УДК 539.3

M.H. Stashchuk[1], M.I. Dorosh[1], M.B. Sokil[2]
[1] Karpenko Physico-Mechanical Institute
of National Academy of Sciences of Ukraine,
[2] National university "Lviv polytechnic"

THE ANALYSIS OF SERVICE ABILITY OF POLYETHYLENE PIPES AND TANKS WITH THE HOLLOW WALL BY STRENGTH CRITERION

Запропоновано методику оцінки напружено-деформованого стану трубчастих елементів зі стільниковою будовою стінки. У результаті записано систему диференціальних рівнянь для відшукання зусиль та моментів, що виникають у стінці стільника. Встановлено ключові рівняння для оцінки компонент напружень. Наведено рекомендації до розрахунку та проектування довгої стільникової труби та вертикального стільникового резервуара, що розміщений у ґрунті.

Methods of the estimation of the stress-strain state of tubular elements with cellular wall structure are suggested. As a result, a system of differential equations for the search of the forces and moments which arise in the cell wall is written. Key equations for the estimation of the stress components are established. Recommendations on the calculation and designing of a cellular long tube and a vertical cell reservoir, located in soil, are presented.

Introduction. At present polymers and high-density polyethylene (HDPE) in particular are the most common materials for production of low-pressure industrial pipelines [1]. Thin-walled structures made of these materials possess a number of advantages in comparison with the metal ones [2]. Tests have demonstrated that the expected lifetime of polyethylene pipes is 50-100 years [3, 4, 5].

To decrease the weight of large diameter polyethylene pipes and tanks their walls are made as a hollow structure [6, 7]. In practice the polyethylene pipes and tanks with a tubular profile wall are used [7, 8]. The production technology of the thin-walled elements with a tubular profile wall foresees a continuous process of winding of infinitely long polyethylene pipes of a diameter of 20 ÷ 110 mm, using special devices-drums, with their synchronous extrusion welding of coils from the internal and outer sides. This technology allows us to produce a tubular construction of a diameter of 6000 mm.

A hollow pipe structure differs significantly from the solid-wall structure. Therefore, calculations for this type of buried thin-walled structures are different from those for the classical pipes [1, 2] and they should consider a wall hollowness [6, 7, 9, 10, 11]. To predict the durability and serviceability of polyethylene thin-walled structures with a hollow wall structure it is necessary to determine the stress-strain state and also to set the allowable strength parameters in such constructions. Based on the above mentioned the final formulas for engineering calculations are proposed and the appropriate practical recommendations are developed.

General methodology of the problem solution. Polyethylene is a viscoelastic material. Its behavior in the strained state depends on the external load, temperature and operating time. When making a design of the polyethylene tube constructions these characteristics must be taken into consideration. Investigations of the strength of polyethylene solid-wall pipes under action of the internal pressure depending on temperature and operation time enabled the development of the well-known international standards [3]. Similar studies were conducted in [4, 5]. According to the results of these studies it is possible to establish the allowable loads for the HDPE thin-walled structures with a tubular profile wall under condition

$$\max\left\{|\sigma_1|,|\sigma_2|,|\sigma_3|\right\} \le MRS, \tag{1}$$

where σ_i are principal stresses in the hollow construction wall, *MRS* is the minimum required strength of HDPE [3, 4].

When applying condition (1) for assessing the boundary state of the investigated constructions it is necessary to evaluate their stress-strain state. This paper proposes the method for assessing the stress-strain state of the polyethylene pipe of the underground hollow-wall constructions with account of the real operating conditions.

Basic equations for assessing the stress-strain state of cylindrical thin-walled structures with a tubular profile wall. A ratio of wall thickness *d* of the investigated thin-walled structure (Fig. 1) and its diameter *D* is $d/D<1/10$, where *d* is a diameter of the polyethylene tube, used to build a construction. Based on the diameters ratio let us use the shell theory apparatus for the stress-strain state evaluation [12]. A chart of the hollow wall structure is shown in Fig. 2.

a *б*

Fig. 1. Cylindrical thin-walled structures with a tubular wall profile: a – a long pipe b – a vertical tank (reproduced by permission of Corporation Energoresurs Invest, http://en.energoresurs.com/).

