Definition of Dynamic Systems Dimension on an Output Signal through Correlation Dimension in a Pseudo-Phase Space

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***Abstract*--The universal method is described which allows to receive a reliable estimation of a nonlinear system dimension only by one scalar output signal, at this the estimation does not depends on what an output signal of the system is used.**

I. INTRODUCTION

At mathematical models of nonlinear dynamic systems making the tentative estimation of dimension of a system is very useful. Such estimation allows to avoid the excessive complexity of the model and the incorrectness concerned with it. The widespread methods evaluate the dimension with reference indications of transient process, their formalization is difficult and they are effective for the low order systems, linear as a rule.

In this paper the universal method is described which allows to receive a reliable estimation of a nonlinear system dimension only by one scalar output signal, at this the estimation does not depends on what an output signal of the system is used.

II. THEORETICAL SUBSTANTIATION

Let the output signal of the system depends on a vector of a variable state

$$x = (x_1,x_2,..,x_N). \qquad (1)$$

Since the phase variable systems $x_1,x_2,..,x_N$ are normally inaccessible for the direct observation, we shall consider the system behavior in a pseudo-phase space with coordinates

$$y(t),\ dy(t)/dt,\ d^2y(t)/dt^2,...,\ d^{M-1}y(t)/dt^{M-1}, \qquad (2)$$

where $y(t)$ is the output signal of the system.

The discrete analog of a pseudo-phase space (2) looks like

$$y(t),\ y(t+\tau),\ y(t+2\tau),...,\ y(t+(M-1)\tau), \qquad (3)$$

where the step τ can be estimated by minimum of an autocorrelation function [1]:

$$\min_{\xi=\tau} \int_{-\infty}^{+\infty} y(t)y(t-\xi)dt\ . \qquad (4)$$

The phase portrait of the system in a pseudo-phase space can be constructed in such away. Let the output signal $y(t)$ is preset by a sequence of discrete values $\{y(k\tau)\}$ with constant step τ, where k is a natural number. Then the pseudo-phase trajectory will be formed with a sequence of points y_k with coordinates

$$y_k = (y(k\tau),\ y(k\tau+\tau),$$

$$y(k\tau+2\tau),\ y(k\tau+3\tau),...,\ y(k\tau+(M-1)\tau)), \qquad (5)$$

where M is a dimension of a pseudo-phase space.

It is known, that the topological features of the system phase portraits under the certain conditions are constant in the space of a variable mode (1) and in pseudo-phase spaces (2) and (3) [1,2]. The correspondence between dimension of a phase space N and dimension of a phase space M is given by the theorems formulated and proved by Takens F. [2]. The principal for us result of these theorems looks thus: a pseudo-phase space of the system is topologically identical to the phase space, if M=2N+1.

To estimate the system dimension is possible with the conception of fractal dimension for the given phase trajectory [1]. The different approaches exist for definition of a fractal dimension with approximately identical results. Let's take advantage of correlation dimension d_m conception for given dimension m of a pseudo-phase space [1]. Let's designate

$$s_{ij} = |y_i - y_j|$$

and define the correlation function in this way:

$$C(r) = \lim_{K\to\infty} 1/K^2 \times (\text{pairs points, for which } s_{ij} < r),$$

where K is the amount of points through all the phase trajectory.

Practically for definition $C(r)$ the following formula is used

$$C(r) = \lim_{K\to\infty} \frac{1}{K^2} \sum_{i=1}^{K} \sum_{\substack{j=1 \\ i\neq j}}^{K} H(r - s_{ij}),$$

where $H(s) = \begin{cases} 1, & s > 0 \\ 0, & s < 0 \end{cases}$, and K is limited from above.

Normally $\lim_{r\to 0} C(r) \approx r^{d_m}$, therefore correlation dimension d_m can be defined by the inclination of a straight line in co-ordinates (ln$C(r)$, ln$r$):

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$$d_m = \lim_{r \to 0} \frac{\ln C(r)}{\ln r}. \qquad (6)$$

Takens has proved, that the dimension d_m increases with m augmentation till the upper limit is reached at $m=M$ [2].

