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Fast parametrized biorthogonal transforms

Streszczenie. *W pracy zaproponowano schemat syntezy szybkich parametryzowanych przekształceń biortogonalnych oparty o diagramy przepływowe dla szybkich przekształceń ortonormalnych i dwupunktowe biortogonalne operatory motylkowe.*

Abstract. *In this paper the authors propose the technique of synthesis of fast parametrised biorthogonal transforms that takes advantage of data-flow graphs for calculation of fast orthonormal transforms and two-point biorthogonal butterfly operators.*

Słowa kluczowe: szybkie parametryzowane przekształcenia liniowe
Keywords: fast parametrised linear transforms

Introduction

In recent years, there could be observed a considerable growth of techniques for synthesis of parametrized linear orthonormal transforms together with the development of fast algorithms for their computations. The parametrized transforms in contrast to transforms with fixed base vectors possess the fundamental advantage of the ability to adapt to statistical characteristics of input signals. For instance in contribution [1] the authors presented a fast Haar-type transformation where the first base vector is adjustable to any *a priori* assumed form. In paper [2] a fast two-stage transformation of order N was proposed where the full N-vector base can be adapted by modification of the $N/2\log_2 N$ number of distinct parameters and as the optimization tool the steepest decent gradient technique is exploited. In papers [3] and [4] we find the formulation of parametrized variants of known fast discrete Fourier, Hartley and Slant-Hadamard transformations.

In this contribution the authors propose the technique for synthesis of fast parametrized biorthogonal transforms that takes advantage of data-flow diagrams for the calculation of fast orthonormal transforms.

Fast parametrized biorthogonal transform

The proposed technique of synthesis of fast biorthogonal parametrized transforms makes use of the following two-point biorthogonal butterfly operators introduced in paper [5]:

$$A_{ij} = \begin{bmatrix} a_{ij} & b_{ij} \\ c_{ij} & d_{ij} \end{bmatrix}, A_{ij}^{-1} = \frac{1}{\det(A_{ij})} \begin{bmatrix} d_{ij} & -b_{ij} \\ -c_{ij} & a_{ij} \end{bmatrix}.$$

In order to ensure the invertibility of A_{ij} matrix we assume that $\det(A_{ij}) \neq 0$. The proposed synthesis technique allows to take the advantage of arbitrary multistage structure of data-flow diagram where two-point orthonormal operators are replaced with biorthogonal operators A_{ij}. Then forward transform U is constructed as the product of U_i matrices composed of butterfly operations A_{ij} realized on successive stages, i.e. $U = U_{n-1} \ldots U_1 U_0$ where i is the stage index and j is the index of operator on i-th stage. The inverse transformation U^{-1} is obtained as the product of inverse matrices U_i^{-1} multiplied in reverse order. It means that $U^{-1} = U_0^{-1} U_1^{-1} \ldots U_{n-1}^{-1}$ and U_i^{-1} matrices hold the inverse biorthogonal operators A_{ij}^{-1}. As a result of application of biorthogonal operators the norms and the angles between base vectors of U transform can take any values but always different to zero. The only possible restrictions are imposed by the connection-reduced structure of the fast data-flow diagram. The pair of U and U^{-1} transforms ensures the perfect reconstruction of signals. Then we name U the *biorthogonal transform*. Since the constructing matrices U_i are sparse and butterfly operators A_{ij} require the same number of 4 multiplications and 2 additions just like orthonormal operators then by analogy to the case of orthonormal transforms the resulting transformation U can be regarded as the *fast* one. It should be noted that each operator A_{ij} is described with four adjustable parameters what finally results in the adaptable *parametrized* transform.

In this paper in order to simplify the practical realization of inverse transformation we assume the following condition $\det(A_{ij}) = \pm 1$ to be fulfilled by each of A_{ij} operators.

Conclusions

It is obvious that biorthogonal linear transforms include the subclass of orthogonal transformations. However they do not undergo the restrictions of orthogonality and unit norm of base vectors. For this reason the possible range of applications of biorthogonal transforms is considerably larger and includes directly the issues of classification of signals, generalized Wiener filtering [6] and the tasks of blind source separation of signals realized on the basis of Independent Component Analysis techniques [7].

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