Volodymyr EMETS, Jan ROGOWSKI

Institute of Computer Science Technical University of Lodz

Iterative regularization method of 3D scattering geometry reconstruction

Abstract. *The problem of the shape reconstruction of a scatterer from the scattered field measured outside under the illumination of an incident plane wave is considered. Theoretically, the inversion algorithm is derived using integral equation formulations together with the Newton - Kantorovich iterative regularization technique. Numerical results show that the good reconstruction is obtained with the only one wave number of the incident wave. The effects of noise on the imaging reconstruction were also investigated.*

Inverse scattering problems are concerned with the reconstruction of the objects from the knowledge of the scattered waves and can be studied nowadays using powerful numerical techniques. This problems have attracted the increasing attention because of interest in noninvasive measurement and remote sensing. However, there are two major theoretical difficulties in solving this problem: uniqueness and ill-posedness. As the measured data are contaminated by noise or not sufficient, the reconstruction is quite difficult. Reviews of studies of inverse acoustic and electromagnetic scattering problems can be found in [1-5]. Today, basically three different categories of numerical methods for the treatment of the full nonlinear scattering problem are known: iterative methods, decomposition methods and sampling/probe methods. Iterative reconstruction techniques can use all the knowledge which is available about the problem. They need to use only minimal data that are necessary to ensure at least local uniqueness of the solution. For inverse obstacle scattering, the methods work when a scatterer is illuminated by one time-harmonic wave coming from different directions. This study proposes an iterative regularisation method for reconstruction of the 3D scatterer geometry from the far-field measurement. Scattering of the incident field by dielectric body with general geometry may be calculated using a Lippmann-Schwinger type integral equation. By applying the discrete dipole approximation method to the solution of this equation, the investigation area is divided into cells small enough that the field value and the dielectric constant in each cell can be taken as constants. Thus, the scattered fields obtained in the direct problem can be regarded as good simulated values of the measured data. Assuming that scattered data are given for several backscattering directions for the only one wave number of the sounding plane wave, the solution of the inverse problem is given as the limit of the Newton-Kantorovich iterative process.

On Fig. 1-2 the results of the reconstruction of the revolution surfaces for $k = 0.5m^{-1}$ (m is a unity of length) and number n of iteration step for the sounding directions $\mathbf{l} = (\sin\theta_i, 0, \cos\theta_i)$, $\theta_i = \arccos\left[2i/(N_l+1)-1\right]$, $i = 0, 1, \ldots, N_l+1$ are given. On Fig. 1 the true curve is described by the formula $\rho(\theta) = 1 - 0.3P_4(\cos\theta) - 10^{-4}\left[P_2(\cos\theta) - P_6(\cos\theta)\right]$ where $P_n(\cos\theta)$ are the Legendre polynomials and (ρ, θ) are the cylindrical coordinates. The number of the sounding directions is $N_l = 3$. The numerical computations show that an augmentation of the number of the sounding directions leads to an augmentation of exactness of the scatterer surface reconstruction within the limits of the measuring errors being considered, that immediately ensues from the uniqueness theorem. The result of the reconstruction of the surface $\rho(\theta) = 1 - 0.3\left[P_2(\cos\theta) - P_1(\cos\theta)\right]$ is shown on Fig. 2 for $N_l = 9$ and for the same number of iterations as on Fig.1.

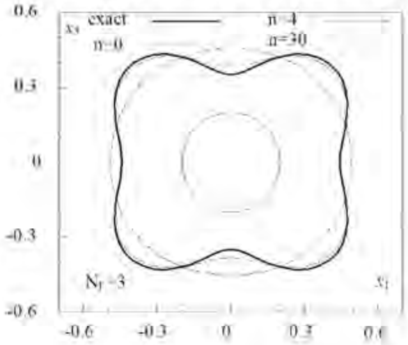

Fig. 1.

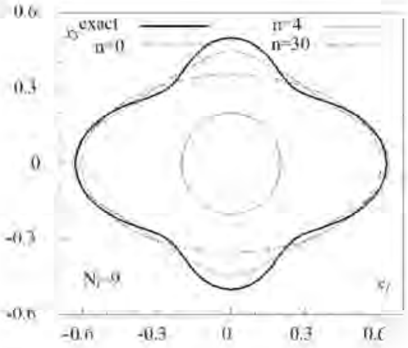

Fig. 2.

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Autorzy: prof. dr hab. Włodzimierz Jemec
email: wfemets@ics.p.lodz.pl
dr inż. Jan Rogowski
email: jasiorog@ics.p.lodz.pl
Politechnika Łódzka, Instytut Informatyki