be equal to average values over the given temperature interval) a two-stage formulation of basic relations for numeric description of the parameters that feature electromagnetic, thermal, and mechanic processes in the bodies exposed to electromagnetic pulses was proposed [2, 3, 7, 8].

At the first stage we write down the equations for determining the EMF parameters and expressions for heat and ponderomotive forces as functions of the electromagnetic parameters.

At the second stage we utilize the dependencies describing the mechanical and thermal parameters for given initial and boundary conditions consisting of temperature T and components σ_{ij} of the stress tensor $\hat{\sigma}$, where heat sources and volumetric forces are represented by Joule heat loses Q and ponderomotive forces found at the first stage.

For a known temperature T and components σ_{ij} of the stress tensor $\hat{\sigma}$ we analyze the parameters of ongoing physical and mechanical processes and their characteristics depending on the pulse electromagnetic loads. Using the condition [2-4, 8]

$$\sigma_i=\sqrt{3I_2(\hat{\sigma})-I_1^2(\hat{\sigma})}\Big/\sqrt{2}\le\sigma_d,$$

(where σ_i is the stress intensity; $I_j(\hat{\sigma})$, $j=1,2$ are the invariants of stress tensor; σ_d is the elastic deformation limit) we determine the permissible EMF parameters that ensure the load-carrying capability of the bodies under consideration.

Solving such a complex problem, even for bodies with simple geometric configurations is associated with considerable mathematical difficulties. In order to utilize approximate approaches to solving constituent problems (e.g. thermo-elasticity at the second stage) the corresponding mathematical physics problems are formulated for the key functions $\vec{H}$, T, $\hat{\sigma}$. Note that the formulation of direct problems with regard to these key functions allows us effectively use an approximate method of solution, i.e. a method of polynomial approximations, and improve their accuracy.

At the first stage of determining the EMF parameters in the conductive body under study we finding the magnetic field intensity vector $\vec{H}(\vec{r},t)$ on the basis of Maxwell's equations while neglecting displacement currents:

$$\Delta\vec{H} - \sigma_0\mu\frac{\partial\vec{H}}{\partial t} = 0,\ Div\vec{H} = 0.$$

A unique solution may be found for given initial conditions, namely absence of EMF at the initial time point $t = 0$:

$$\vec{H}(\vec{r},0) = 0$$

and given values of $\vec{H}(\vec{r},t)$ on the body surface (boundary conditions). In such a case the problem is reduced to solving only one equation

$$\Delta\vec{H} - \sigma_0\mu\frac{\partial\vec{H}}{\partial t} = 0 \quad (1)$$

for zero initial and corresponding boundary conditions in the form of tangential components of $\vec{H}(\vec{r},t)$ on the body surface.

For the found components of the vector $\vec{H}(\vec{r},t)$ the expressions for Joule heat Q and ponderomotive forces $\vec{F}$ are as follows:

$$Q = \left[rot\vec{H}(\vec{r},t)\right]^2 \Big/ \sigma_0,$$

$$\vec{F} = \mu rot\vec{H}(\vec{r},t)\times\vec{H}(\vec{r},t). \quad (2)$$

At the second stage of determining the parameters describing the thermo-elastic condition of the temperature T and stress tensor $\hat{\sigma}$ are chosen as key functions [2, 3, 7, 8]. Assuming that at the initial time point $t = 0$ the displacement $\vec{U}$ and its velocity $d\vec{U}/dt$ are equal to zero, and the temperature T is equal to T_0, we receive the initial conditions [2, 3, 8]

$$T(\vec{r},0) = 0,\ \hat{\sigma}(\vec{r},0) = 0,$$

$$\frac{1}{2G}\frac{\partial\hat{\sigma}(\vec{r},0)}{\partial t} + \left[\alpha\frac{\partial T(\vec{r},0)}{\partial t} - \frac{\nu}{E}\frac{\partial\sigma_*(\vec{r},0)}{\partial t}\right]\hat{I} = 0. \quad (3)$$

where T is the temperature deviation from its initial value T_0; $\sigma_* \equiv I_1(\hat{\sigma}) = \sigma_{\hat{e}\hat{e}} = \sigma_{11} + \sigma_{22} + \sigma_{33}$; $G = E\big/\left[2(1+\nu)\right]$ is the shear modulus; α, ν are the coefficient of linear thermal expansion and Poisson coefficient; E is Young's modulus; $\hat{I} \equiv \{\delta_{i\hat{e}}\}$ is a unit tensor; $\delta_{i\kappa}$ are the Kronecker symbols.

The temperature T and stress tensor components $\sigma_{i\kappa}$ determined by dependencies (2) may be represented as the sum of two constituents [2, 3, 7, 8]:

$$T = T^Q + T^F,\ \sigma_{i\kappa} = \sigma_{i\kappa}^Q + \sigma_{i\kappa}^F,$$

where T^Q, $\sigma_{i\kappa}^Q$ and T^F, $\sigma_{i\kappa}^F$ are the constituents caused by Joule heat and ponderomotive forces, respectively.

