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A lattice structure for biorthogonal two channel filter banks

Abstract. *A lattice structure for implementation of biorthogonal wavelet transforms of degree K/K i K/(K-2) has been proposed.*

Keywords: biorthogonal wavelet transform, biorthogonal lattice structure

Introduction

Wavelet transform belongs to the fundamental methods of digital signal processing [1-3].

Among its models that are easiest in implementation is the lattice model of the wavelet transform [2,3]. It is based on poliphase factorization od paraunitary matrix $\mathbf{H}_p(z)$ as a product of elementary basic matrices.

$$\text{(1)} \qquad \mathbf{H}_p(z) = \mathbf{\Lambda C}_J \mathbf{Z C}_{J-1} \mathbf{Z C}_3 \mathbf{Z} ... \mathbf{C}_2 \mathbf{Z C}_1 ,$$

where: $J=(K-1)/2$, K is an odd number indicating degree of filters, $c_k = \cos(\alpha_k)$, $s_k = \sin(\alpha_k)$,

$$\mathbf{C}_k = \begin{bmatrix} c_k & s_k \\ -s_k & c_k \end{bmatrix}, \ \mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}, \ \mathbf{\Lambda} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

For more general and very important case of biorthogonal wavelet transform, often applied in data compression, lattice model of the transform similar to (1) is so far unknown. The lattice structure proposed in [4] is only a fragment of analysis or synthesis stage. Closest to (1) seems the lattice structure proposed in [5,6]. However, it is necessary to point out that in [4-6] the application of lattice structure for the factorization of biorthogonal filter bank, has not been discussed. Hence, the construction of lattice model of the biorthogonal wavelet transform is a current and important issue.

Lattice model of biorthogonal filter bank

Let us consider a two-channel bank of biorthogonal filters (Fig. 1) composed of analysis filters H, G and synthesis filters S, R of degree K.

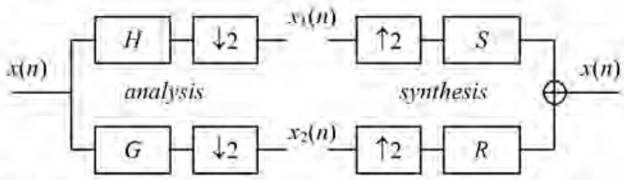

Fig.1. Biorthogonal filter bank for analysis and synthesis stage

Lattice model for the analysis stage of biorthogonal two-channel filter bank (Fig. 2) is based on the matrix equation

$$\text{(2)} \qquad \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} = \mathbf{H}_n(z) \begin{bmatrix} x(2n) \\ x(2n+1) \end{bmatrix}$$

and factorization of matrix $\mathbf{H}_n(z)$

$$\text{(3)} \qquad \mathbf{H}_n(z) = \mathbf{D}_J \mathbf{Z D}_{J-1} \mathbf{Z} ... \mathbf{C}_3 \mathbf{Z D}_2 \mathbf{Z D}_1 ,$$

where: $\det(\mathbf{D}_k) = \pm 1$, $\mathbf{D}_k = \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix}$ [5,6].

The main difference between factorization (1), (3) and orthogonal as well as biorthogonal lattice structure lies in basic matrices $\mathbf{C}_k$ i $\mathbf{D}_l$.

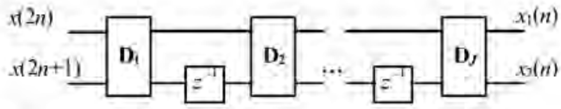

Fig.2. Biorthogonal lattice structure for analysis stage

Lattice model for the synthesis stage of biorthogonal two-channel filter bank is based on the matrix equation

$$\text{(4)} \qquad \begin{bmatrix} x(2n) \\ x(2n+1) \end{bmatrix} = \mathbf{H}_n^{-1}(z) \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix}$$

and factorization

$$\text{(5)} \qquad \mathbf{H}_n^{-1}(z) = \mathbf{D}_1^{-1} \mathbf{Z}^{-1} \mathbf{D}_2^{-1} ... \mathbf{Z}^{-1} \mathbf{D}_{J-1}^{-1} \mathbf{Z}^{-1} \mathbf{D}_J^{-1} ,$$

where: $\mathbf{D}_k^{-1} = \dfrac{1}{\det(\mathbf{D}_k)} \begin{bmatrix} d_k & -b_k \\ -c_k & a_k \end{bmatrix}$, $\mathbf{Z}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix}$.

The relationship between biorthogonal filter banks of wavelet transform and mentioned above lattice structures is given by the following lemma and theorem.

Lemma. Lattice structure (2), (3) or (4), (5) is respectively model of analysis or synthesis stage of biorthogonal two-channel filter bank.

Theorem. The analysis and synthesis stage of any biorthogonal two-channel filter bank degree K/K or $K/(K-2)$ can be expressed in a form of a lattice structure based on factorizations $\mathbf{H}_n(z)$ or $\mathbf{H}^{-1}{}_n(z)$.

Conclusions

Lattice structure (2), (3) and (3), (4) is a model that makes it possible to implement a broad class of biorthogonal wavelet transforms, including those applied to data compression standards with symmetric filters 7/5 i 9/7.

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