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## Axial symmetry two dimensional model of a ferromagnetic mass accelerator

**Abstract.** Mathematical model of ferromagnetic mass accelerator is represented as the equations of lumped electrical circuit and magnetic field. This model takes into account the symmetry of magnetic field accelerator, the nonlinearity of ferromagnetic core characteristic  $B(H)$ , eddy currents.

**Streszczenie.** Model matematyczny akceleratora mas ferromagnetycznych jest przedstawiony jako równania skupionego obwodu elektrycznego oraz pola magnetycznego. Ten model uwzględnia symetrię akceleratorowego pola magnetycznego, nieliniowość charakterystyki rdzeniu ferromagnetycznego  $B(H)$ , prądy wirowe.

**Keywords:** coil, eddy current, electromagnetic accelerator, ferromagnetic core (armature).

**Słowa kluczowe:** cewka, prądy wirowe, akcelerator elektromagnetyczny, rdzeń ferromagnetyczny (armatura).

Object of research comprises the following main components: the pulse voltage source, stationary inductance coil that generates a magnetic field, and movable ferromagnetic core (Fig.1). Magnetic field coil has an axial symmetry, and therefore the mathematical model of the accelerator is a two-dimensional in the cylindrical coordinate system. This model takes into account the effect of moving core, the nonlinearity of its characteristics  $B(H)$  and eddy currents in the magnetic core material.

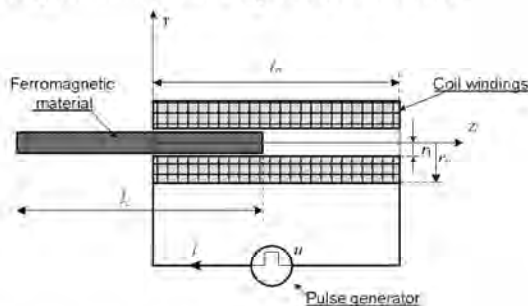


Fig.1. Illustration of axial symmetry of the accelerator

Since the ferromagnetic core is moving, it is not modeled as a separate region, and is represented as moving magnetic permeability distribution that is interpolated on a fixed mesh (Fig2). Therefore, research area is separated into two subdomains  $R1$  (core and environment) and  $R2$  (coil windings).

In the  $R1$  subdomain we can write magnetic field equation within the moving ferromagnetic core as

$$(1) \quad \gamma \frac{\partial \mathcal{A}}{\partial t} + \nabla \times (\mu^{-1} \nabla \times \mathcal{A}) + \gamma (-\mathbf{v} \times \nabla \times \mathcal{A} + \nabla \varphi) = 0$$

where  $\mathcal{A}$  is the magnetic vector potential,  $\varphi$  is the electric potential,  $\gamma$  – electrical conductivity of the environment,  $\mathbf{v}$  is the armature velocity (assuming a stationary observer) and  $\mu$  is the magnetic permeability.

In section  $R2$ , where the winding coil, the magnetic field equation has the form (2)

$$(2) \quad \nabla \times (\mu^{-1} \nabla \times \mathcal{A}) = \frac{wi}{l_w (r_c - r_i)} \mathbf{e}_\varphi$$

where  $w$  is the number of turns in the coil,  $i$  is the coil current,  $l_w (r_c - r_i)$  is the dimensions shown in Fig.1 and  $\mathbf{e}_\varphi$  is the azimuthal unit vector in circular cylindrical coordinates.

Electrical circuit, composed of a voltage source and the coil, described the second law of Kirchhoff

$$(3) \quad \frac{\partial \psi}{\partial t} + Ri - u = \int_l \frac{\partial \mathcal{A}}{\partial t} dl + Ri - u = 0$$

where  $\psi$  is the linkage,  $\Phi = \int_l \mathcal{A} dl$  is the magnetic flux,  $R$  is the electric resistance coils,  $i$  is the the coil current and  $u$  is the pulsed voltage source.

Boundary conditions are like magnetic

$$(4) \quad \mathbf{n} \times \mathcal{A} = 0$$

Lorentz force accelerates the ferromagnetic core. Its movement is described as a differential equation

$$(5) \quad m \frac{d^2 z_0}{dt^2} = F_z$$

where  $m$  is the mass of the ferromagnetic core,  $z_0$  is the moving coordinate core and  $F_z$  z-component of Lorentz force.

Lorentz force is calculated by the expression

$$(5) \quad \mathbf{F} = \int_V (\mathbf{J} \times \mathbf{B}) dV$$

where  $\mathbf{J}$  is the vector current density in the core,  $\mathbf{B}$  is the magnetic flux density in the core and  $V$  is the core volume.

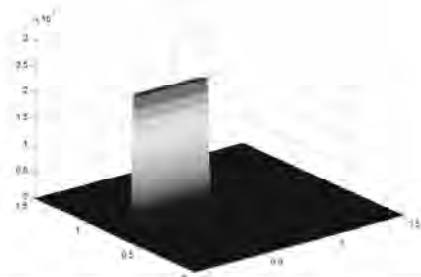


Fig.2. Distribution of magnetic permeability in  $R1$

### REFERENCES

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