# MATHEMATICAL MACROMODELING OF DYNAMIC SYSTEMS AND EXAMPLES OF MACROMODELS

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***The summary* - The problems of macromodeling of continuous nonlinear dynamic systems with lumped constant parameters are considering. The common modeling structure in the form of the system of ordinary differential equations is shown. The regularised methods of its identification are described. The examples of macromodels of different complexity and nature are given.**

*Keywords* - **macromodelling, regularization, self-oscillator model, prognostic model of the rate of exchange.**

## I. THEORETICAL SUBSTANTIATION

We designate as macromodels the mathematical models of arrangements and systems which can be more simple than original but represent enough exactly the appreciable external peculiarities of modeling objects behavior.

On the whole the task of model construction is composed in: selection of the valid macromodels structures in the form of system of differential equations of the define class; normalized identification of the macromodels.

In the general case the mathematical macromodel is the operator, which connects the input, output and internal variables:

$$\Phi(\bar u(t),\bar y(t),\bar x(t);\bar p)=0\,, \qquad (1)$$

where: $\bar u=(u_1,\dots,u_r)$, $\bar y=(y_1,\dots,y_s)$, $\bar x=(x_1,\dots,x_n)$ are the vectors of input, output and internal values; $\bar p=(p_1,\dots,p_n)$ – vector of operator parameters.

To identificate the mathematic model of the system means to find under the known vectors $\bar u(t\in[t_0,t_1])$ and $\bar y(t\in[t_0,t_1])$ the vector $\bar p$ of the model $\Phi(\bar u, \bar y, \bar x; \bar p)$ such one in order to:

$$\min_{\bar p}\left\|\bar y(t)-\tilde{\bar y}(t;\bar p)\right\| \quad \text{for all} \quad t\in[t_0,t_1]\,, \qquad (2)$$

where $\tilde{\bar y}(t;\bar p)$ is the solution of the equation (1).

In practice the vectors $\bar u(t)$ and $\bar y(t)$ are given on the discrete set of values of the $\{t_k\}$, $k$=1,...,$M$. In this case the criterion of identification (2) looks as:

$$\min_{\bar p}\left\|\bar y(t_k)-\tilde{\bar y}(t_k;\bar p)\right\|\ , \quad k=1,...,M. \qquad (3)$$

The selection of the definite norm determines the method of the optimization task solution (3). The routine method is the least-squares method

$$\min_{\bar p}\sum_{k=1}^{M}(\bar y(t_k)-\tilde{\bar y}(t_k;\bar p))^2\ , \quad k=1,...,M \qquad (4)$$

and Tchebyshev minimax method

$$\min_{\bar p}\max_{k=1,M}\left|\bar y(t_k)-\tilde{\bar y}(t_k;\bar p)\right|, \quad k=1,...,M\,. \qquad (5)$$

If $\tilde{\bar y}(t_k;\bar p)$ is a linear function of components of the parameters vector $\bar p$, then (4) comes to the solution of $s$ systems of linear algebraic equations of $N$ order, and (5) solves by linear programming methods.

Macromodeling lies in the selection of mathematical operator structure in (1) and in the identification the parameters of the selected structure by the given sets $\{\ \bar u(t_k)\ \}$, $\{\ \bar y(t_k)\ \}$, $k$=1,...,$M$. At that the mathematical operator in (1) has to be the most simple.

The theorem which validates the common structure of dynamic systems macromodels is proved [1].

<u>Theorem</u>. Let the non-linear system admits the description with ordinary algebro-differential equations

$$\begin{aligned}\dot{\bar x} &= \bar f(\bar x,\bar u,t),\\ \bar y &= \bar\varphi(\bar x,\bar u,t),\end{aligned} \qquad (6)$$

where: $\bar u(t),\bar x(t),\bar y(t)$ are vectors of the input, states and output variables; vector-function $\bar\varphi(\bar x,\bar u,t)$ is differentiated by $t$; total dimensionality of vectors $\bar x(t),\bar y(t)$ is equal to $k$.

The equivalent by input-output system is that one, which consists of linear stationary dynamic and non-linear non-stationary non-dynamic subsystems according to equations

$$a(\lambda)\bar y = D(\lambda)\bar\chi(\bar y,\bar y^{(1)},...,\bar y^{(k-1)},\bar v,\bar v^{(1)},...,\bar v^{(k-2)},\bar u,\dot{\bar u},t). \qquad (7)$$

The matrix of the transfer functions $W(\lambda)=D(\lambda)/a(\lambda)$ describes the linear subsystem with output vectors $\bar y,...,\bar y^{(k-1)},\bar v,...,\bar v^{(k-2)}$ and input vector $\bar v$, and the non-linear vector-function $\bar\chi(.)$ responds to the non-linear subsystem with input vector $\bar y,...,\bar y^{(k-1)},\bar v,...,\bar v^{(k-2)},\bar u,\dot{\bar u}$ and output vector $\bar v$.