In fact, the polyethylene construction works under linear-elastic deformation. This is consistent with condition (1), in which parameter *MRS* is less than the yield strength of HDPE. The polyethylene creep is not taken into account.

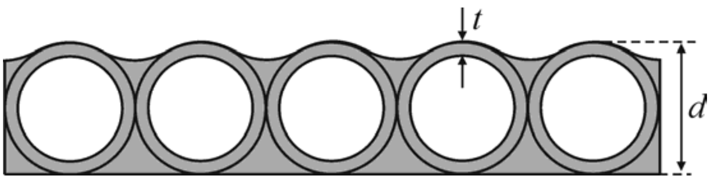

Fig. 2. Wall of a thin-walled construction

The system of differential equations for cylindrical orthotropic shells. Let us model a HDPE thin-walled structure by a cylindrical shell (Fig. 3). The shell is considered to be structurally orthotropic. Structural orthotropy means that anisotropic properties are caused by the hollow structure of the polymer product.

A shell is referred to the coordinate system *xOy* (Fig. 3). The location of the point of the shell median surface is characterized by coordinates *x* and $y=R\varphi$, where *x* is the distance of the point along a generatrix from the initial equatorial section, $R=D/2$ is the radius of the cylindrical shell, φ is the angle between the initial and arbitrary median planes. Axis *Oz* is directed outward along a normal to the median surface. It is considered that the main directions of elasticity at each point of the shell coincide with the direction of the coordinate lines *Ox*, *Oy*, *Oz*.

Let us investigate the stress-strain state of such a shell. The shell differential equations of equilibrium are similar to the equations of the theory of ordinary shells [12, 13].

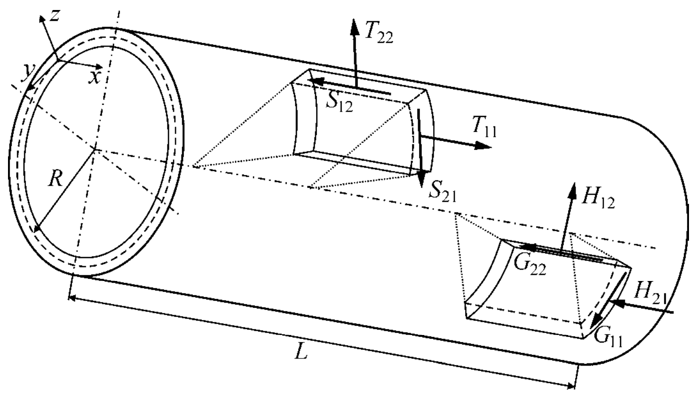

Fig. 3. Geometry and coordinate system of a cylindrical orthotropic shell

Deformation components ε_{ii}, ω, χ_{ii}, τ of the median surface are expressed by displacement components u, υ , w similarly to the orthotropic shells [12, 13]

A cellular structure is accounted by the corresponding presentation of the state equations of the considered shell. The equation of state, relating forces T_{11}, T_{22}, S_{12}, S_{21} and moments G_{11}, G_{22}, H_{12}, H_{21} of the strain components ε_{ii}, ω, χ_{ii}, τ of the medium surface, is written similarly to the equations of the theory of orthotropic shells [12, 13].

$$T_{11} = B_{11}^{*}\varepsilon_{11} + \nu_2 B_{11}^{*}\varepsilon_{22}\,, \quad G_{11} = -D_{11}^{*}\chi_{11} - \nu_2 D_{11}^{*}\chi_{22}\,, \; \left(1 \rightleftarrows 2\right),$$

$$S_{12} = B_{12}^{*}\omega\,, \quad S_{21} = B_{12}^{*}\omega + D_{12}^{*}\frac{\tau}{R}\,, \quad H_{12} = H_{21} = D_{12}^{*}\tau\,. \qquad (2)$$

The positive direction forces T_{ii} , S_{ji} and moments G_{ii} , H_{ji} are shown in Fig. 3.

Such a presentation allows us in a particular case to get known results for pipes with a solid wall. However, extensional rigidities B_{ij}^{*} , flexural rigidities D_{ij}^{*} in equation (2) which are established for tubes with a solid wall simultaneously with formulation of equations (2) in the case of cells, are unknown values. They determine the appropriate structural orthotropy. The value of these characteristics can be set by the experimental or numerical methods. Rigidities B_{ij}^{*} and D_{ij}^{*} depend on the structure of the polyethylene construction wall, i.e the diameter of polyethylene pipe d of its thickness t and the mechanical properties of polyethylene (elasticity modulus E and Poisson's ratio v).