It was experimentally proven [5, 9] that in the case of a pulsed EMF that belongs to the class of "non-destructive" pulsed EMF, a body exposed to its action heats up adiabatically, i.e. its temperature at any point depends only on the amount of EMF energy that has been irreversibly absorbed by an appropriate elementary volume (Joule heat). Under those conditions, the temperature field T^Q can be described by the equation

$$\frac{\partial T^Q}{\partial t} = \frac{\kappa}{\lambda}Q$$

where κ, λ are the temperature coefficient and thermal conductivity. Therefore, in the considered case the temperature constituent T^Q can be found from the expression:

$$T^Q(\vec{r},t) = \frac{\kappa}{\lambda}\int_0^t Q(\vec{r},t)\,dt$$

We assume that the body is free from power load. In such case, using the thermo-elasticity equations formulated in terms of temperature and stress tensor components [2, 3, 7, 8] for determining the components $\sigma_{i\kappa}^Q$ of the stress tensor constituent $\hat{\sigma}^Q$ caused by the temperature T^Q, we obtain the following system of equations [2, 3, 8]

$$Def\left(Div\hat{\sigma}^Q\right) = \rho\frac{\partial^2}{\partial t^2}\left[\frac{1}{2G}\hat{\sigma}^Q + \left(\alpha T^Q - \frac{\nu}{E}\sigma_*^Q\right)\hat{I}\right] \quad (4)$$

The system (4) is solved utilizing initial conditions

$$\hat{\sigma}^Q(\vec{r},0) = 0,$$

$$\frac{1}{2G}\frac{\partial\hat{\sigma}^Q(\vec{r},0)}{\partial t} + \left[\alpha\frac{\partial T^Q(\vec{r},0)}{\partial t} - \frac{\nu}{E}\frac{\partial\sigma_*^Q(\vec{r},0)}{\partial t}\right]\hat{I} = 0 \quad (5)$$

and boundary conditions

$$\hat{\sigma}^Q\vec{n} = 0 \text{ for } \vec{r} = \vec{r}_0,$$

where ρ is the density of the body material; Def – deformator.

When finding the temperature T^F and stress $\sigma^F_{\kappa j}$ constituents caused by the action of ponderomotive force $\vec{F}$, we take into account the fact that the thermal perturbation in the conductive body is resulted from the deformation due to dynamic power (ponderomotive force $\vec{F}$), which also has a pulsed character. In such case the deformation process of the conductive bodies may be considered adiabatic [5, 9] and the temperature T^F increment is determined at small thermal perturbations ($\frac{T^F}{T_0}$ << 1) by the formula [5, 6],

$$T^F = -\frac{(3\lambda_* + 2\mu_*)\alpha\kappa T_0 \varepsilon^F_{\kappa\kappa}}{\lambda},$$

where $\varepsilon^F_{\kappa\kappa}$ is the first invariant of the tensor of deformations $\hat{e}^F$ caused by the action of ponderomotive force; $\lambda_*, \mu_* \equiv G$ are the isothermal Lame coefficients Lame.

Taking into account that in the case of adiabatic deformation of the body Hooke's law is given by [6]

$$\sigma^F_{i\kappa} = 2\mu_* \varepsilon^F_{i\kappa} + \lambda_s \varepsilon^F_{\kappa\kappa}\delta_{i\kappa},$$

where

$$\lambda_s = \lambda_* + \frac{(3\lambda_* + 2\mu_*)^2 \alpha\kappa T_0}{\lambda} \equiv \frac{\nu E\left(1 + \varepsilon_*(1-\nu)/\nu\right)}{(1+\nu)(1-2\nu)}$$

is the adiabatic Lame coefficient;

$$\varepsilon_* = \frac{(3\lambda_* + 2\mu_*)^2 \alpha^2 \kappa T_0}{(\lambda_* + 2\mu_*)\lambda} \equiv \frac{\alpha^2 \kappa E T_0 (1+\nu)}{(1-\nu)(1-2\nu)\lambda}$$

Is the parameter of deformation and temperature fields coupling, the expression of volumetric deformation of the body takes the form $\varepsilon^F_{\kappa\kappa} = \frac{\sigma^F_{\kappa\kappa}}{3\lambda_s + 2\mu_*}$.

Then the temperature T^F increment with respect to the initial temperature T_0 can by calculated by [2,3,7,8]:

$$T^F = -\frac{\left[(3\lambda_* + 2\mu_*)\alpha\kappa T_0 \sigma^F_{\kappa\kappa}\right]}{\left[\lambda(3\lambda_s + 2\mu_*)\right]} \equiv$$

$$= -\frac{\alpha\kappa T_0}{\left[1 + 3\varepsilon_*(1-\nu)/(1+\nu)\right]\lambda}\sigma^F_{\kappa\kappa} \qquad (6)$$

