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Accuracy of the Hilbert matrix determinant calculation

Streszczenie. *Zaprezentowano nowe podejście do obliczenia dokładności wyznacznika macierzy Hilberta. Nowa metoda bazowana jest na wykorzystaniu pojęcia dobroci liczb. Dla sprawdzenia poprawności metody dokonano jej porównanie z badaniami statystycznymi.*

Abstract. *The new approach to determine the accuracy of calculation of the Hilbert matrix determinant is presented. The key element of the new method is the estimation of the accuracy of number based on so called quality of the number. In order to validate the new approach comparison with statistical calculation was used*

Słowa kluczowe, macierz Hilberta, metoda Gaussa, dobroć liczby.
Keywords: Hilbert's matrix, Gauss method, quality of the number

The Hilbert matrices are canonical examples of ill-conditioned matrices, making them notoriously difficult to use in numerical computation. Gaussian elimination (GE) is the standard method for solving both a system of linear equations and of determinant. As such, it is one of the most ubiquitous numerical algorithms and plays a fundamental role in scientific computation (in circuit theory, for instance). The accuracy of the solution of determinant of a Hilbert matrix by GE is the target of serious scientific research [1].

The key element of the new method is the estimation of the quality of the number (Qc) [2]. In general case Qc is a real number. The goal of the computer experiments is examination of the new approach. The first time the quality of the determinant of the Gilbert matrix obtained with the new method is calculated. After that the same determinant is calculated statistically with random change with normal distribution of the start elements of the matrix. The graph of associated probability density function is bell-shaped and is known as the Gaussian function (Fig.1a). The quality of the determinant is calculated by the formula (Fig.1b).

The experiments were conducted with different values of the alteration of the matrix H_7 elements: $H_7[i,j]\cdot(1\text{-}4\cdot10^{m})$, $H[i,j]\cdot(1\text{-}3.6\cdot10^{m}),\ldots, H[i,j]\cdot(1+4\cdot10^{m})$ in 20 points, $m = -9,-10,\ldots,-15$. The comparison of the values of the determinant of the Gilbert matrix that were obtained by the two methods has confirmed the adequacy of the new approach.

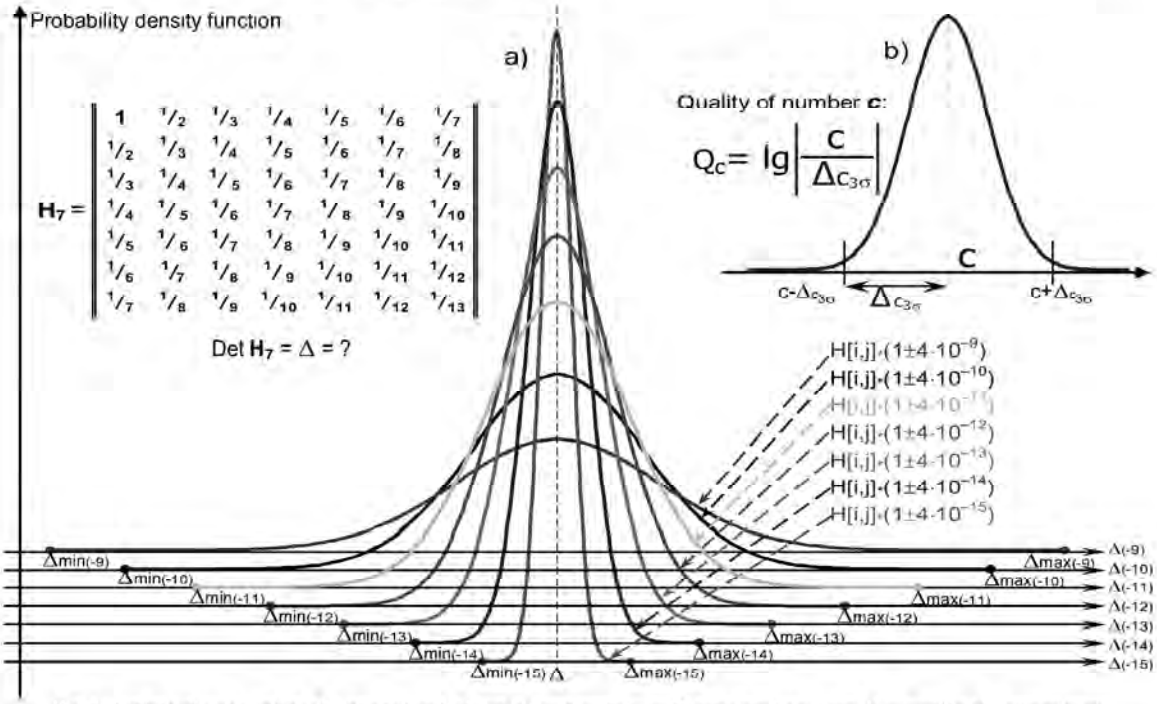

Fig.1 Statistical calculation of determinant of the Hilbert matrix: a- Gaussian functions, b – quality of determinant

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