So, the algorithm of the system dimension estimation by the output signal is the following:

– to define a necessary discretization step by (4);

– to create on the signal discrete values the pseudo-phase spaces with different dimensions m (5);

– to calculate the correlation dimension d_m values by (6) for series of values of dimension m of the pseudo-phase space;

– to define $m=M$, at which d_m becomes maximum;

– to estimate the dimension of the system N by the formula N=(Ì-1)/2.

III. EXAMPLE OF A METHOD USE

Let's consider the example of such a method use of the dimension estimation.

Some generator described with five differential equations of the first order is studied [3]. The calculated by an algorithm dependence of correlation dimension d_m on the pseudo-phase space m dimension for the generator output signal of is the following:

m	2	3	4	5	6	7	8	9	10	11
d_m	0.960	0.963	1.036	1.062	1.096	1.145	1.145	1.143	1.140	1.164

The plot of this dependence is shown on Fig.1.

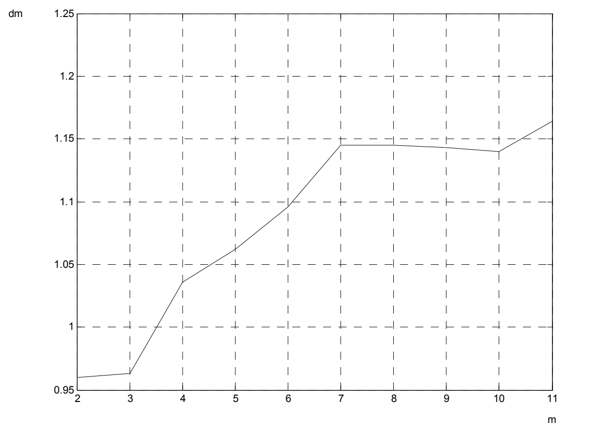

Fig. 1. The plot of correlation dimension dependence on pseudo-phase space

As we see, the correlation dimension achieves the local maximum, if the dimension of the pseudo- phase space m is equal to 7. By Takens [2], it corresponds to the third order system. So, the generator model which is described with a system of differential quintets, mainly has a behavior as the system of the third order. However for m=11 the value d_m again appreciably increases. It means that to reproduce all details of the object reaction is possible only by the model of the fifth order. Such conclusions are confirmed by immediate mathematical simulation of the generator by the output signal [3].

The values of correlation dimension were evaluated with a special PASCAL-program. On Fig.2 the dependences ln(C(r)) on ln(r) for different values of pseudo-phase space m dimension are shown. According these plots the root-mean-square linear approximations are constructed and with the corresponding straight lines inclinations the values of correlation dimension mentioned above are calculated.

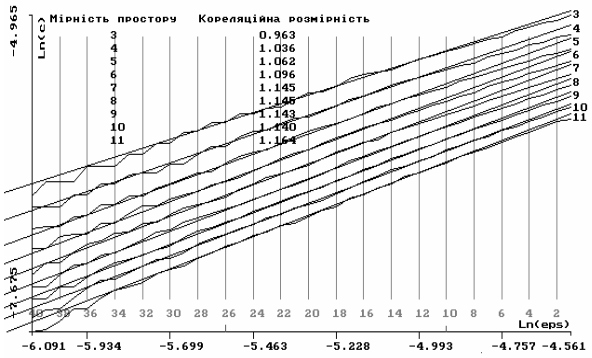

Fig. 2. Dependences ln(C(r)) from ln(r) for the generator

Since it is complicated to apply immediately the formula (6) because of numerical errors at r>0 increase, the following empirical method is used. The parts of the curves close to linear are marked the abscissa axis. On these parts the straight lines of approximation under the least squares method are constructed. By the inclinations of these straight lines the values of correlation dimensions are calculated.

IV. CONCLUSIONS

The appropriate PASCAL-program is used with success for the evaluation of correlation dimensions and estimation of dimension of numerous simulation objects. In particular, on the test examples of strange attractors with well known values of dimensions [1] the evaluation of correlation dimensions is checked.

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