On the basis of thermo-elasticity equations and boundary conditions for the unloaded body surface we conclude that the stress $\hat{\sigma}^F$ satisfies the following equation [2, 3, 7, 8]:

$$Def\left(Div\hat{\sigma}^F + \vec{F}\right) = \rho\frac{\partial^2}{\partial t^2}\left[\frac{1}{2G}\hat{\sigma}^F - \frac{\nu\nu_*}{E}\sigma^F_* \hat{I}\right] \qquad (7)$$

as well as initial conditions

$$\hat{\sigma}^F(\vec{r},0) = 0, \quad \frac{\partial\hat{\sigma}^F(\vec{r},0)}{\partial t} = 0$$

and boundary conditions

$$\hat{\sigma}^F \vec{n} = 0 \text{ for } t>0 \text{ and } \vec{r} = \vec{r}_0,$$

where $\nu_* = 1 + \frac{(1-\nu)(1-2\nu)\varepsilon_*}{\nu(1+\nu)\left(1 + 3\varepsilon_*(1-\nu)/(1+\nu)\right)}$.

As an example, we use this model to formulate the thermo-mechanics problem for a conductive layer.

3. Dynamic thermo-mechanics problem for a conductive layer exposed to pulsed electromagnetic action.

Formulation of the problem. Let us consider a conductive layer of constant thickness $2h$ and position the Cartesian coordinates (x, y, z) in such a way that the plane xOy coincides with the medial plane of the layer.

The layer's material layer is homogeneous, isotropic and non-ferromagnetic. Its physical and mechanical properties are constant (they are equal to average values over the heating period). The layer is exposed to the action of non-stationary (arbitrarily varying with time) EMF that is given by the values of tangential component H_y of the magnetic field intensity vector $\vec{H} = \{0; H_y; 0\}$ on both sides of the layer $z = \pm h$.

The system of initial equations and relations. In the case when all the key functions задачі $H_y(z,t)$, $\sigma_{zz}(z,t)$ depend only on the thickness coordinate z and time t, the non-zero component $H_y(z,t)$ of the magnetic field intensity $\vec{H}$ is described by Maxwell's equation

$$\frac{\partial^2 H_y}{\partial z^2} - \sigma_0\mu\frac{\partial H_y}{\partial t} = 0, \qquad (8)$$

boundary conditions

$$H_y(\pm h, t) = H^{\pm}_y(t) \qquad (9)$$

and zero initial condition

$$H_y(z,0) = 0.$$

Specific density of Joule heat Q and the ponderomotive force $\vec{F}$ can be determined, using the function H_z, in the following form:

$$Q = \frac{1}{\sigma_0}\left(\frac{\partial H_z}{\partial z}\right)^2, \quad \vec{F} = \left\{0; 0; F_z = -\mu\frac{\partial H_z}{\partial z}H_z\right\}. \qquad (10)$$

The temperature constituent T^Q can be found by time integration of the expression $Q(z,t)$, and the stress constituent σ^Q_{jj} ($j = x, y, z$; repeated indices are excluded from summation indices) caused by Joule heat $Q(z,t)$ can be found from the system [3, 8]

$$\frac{\partial^2\sigma^Q_{zz}}{\partial z^2} - \frac{1}{\tilde{n}_1^2}\frac{\partial^2\sigma^Q_{zz}}{\partial t^2} = \alpha\rho\frac{1+\nu}{1-\nu}\frac{\partial^2 T^Q}{\partial t^2}$$

$$\sigma_{xx}^{Q}=\sigma_{yy}^{Q}=\frac{\nu}{1-\nu}\sigma_{zz}^{Q}-\frac{\alpha E}{1-\nu}T^{Q} \qquad (11)$$

supplemented by initial conditions

$$\sigma_{zz}^{Q}(z,0)=0, \quad \frac{\partial\sigma_{zz}^{Q}(z,0)}{\partial t}=-\frac{\alpha E}{1-2\nu}\frac{\partial T^{Q}(z,0)}{\partial t} \qquad (12)$$

and boundary conditions

$$\sigma_{zz}^{Q}(\pm h,t)=0\,. \qquad (13)$$

Correspondingly, the stress constituent σ_{jj}^{F} $(j=x,y,z)$ and the temperature constituent T^{F} caused by action of the ponderomotive force $\vec{F}=\{0,0,F_z(z,t)\}$ are described by the relations [3, 8]:

$$\frac{\partial^2\sigma_{zz}^{F}}{\partial z^2}-\frac{1}{c_{1a}^2}\frac{\partial^2\sigma_{zz}^{F}}{\partial t^2}=-\frac{\partial F_z}{\partial z}\,,$$

$$\sigma_{xx}^{F}=\sigma_{yy}^{F}=\frac{\nu}{(1-\nu)}\sigma_{zz}^{F}\,,$$

$$T^{F}=-\frac{\alpha T_0}{\lambda(1-\nu)}\frac{\kappa(1+2\nu)\sigma_{zz}^{F}}{\left[1+3\varepsilon_*(1-\nu)/(1+\nu)\right]}, \qquad (14)$$

supplemented by initial conditions

$$\sigma_{zz}^{F}(z,0)=0, \quad \frac{\partial\sigma_{zz}^{F}(z,0)}{\partial t}=0 \qquad (15)$$

and boundary conditions

$$\sigma_{zz}^{F}(\pm h,t)=0 \qquad (16)$$

where $c_{1a}=c_1\sqrt{1+\varepsilon_*}$ is the adiabatic velocity of elastic expansion waves in the layer.