The value of rigidities B_{ij}^{*} , D_{ij}^{*} and Poisson's ratio v_i are evaluated by the numerical experiment. For this purpose the rigidities in equation (2), are written in the form:

$$B_{ii}^{*} = f_{ii}\frac{Ed}{1-\nu_1\nu_2}\,,\; B_{12}^{*} = f_{12}\frac{Ed}{2(1+v)}\,,\; D_{ii}^{*} = p_{ii}\frac{Ed^3}{12(1-\nu_1\nu_2)}\,,\; D_{12}^{*} = p_{12}\frac{Ed^3}{12(1+v)}\,. \qquad (3)$$

Here v_i (i=1,2) are Poisson's ratios in the x and φ direction respectively for an orthotropic shell which values are found below, f_{ij}, p_{ij} are functions that are determined by the cellular structure. Values f_{ij}, p_{ij} characterize the decrease of extensional rigidities and flexural rigidities to compare to the same values for the shell with a solid wall of thickness d. Thus the problem arises to establish the Poisson's ratio v_i and functions f_{ij}, p_{ij}. When accepting f_{ij}=1, p_{ij}=1 and v_1=v_2=v the corresponding rigidity values for the shell with a solid wall of thickness d are obtained.

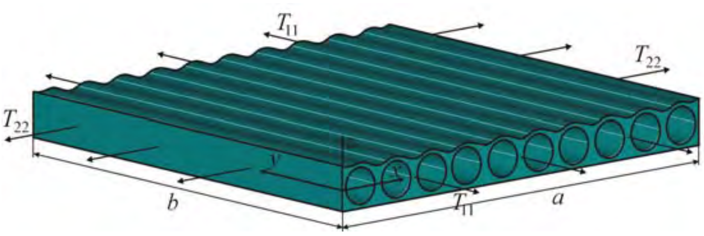

Fig. 4. A chart of a hollow plate

Determination of Poisson's ratios v_i and functions f_{ij}, p_{ij}. Poisson's ratios ν_i are found from the ratio of longitudinal ε_i and transverse ε_i' deformations. Longitudinal ε_i and transverse ε_i' deformations are evaluated under specimen tension. The specimen is a rectangular plate (Fig. 4).

Longitudinal ε_i and transverse ε'_i deformations are determined numerically using the finite element method [14]. Two cases are considered: the evenly distributed forces are applied in the direction *Ox* (T_{11}) and in the direction *Oy* (T_{22}) (Fig. 4). In the first case, the calculations will be related with evaluation of the Poisson's ratio v_1, and in the second – of v_2. The model of the investigated object is shown in Fig. 5. It takes into account the geometry of the cellular structure. The numerical analysis was done using the NASTRAN [15] finite element program system.

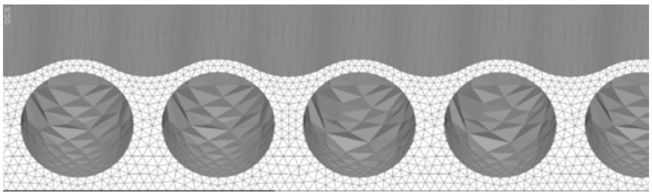

Fig.5. Finite element model of a rectangular hollow (cellular) specimen

When applying this program the displacement u_i (displacement in the direction *Ox*, in this case i=1 corresponds to forces T_{11}, and i=2 – to force T_{22}) and v_i.(towards *Oy*) of the medium surface of the hollow specimen is established. Using the obtained displacements determine the longitudinal ε_i and transverse ε'_i relative strains of the cellular plate specimen by the relation

$$\varepsilon_1 = \left|u_1(a) - u_1(0)\right|/a\,, \quad \varepsilon'_1 = \left|\upsilon_1(a) - \upsilon_1(0)\right|/a\,;$$

$$\varepsilon_2 = \left|\upsilon_2(b) - \upsilon_2(0)\right|/b\,, \quad \varepsilon'_2 = \left|u_2(b) - u_2(0)\right|/b\,. \tag{4}$$

Then Poisson's ratios v_1 and v_2 will be

$$\nu_1 = \varepsilon'_1/\varepsilon_1\,, \quad \nu_2 = \varepsilon'_2/\varepsilon_2\,. \tag{5}$$

By analyzing numerical calculations carried out on the basis of relations (4) and (5), the Poisson's ratios will be represented as

$$\nu_1(d/t) = h_1(d/t)\nu\,, \quad \nu_2(d/t) = \nu\,, \tag{6}$$

where v is the Poisson's ratio of the polyethylene and function $h_1(d/t)$ depends significantly on parameter d/t. Dependence h_1 on the parameter d/t is shown in Fig. 6.