The system (7) responses to the block diagram on Fig.1

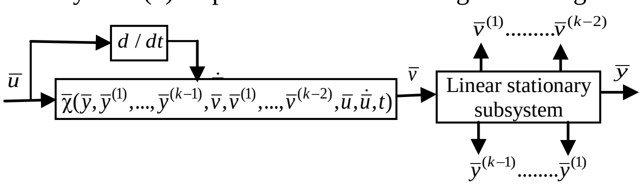


Fig.1. The structure of non-linear system

In [1] the communications of the structure (7) with different known structures of the mathematical models are analyzed. In particular the Hammerstein model is a special case of the structure (7), and the Wiener model do not arise from it.

The practical value of structure (7) depends on the presence of the identification methods. The identification of the non-linear systems structures consists in the identification of the linear dynamic subsystem, determination of the internal vector $\bar v$ and approximation of non-linear vector-function $\bar\chi(.)$. There are two methods of macromodels identification: long time known method of non-linear statics and small-signal dynamics; relatively new method of the inverse linear subsystem [1].

So the goal of the macromodeling after the structure (7) comes to the approximation of the linear dynamic system

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(more simple task) and non-linear vector function of many arguments $\overline{\chi}(.)$ (the task is significantly complicate).

For the second approximation the choice of the basis-functions is very important. The multidimensional power polynomials are spread:

$$\chi(\overline{\xi}) \approx \sum_{i=0}^{r} \sum_{j=0}^{r} \dots \sum_{k=0}^{r} c_{ij\dots k} \xi_1^i \xi_2^j \dots \xi_n^k, \quad i+j+\dots+k \le r\,. \qquad (8)$$

The basis for this gives the known theorem of Stone-Weyerstrass, which determines the principal attainability of any accuracy of approximation when the power $r$ of the multidimensional power polynomial increases (8).

However the immediate solution of the tasks (4) or (5) at the approximation (8) reveals the incorrectness from the viewpoint of Hadamard for sufficiently large $r$. In particular the small shanges (noises) of the vectors $\overline{\xi}$, $\overline{v}$ cause the significant changes of the values $c_{ij\dots k}$.

There exist the methods of incorrectness elimination which are based on the theory of regularization of incorrect tasks [2]. In particular the author elaborated the methods of regularization based on the spectrum of the compound signal splitting [1], reduction of approximating polynomial [6], choice of the approximating basis [5], use of the Lyapunov second method [7].

## II. Examples of the Macromodelling

Author synthesized many original models of the non-linear dynamic systems of the different complexity and nature: electronic oscillators and detectors; rotating synchronous generator; realization of the random process; the operating amplifier; block of TV chromaticity, prognostic macromodels of finance and economic system at rate of exchange etc. [1,3-8].

Let us use the method inverse linear subsystem for the development of the macromodel of generator of monoharmonic signal.

First let us choose the linear subsystem. We can choose it in the form of conservative linear system with complex eigenvalues which in pairs correspond to the known frequency components of the signal generator. Then considering the requirements of the transit process, the required form of the signal, stability of the boundary cycle or others, remains to choose the non-linear vector-function.

The diagram of the generator on one transistor which is the subject of modeling is shown on Fig.2 with MICROCAP language.

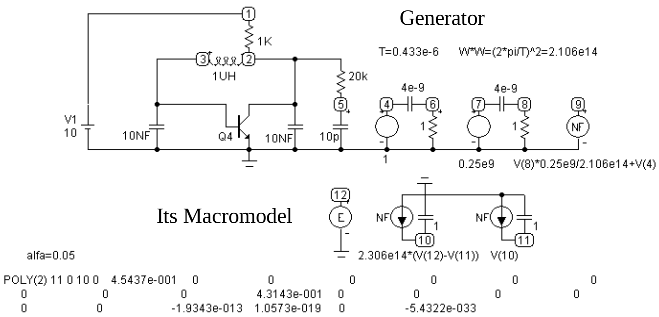


Fig.2. The diagram of the generator with the differentiating links and macromodel diagram

The equations of macromodel are the following:

$$\begin{aligned} &\dot{y} = y_1; \\ &\dot{y}_1 = -\omega^2 y + \omega^2 v; \qquad , \\ &v = \sum_{i,j=0}^{5} k_{ij} y^i y_1{}^j; \quad i+j \le 5. \end{aligned} \qquad (9)$$

where two first equations describe the linear dynamic subsystem and the last one – the non-linear subsystem.

The diagram of the equations is shown on Fig. 3 and it completely corresponds to the structure shown on Fig.1.