For different kind of layer fixation other boundary condition should be formulated [6].

4. The methodology of obtaining solutions to initial-boundary problems.

To find the numerical solutions of the problem, i.e. the values of electromagnetic field characteristics, temperature and stress we apply the technique based on approximation of the sought functions $\Phi(z,t)=\{H_y,\sigma_{zz}^{Q},\sigma_{zz}^{F}\}$ with respect to the thickness coordinate z by cubic polynomials [2, 3, 8] in the form

$$\Phi(z,t)=\sum_{i=1}^{4}a_{i-1}^{\phi}(t)z^{i-1}\,, \qquad (17)$$

where the coefficient $a_{i-1}^{\phi}(t)$ of approximation polynomials (17) are determined by the expression

$$a_{i-1}^{\phi}(t)=a_{i-1,1}^{\phi}\Phi_1(t)+a_{i-1,2}^{\phi}\Phi_2(t)+a_{i-1,3}^{\phi}\Phi_3^{+}(t)+a_{i-1,4}^{\phi}\Phi_4^{-}(t)$$

where $\Phi^{\pm}(t)$ are the values of the sought functions on the layer's planar surfaces;

$$\Phi_s(t)=\frac{s}{(2h)^s}\int_{-1}^{1}\Phi(z,t)z^{s-1}dz, \quad s=1,2\,. \qquad (18)$$

are their integral characteristics with respect to the thickness coordinate z

To obtain the integral characteristics $\Phi_s(t)$ we should integrate the function $\Phi(z,t)$ defined in the section 3 applying the formula (18) and taking into account the expression (17).

The transformations having been completed, we obtain a set of equation that serves for determination of integral characteristics $H_{ys}(t)$ and $\sigma_{zzs}(t)$ of the component H_y of magnetic field intensity vector and of the components of stress tensor. As a result we have obtained the following expressions for the integral characteristics $H_{ys}(t)$ $(s=1,2)$ of the component $H_y(z,t)$ of magnetic field intensity vector:

$$\frac{dH_{y1}}{dt}+\frac{3}{m_{0*}}H_{y1}=3\left(H_y^{+}+H_y^{-}\right),$$

$$\frac{dH_{y2}}{dt}+\frac{15}{m_{0*}}H_{y2}=5\left(H_y^{+}-H_y^{-}\right); \qquad (19)$$

for the integral characteristics $\sigma_{zzs}^{Q}(t)$ $(s=1,2)$ of the components $\sigma_{zz}^{Q}(z,t)$ of the stress tensor constituent $\hat{\sigma}^{Q}$:

$$\frac{d^2\sigma_{zz1}^{Q}}{dt^2}+\frac{3c_1^2}{h^2}\sigma_{zz1}^{Q}=-\frac{\alpha E}{1-2\nu}\frac{d^2T_1^{Q}}{dt^2},$$

$$\frac{d^2\sigma_{zz2}^{Q}}{dt^2}+\frac{15c_1^2}{h^2}\sigma_{zz2}^{Q}=-\frac{\alpha E}{1-2\nu}\frac{d^2T_2^{Q}}{dt^2}\,, \qquad (20)$$

and for the integral characteristics $\sigma_{zzs}^{F}(t)$ $(s=1,2)$ of the components of stress tensor $\sigma_{zz}^{F}(z,t)$ of the constituent $\hat{\sigma}^{F}$:

$$\frac{d^2\sigma_{zz1}^{F}}{dt^2}+\frac{3c_1^2}{h^2}\sigma_{zz1}^{F}=\frac{c_1}{h}\left[F_z(1,t)-F_z(-1,t)\right],$$

$$\frac{d^2\sigma_{zz2}^{F}}{dt^2}+\frac{15c_1^2}{h^2}\sigma_{zz2}^{F}=$$

$$=\frac{c_1}{h}\left[F_z(1,t)+F_z(-1,t)-\int_{-1}^{1}F_z(z,t)dz\right] \qquad (21)$$

where $m_{0*}=\sigma_0\mu h^2$.