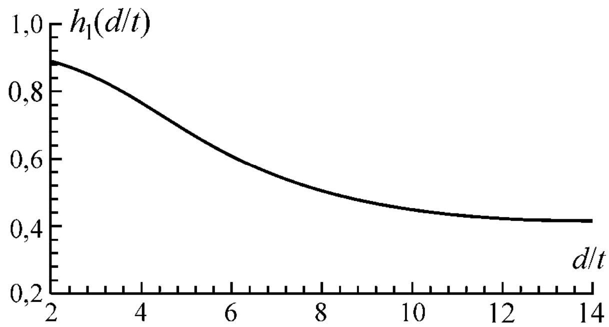

Fig. 6. Dependence of h_1 on d/t parameter

Rigidities B^*_{11}, D^*_{11} are evaluated using the results obtained for the stress-strain state of the hollow plate elements. Rigidities B^*_{11}, D^*_{11} and the corresponding functions f_{11}, p_{11} in equation (3), are evaluated numerically by the finite element method. For this purpose the first two relations (2) are written as

$$T_{11} = B^*_{11}\varepsilon_{11}\,,\; G_{11} = -D^*_{11}\chi_{11}\,, \tag{7}$$

where for the cellular plate element the deformation components $\varepsilon_{11} = \partial u/\partial x$ and $\chi_{11} = \partial^2 w/\partial x^2$, and component $\varepsilon_{22} = \partial\upsilon/\partial y \equiv 0$, $\chi_{22} = \partial^2 w/\partial y^2 \equiv 0$. Two last relations are true for the plane deformation conditions [16] of the elastic body. Therefore we consider a hollow plate element which is in the plane strain conditions (Fig. 2) and loaded first by force T_{11} and then by a separate moment G_{11}.

Using the finite element program NASTRAN [15] a set of numerical experiments to determine the stress-strain state of cellular elements under plane strain conditions were performed. The chart of the finite element model of the studied object is shown in Fig. 7. It takes into account the geometry of the cellular structure.

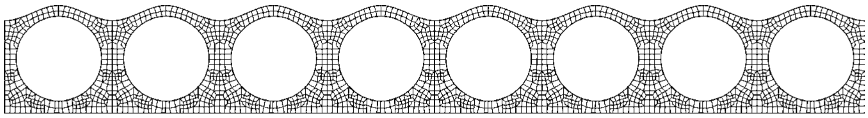

Fig. 7. A chart of the finite element model

From the data obtained for displacements u and w of the medium surface of the cellular lamellar specimen the components of strain ε_{11}, χ_{11} are set by formulas

$$\varepsilon_{11} = \left(u(a) - u(0)\right)/a, \quad \chi_{11} = \left(w(a) - 2w(a/2) + w(0)\right)/(a/2)^2. \tag{8}$$

Using relation (7) and taking into account expressions (8), the corresponding dependences of rigidities B_{11}^* and D_{11}^* are established.

By varying the parameter d/t one can find the corresponding dependences of rigidities B_{11}^*, D_{11}^* on parameter d/t, i.e. functions $f_{11}(d/t)$, $p_{11}(d/t)$. Below the dependence of these functions on parameter d/t is presented.

Setting rigidities B_{22}^* and D_{22}^* (in the circumferential direction) is less time consuming. So, for a hollow beam the relationships between its relative elongation and axial force N and also between curvature χ and bending moment M are as follows [17]

$$N = FE\varepsilon, \qquad M = I_x E\chi, \tag{9}$$

where F and I_x are area and moment of inertia of the beam cross-section .