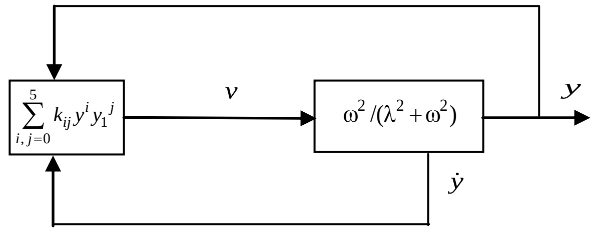


Fig.3. The diagram of the macromodel equations

The electric circuit of the macromodel which consists of two integrators composed of ideal current sources and unitary capacities and non-linear voltage source E, polynomial coefficients of which are under the macromodel diagram, is shown on Fig.2.

After the generator output signal (the voltage of the node 5 on Fig.2) the period of the oscillations after the transit process finishes $T$=0.433e-6 is determined. The correspondent value of frequency square $\omega^2$=2.106e14 is the only parameter for linear conservative subsystem of the second order with unit transfer in statics:

$$\ddot{y} + \omega^2 y = \omega^2 v\,. \qquad (10)$$

For calculation of the internal signal $v$ as the output one of the inverse linear subsystem the second derivative $\ddot{y}$ (the voltage of the node 8 on Fig.2) is calculated by the successive differentiation. Then the macromodel internal signal $v$ (the voltage of the node 9) is calculated by the equation (10).

The sequence of $M$ values of signals $y$, $\dot{y}$ and $v$, to which the voltages of the nodes 4, 7 and 9 on Fig .2 correspond, is used for normalized by Tikhonov calculations of the two-dimensional power polynomial of the non-linear function in model (9) correspondingly to (11).

$$\min_{k} \left( \sum_{m=1}^{M} \left( v_m - \sum_{i,j=0}^{5} k_{ij} y_m^i \dot{y}_m^j \right)^2 + \alpha \sum_{i,j=0}^{5} k_{ij}^2 \right), \quad i+j \le 5\,. \qquad (11)$$

The combined method of the approximating polynomial reduction founded in [1] was used. The idea of reduction lies in the reveal and elimination of the “unnecessary” terms of the approximation power polynomial. For that the small random excitations are introduced in the multitude of the signals readings and the task (11) is solving twice: without excitations and with excitations. That term of the polynomial is deleted which coefficient received the largest relative deviation owing to excitation

$$\delta_{ij} = \left| (\widehat{k}_{ij} - k_{ij}) / \widehat{k}_{ij} \right|\,, \qquad (12)$$

where $\widehat{k}_{ij}, k_{ij}$ are the approximation coefficients of the excited and unexcited tasks (11).

The multiple repetition of reduction procedure decreases the coefficient vector length and can improve significantly the macromodel quality (9) from the point of view of the given dependence representation. The reduction stops when between the relative derivations (12) there are no such ones which significantly exceed others and the quality of the model is satisfactory. In our task the reduction decreases the quantity of approximation coefficients of the non-linear function from 20 to 5.

The transition process in the generator and in the macromodel is shown on Fig. 4. On the top the projection of the phase-plane portrait onto the plane (V(4), V(4)`) and the phase-plane portrait of the macromodel.

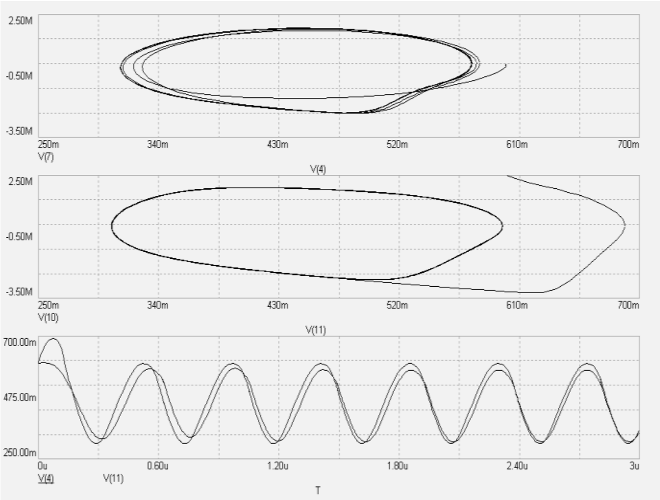


Fig.4. The transition in the generator and macromodel

The established mode represents with maximum relative error equal to 6%.