The equation (19) should be solved by zero initial conditions for the function $H_{ys}(t)$, the equation (20) should be solved by the initial conditions:

$$\sigma_{zzs}^{Q}(0)=0, \quad \frac{d\sigma_{zzs}^{Q}(0)}{dt}=-\frac{\alpha E}{1-2\nu}\frac{dT_s^{Q}(0)}{dt}, \qquad (22)$$

and the equation (21) should be solved by the initial conditions:

$$\sigma^F_{zzs}(0)=0,\quad \frac{d\sigma^F_{zzs}(0)}{dt}=0. \qquad (23)$$

To solve the obtained set of equations (so called Cauchy problems) with regard to the integral characteristics of the sought functions we can apply Laplace transforms with respect to time variable t and taking into account the corresponding initial conditions. As a result, the following expressions have been obtained:

– for the integral characteristics $H_{ys}(t)$, $(s=1,2)$ of the component $H_y(z,t)$ of magnetic field intensity vector

$$H_{y1}(t)=e^{-3/(\sigma_0\mu h^2)t}\int_0^t e^{3/m_{0*}\tau}3\left[H^+(\tau)+H^-(\tau)\right]d\tau,$$

$$H_{y2}(t)=e^{-15/(\sigma_0\mu h^2)t}\int_0^t e^{15/m_{0*}\tau}5\left[H^+(\tau)-H^-(\tau)\right]d\tau; \qquad (24)$$

– for the integral characteristics $\sigma^Q_{zzs}(z,t)$ of the components $\sigma^Q_{zz}(z,t)$ of the stress tensor constituent $\hat{\sigma}^Q$

$$\sigma^Q_{zz1}(t)=-\frac{\alpha E}{1-2\nu}\int_0^t\frac{dT_1^Q(\tau)}{d\tau}\cos\left(\omega_1^*(t-\tau)\right)d\tau,$$

$$\sigma^Q_{zz2}(t)=-\frac{\alpha E}{1-2\nu}\int_0^t\frac{dT_2^Q(\tau)}{d\tau}\cos\left(\omega_2^*(t-\tau)\right)d\tau; \qquad (25)$$

– for the integral characteristics $\sigma^F_{zzs}(z,t)$ of the components $\sigma^F_{zz}(z,t)$ of the stress tensor constituent $\hat{\sigma}^F$

$$\sigma^F_{zz1}(t)=\frac{c_1}{\sqrt{3}}\int_0^t\left[F_z(1,\tau)-F_z(-1,\tau)\right]\sin\left(\omega_1^*(t-\tau)\right)d\tau, \qquad (26)$$

$$\sigma^F_{zz2}(t)=\frac{c_1}{\sqrt{15}}\int_0^t\left[F_z(1,\tau)+F_z(-1,\tau)-\right.$$
$$\left.-\int_{-1}^t F_z(z,\tau)d\tau\right]\sin(\omega_2^*(t-\tau))d\tau,$$

where $\omega_1^*=\frac{c_1\sqrt{3}}{h}$, $\omega_2^*=\frac{c_1\sqrt{15}}{h}$ are the two first natural frequencies of the layer's oscillations along the thickness coordinate. On the basis of the found functions $H_{ys}(t)$, $\sigma^Q_{zzs}(t)$, $\sigma^F_{zzs}(t)$ we can find the expressions:

– of the component $H_y(z,t)$ of magnetic field intensity vector

$$H_y(z,t)=\frac{3}{4}H_{y1}(t)\left(1-z^2\right)+\frac{15}{4}H_{y2}(t)\left(z-z^3\right)-$$
$$-\frac{1}{4}\left[H_y^+(t)+H_y^-(t)\right]\left(1-3z^2\right)-$$
$$-\frac{1}{4}\left[H_y^+(t)-H_y^-(t)\right]\left(3z-5z^3\right) \qquad (27)$$

– of the component $\sigma^Q_{zz}(z,t)$ of the stress tensor constituent $\hat{\sigma}^Q$

$$\sigma^Q_{zz}(z,t)=\frac{3}{4}\sigma^Q_{zz1}(t)\left(1-z^2\right)+$$
$$+\frac{15}{4}\sigma^Q_{zz2}(t)\left(z-z^3\right); \qquad (28)$$

– of the component $\sigma^F_{zz}(z,t)$ of the stress tensor constituent $\hat{\sigma}^F$

$$\sigma^F_{zz}(z,t)=\frac{3}{4}\sigma^F_{zz1}(t)\left(1-z^2\right)+$$
$$+\frac{15}{4}\sigma^F_{zz2}(t)\left(z-z^3\right), \qquad (29)$$

where the functions $H_{ys}(t)$, $\sigma^Q_{zzs}(z,t)$, $\sigma^F_{zzs}(z,t)$ $(s=1,2)$ have been found from (24)–(26).

Therefore, the general closed form solution to the dynamic one-dimensional boundary problem of thermo-mechanics for the planar conductive layer on the whole time interval of arbitrary homogenous non-stationary electromagnetic action.

5. Investigation of thermo-mechanical behavior and load-carrying capacity of the non-ferromagnetic conductive layer exposed to electromagnetic pulses

Let us consider the homogeneous (with respect to coordinates) pulsed electromagnetic action that is mathematically represented by the function $H_*(t)$ [3, 5, 8, 9] in the form

$$H_*(t)\equiv H_0(t)=kH_0\left(e^{-\beta_1 t}-e^{-\beta_2 t}\right), \qquad (30)$$

where k is the normalization factor, H_0 represents the maximum value of magnetic field intensity vector generated by the electromagnetic pulse on the surface of the conductive body, β_1, β_2 stand for the parameters characterizing the pulse's front rise and decay time.

This expression reflects with sufficient accuracy the generic time dependency of the electromagnetic pulse, which is widely used in the practice of magnetic-pulse treatment of conductive material – it begins with a rapid rise to the maximum and then slowly decays [5, 9]. Such time dependency is caused by the specific nature of capacitor-solenoid systems' operation. The parameter $\beta_2 >> \beta_1$ what provides the afore-described time dependency of the pulse. That EMP time dependency is shown in Fig. 1.

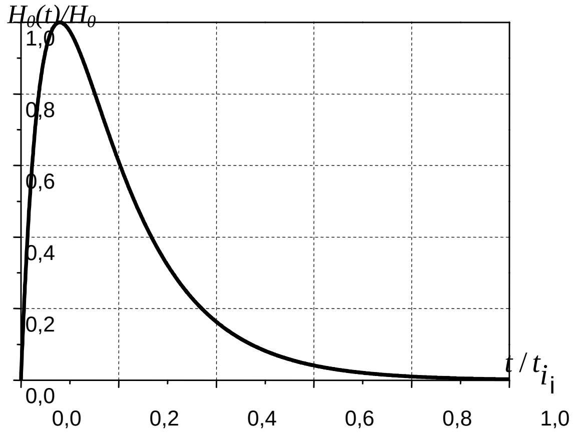

Fig. 1. Time dependency of the electromagnetic pulse.

Let us consider a layer that is exposed to the action electromagnetic pulse (EMP), defined by the values of tangential component H_y on both sides of the layer in the form (30), i.e. $H_y(\pm 1,t) = H_0(t)$.

Numerical studies have been conducted for layers made of the following non-ferromagnetic materials: steel (X18H9T), copper (Cu), and aluminum (Al) of $2h = 2\cdot 10^{-3}$ m thickness.

In Fig. 2 – Fig. 8 there are shown the time dependencies of Joule heat Q, the component F_z of the ponderomotive force $\vec{F}$, the temperature constituents T^Q and T^F, the stress tensor components σ_{zz} i σ_{xx}, and the total stress intensity σ_i in the layer under the action of a pulse with the duration of $t_i = 100$ mcs. The parameters of the pulse: β_1 =69000; β_2 =138000; κ =4. The dependencies 1-3 in Fig. 2 – Fig. 4 and Fig. 8 correspond to the functions' values calculated for z =1; 0,5; 0, respectively. The dependencies in Fig. 5 – Fig. 7 are given for such z-coordinate when their values reach maximum ones (Fig. 5 shows T^F time dependency for $z = 0$; Fig. 6 shows σ^Q_{zz} and σ^F_{zz} time dependencies for $z = 0$; Fig. 7 shows σ^Q_{xx} time dependency for $z = h$ and σ^F_{xx} time dependency for $z = 0$).

Joule heat dependencies have the character of two consecutive pulses – the maximum of the first one is significantly bigger than the maximum of the second one. At the initial instants the ponderomotive force is of compressive nature, later on it is of stretching nature and at $t \approx 0,5 t_i$ decays to zero. The temperature constituent T^Q has its highest values on the layer's sides $z = \pm h$. The time dependency of the temperatures constituent T^F caused by the ponderomotive force corresponds to the time dependency of stress tensor components σ^F_{jj} $(j = z, x, y)$, and its maximum (for these specific EMP parameters) is by two orders less than the maximum of the temperature constituent T^Q caused by Joule heat. That is why the temperatures constituent T^F negligible in comparison with the temperature constituent T^Q.

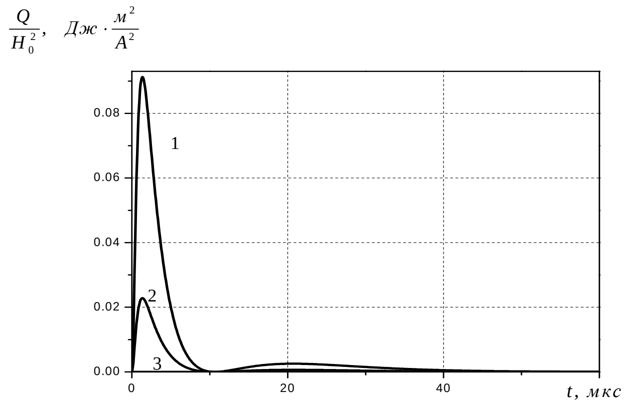

Fig. 2. Time dependencies of Joule heat within the layer.

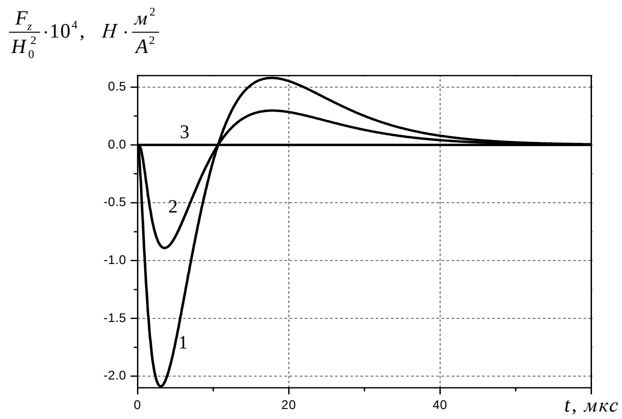

Fig. 3. Time dependencies of the ponderomotive force within the layer.