The combined method of normalization is used for the prognostic macromodels of socio-economic systems. The dependence of the rate of exchange on time is given in the discrete consequence form:

$$y_0(t_i);\ i = \overline{1, m}. \qquad (13)$$

Let us represent this dependence by the reaction of the macromodel with the structure given with a system of ordinary differential equations (14). Because there is no information about a linear dynamic subsystem it is chosen as a sequention of the ideal integrators.

$$\begin{cases} \dot{y}_0 = y_1; \\ \dot{y}_1 = y_2; \\ \vdots \\ \dot{y}_n = \sum\limits_{i_0,\dots,i_n=0}^{r} c_{i_0 \dots i_n} y_0^{i_0} \dots y_n^{i_n}; \qquad i_0 + \dots + i_n \le r; \end{cases} \qquad (14)$$

The parametric identification of the macromodel (14) consists in the calculation of $n$+1 derivatives of the discrete dependence (13) using the spline-interpolation and determination of the of the coefficient approximation vector $\overline{c}$ with normalized after Tikhonov linear task of minimization:

$$\min_{\overline{c}} \left( \sum_{i=1}^{m} \left( \dot{y}_n(t_i) - \sum_{i_0,\dots,i_n=0}^{r} c_{i_0,\dots,i_n} y_0^{i_0}(t_i) \cdot \dots \cdot y_n^{i_n}(t_i) \right)^2 + \alpha \sum_{i_0 \dots i_n = 0}^{r} c_{i_0 \dots i_n}^2 \right) \qquad (15)$$

But the coefficients vector $\overline{c}$ from (15) does not ensure the desired quality of macromodel (14). So the additional normalization of the task (15) is used – the reduction method described in the preceding example.

If it is succeed to represent well the dependence (13) with the help of equations (14), it will have the prognostic properties. The solution of the system (14) outside the range of identification boundary ($t_1$,$t_m$) keeps the dynamic properties of the identification object if the macromodel represents well the given consequence (13). So the prognosis is reliable till the conditions at which the dependence (13) is recorded.

For macromodeling the rate of exchange of US dollar to Ukrainian hryvna during the year 1998 (Fig. 5).

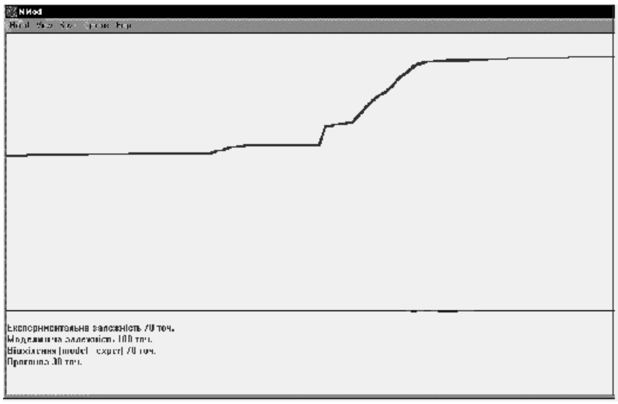

Fig.5. Prognostics diagram of ($US)/(UA hryvna) quotation

The best results were received at $n$=2 and $r$=3 with reduction of polynomial till 17-18 elements. By the observed dependence (70 days) the prognosis for 30 days was realized. As the initial conditions of the system of differential equations the means of derivatives in the last node of the observed dependence were taken.

Usually the prognosis applies to the unknown future. The received model allows to find easy the retrospective prognosis (to the past). It is enough for that to invert the sign of the right part in all equations of the developed macromodel (14) what responds to the time inversion, to realize the integration at chosen initial conditions and to invert the time in the received solution.

## REFERENSES

[1] Матвійчук Я.М. Математичне макромоделювання динамічних систем: теорія та практика. Видавничий центр ЛНУ ім.І.Франка. 2000. –215с.

[2] Тихонов А.Н. и др. Численные методы решения некорректных задач. М., 1990.

[3] Матвийчук Я.Н. Общие структуры макромоделей нелинейных динамических систем // "Электронное моделирование". Междунар. науч.-теор. журнал. №4, 1994. – С.42-48.

[4] Матвийчук Я.Н. Синтез детерминированных систем, воспроизводящих заданные случайные последовательности. "Электронное моделирование". №3, 1998. – С.25-31.

[5] Матвійчук Я.М. Математичне макромоделювання синхронного генератора // "Теоретична електротехніка". Зб. Львів. ун-ту. Вип.53, 1996. –С.156-161.

[6] Я.Матвійчук, А.Курганевич. Регуляризація задачі ідентифікації макромоделей нелінійних динамічних систем методом редукції апроксимаційного базису // "Теоретична електротехніка" Вип.55, 2000. –С.31-36.

[7] Я.Матвійчук. Регуляризація математичних макромоделей за допомогою другого методу Ляпунова "Теоретична електротехніка". Вип.57, 2004. –С.12-18.

[8] Я.Матвійчук, В.Паучок. Регуляризована ідентифікація динамічних прогностичних макромоделей. "Теоретична електротехніка". Вип.57, 2004. –С.31-37.