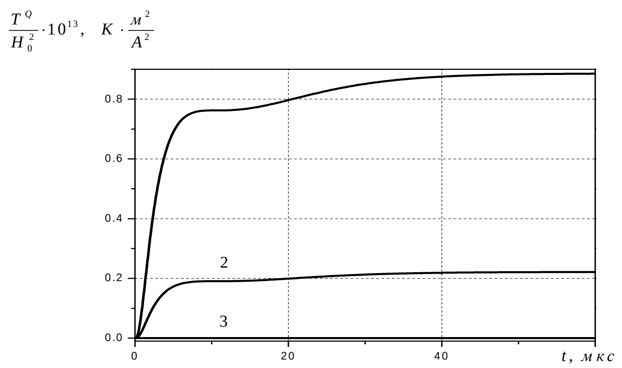

Fig. 4. Time dependencies of the temperature constituent T^Q within the layer.

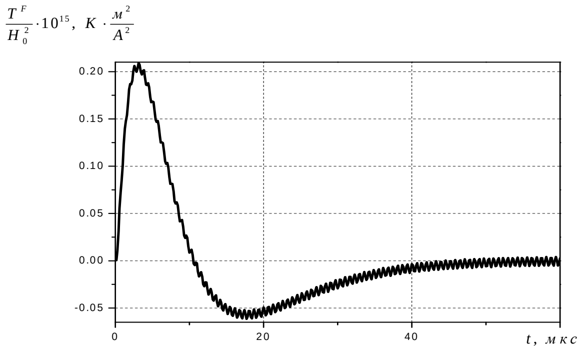

Fig. 5. Time dependencies of the temperature constituent T^F within the layer.

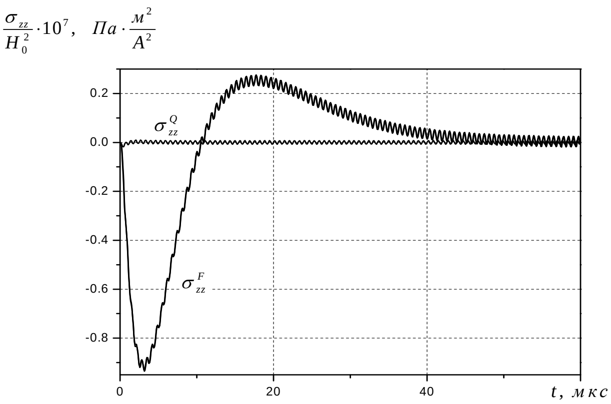

Fig. 6. Time dependencies of normal stress σ_{zz} within the layer.

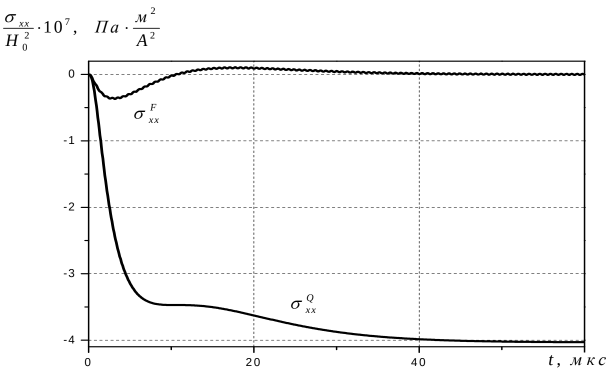

Fig. 7. Time dependencies of tangential stress σ_{xx} within the layer.

At the initial instants the stress tensor component σ_{zz}^F is of compressive nature, later on it gets of stretching nature, and at $t \approx 0,4 \div 0,5 t_i$ it enters natural mode. The maximum values of compressive stress σ_{zz}^F are about three times bigger than the maximum values of stretching stress. At the initial instants the stress tensor component σ_{zz}^Q is of compressive nature, later on it starts to oscillate around zero. Its values are much smaller than those of the stress tensor component σ_{zz}^F. The stress tensor component σ_{xx}^F is of the same character as the stress tensor component σ_{zz}^F, but its maximum values are three times smaller. Stress σ_{xx}^Q is of compressive nature, its time dependency reflects the temperature T^Q time dependency, and its values exceed by one order the values of stress σ_{xx}^F. Stress σ_{zz}^F and stress σ_{xx}^Q are of the same magnitude.

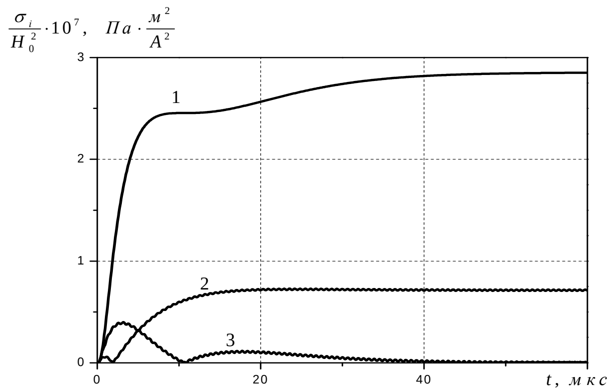

Fig. 8. Time dependencies of the total stress intensity σ_i within the layer.

Fig. 9 illustrates the dependency of total stress intensity $\sigma_i^{\max}$ maximum on the value of H_0 in the steel, aluminum, and copper planar layers of $2h = 2$ mm thickness for two given pulse durations (t_1 = 1000 mcs – thick curves, t_1 = 100 mcs – thin curves). With the help of those dependencies one can find maximum critical values of H_0 ($H_0 = H_0^{кр}$), when the load-carrying capacity of the layer is lost, for given values of dynamic limit of elastic deformation σ_d of the layer's material.

For the layers under study the following results have been obtained:

a) For EMP duration $t_i = 1000$ mcs

– for the steel layer ($\sigma_d = 300$ MPa) – $H_0^{кр}$ is not reached if $H_0 \le 5 \cdot 10^7$ A/m,

– for the copper layer ($\sigma_d = 70$ MPa) – $H_0^{кр} \approx 1,3 \cdot 10^7$ A/m,

– for the aluminum layer ($\sigma_d = 30$ MPa) – $H_0^{кр} \approx 0,65 \cdot 10^7$ A/m.

б) For EMP duration $t_i = 100$ mcs

– for the steel layer ($\sigma_d = 300$ MPa) – $H_0^{кр}$ is reached if $H_0^{кр} \approx 3,3 \cdot 10^7$ A/m,

– for the copper layer ($\sigma_d = 70$ MPa) – $H_0^{кр} \approx 0,8 \cdot 10^7$ A/m,

– for the aluminum layer ($\sigma_d = 30$ MPa) – $H_0^{кр} \approx 0,55 \cdot 10^7$ A/m,

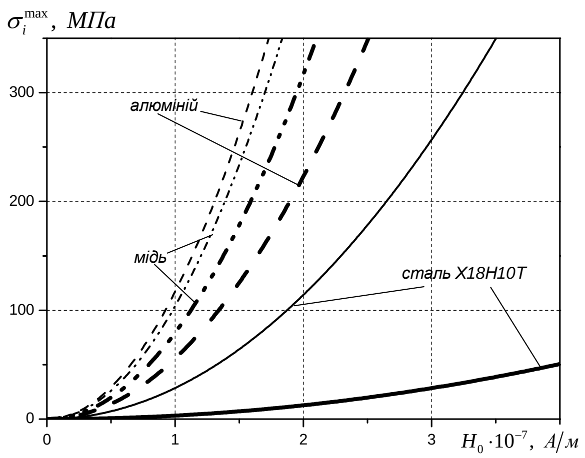

Fig. 9. Dependency of maximum values of the total stress intensity $\sigma_i^{\max}$ on the value of H_0 in the layers made of different material.

Similar dependencies have been also received for non-ferromagnetic layers of different thickness.

6. Conclusions

The obtained dependencies of the total stress intensity maximum values in the investigated conductive plates made of different non-ferromagnetic materials, including stainless steel X18H9T, copper and aluminum, on the maximum value of magnetic field intensity on the surface of the body for different EMP durations allow us to find critical values of EMP physical and temporal parameters – their exceeding leads to loss of load-carrying capacity of the plate as a structural element.

Those qualitative and quantitative patterns of thermo-mechanical behavior of conductive plates under the action of EMP can serve as a basis for development of rational operation modes of structural elements of appliances and devices exposed to pulsed electromagnetic field, in order to maintain their load-carrying capacity.

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ТЕРМОНАПРУЖЕНИЙ СТАН І НЕСУЧА ЗДАТНІСТЬ НЕФЕРОМАГНІТНИХ ЕЛЕКТРОПРОВІДНИХ ПЛАСТИН ЗА ДІЇ ЕЛЕКТРОМАГНІТНИХ ІМПУЛЬСІВ

Роман Мусій

Розглянуто математичну модель пружного деформування неферомагнітних електропровідних тіл за дії імпульсних електромагнітних полів (ЕМП), яка враховує адіабатичний характер процесів нагрівання і деформування електропровідних тіл імпульсними ЕМП. В рамках даної моделі сформульовано динамічну задачу термомеханіки для неферомагнітного електропровідного шару з плоскопаралельними границями за дії імпульсного ЕМП. Запропоновано методику наближеного розв'язування сформульованої задачі, яка використовує апроксимацію розподілів всіх ключових функцій за товщинною координатою кубічним многочленом. Знайдено і чисельно проаналізовано розв'язок розглядуваної задачі за однорідної дії електромагнітного імпульсу (ЕМІ). Досліджено термомеханічну поведінку і несучу здатність неферомагнітних електропровідних пластин за дії електромагнітних імпульсів.

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