Summarizing the results presented by relations (9), in the case of a hollow plate let us write:

$$T_{22} = \frac{FE\varepsilon_{22}}{d(1-\nu_1\nu_2)} + \nu_1 \frac{FE\varepsilon_{11}}{d(1-\nu_1\nu_2)}, \quad G_{22} = \frac{I_x E\chi_{22}}{d(1-\nu_1\nu_2)} + \nu_1 \frac{I_x E\chi_{11}}{d(1-\nu_1\nu_2)}. \tag{10}$$

Here F and I_x are the area and moment of inertia of polyethylene pipe containing a weld in the wall of the construction (Fig. 2).

From relations (2) and (10) the extensional rigidity B_{22}^* and the flexural rigidity D_{22}^* take the form

$$B_{22}^* = \frac{FE}{d(1-\nu_1\nu_2)} = f_{22}(d/t)\frac{dE}{1-\nu_1\nu_2}, \; D_{22}^* = \frac{I_x E}{d(1-\nu_1\nu_2)} = p_{22}(d/t)\frac{d^3E}{12(1-\nu_1\nu_2)}. \tag{11}$$

Accordingly functions $f_{22}(d/t)$ and $p_{22}(d/t)$ are as follows

$$f_{22}(d/t) = F/d^2 = 0{,}933 - \pi(1/2 - t/d)^2,$$

$$p_{22}(d/t) = 12I_x/d^4 = \left[0{,}837 - 3\pi(1/2 - t/d)^4\right] - 12(z_c/d)^2 f_{22}(d/t). \tag{12}$$

Here $z_c = -0{,}12d\left[3{,}73 - 2\pi(1/2 - t/d)^2\right]^{-1}$ is the neutral axis displacement in the wall of HDPE construction.

Variation of functions $f_{ii}=f_{ii}(d/t)$ and $p_{ii}=p_{ii}(d/t)$ that are included in the expressions for rigidities (3), and show the effect of a hollow structure in dependence on the d/t value, is shown graphically in Fig. 8.

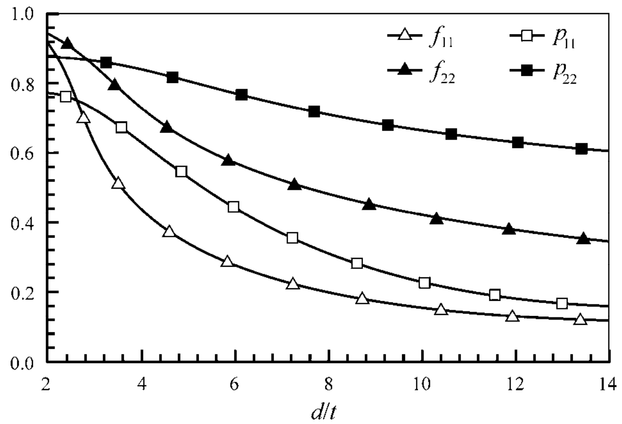

Fig. 8. Dependence of $f_{ii}=f_{ii}(d/t)$ and $p_{ii}=p_{ii}(d/t)$ on d/t

From the results in the plot, it follows that rigidities (B_{11}^*, D_{11}^*) of an orthotropic shell in the axial direction are less to compare with those (B_{22}^*, D_{22}^*) in the direction along its edge. The rigidities of the wall of the cylindrical thin-walled shell with a tubular profile wall for the most common value d/t=10 are written as

$$B_{11}^* = \frac{0,163Ed}{1-0,45\nu^2},\ B_{22}^* = \frac{0,432Ed}{1-0,45\nu^2},\ D_{11}^* = \frac{0,265Ed^3}{12\left(1-0,45\nu^2\right)},\ D_{22}^* = \frac{0,672Ed^3}{12\left(1-0,45\nu^2\right)}. \quad (13)$$

The values of these quantities in the axial direction are about three times smaller than the corresponding rigidities in the circumferential direction.

Assessment of the stress tensor component in the cellular structure wall. To establish the boundary state of the HDPE pipes and tanks by equation (1) it is necessary to estimate the stress state in their wall.

By setting from equations (2), (5), (13) forces T_{ii}, S_{ij} and moments G_{ii}, H_{ij}, calculate stress σ_{ij} in the cell wall. Components σ_{11} can be represented as

$$\sigma_{11}(x,\varphi,z) = \left[\frac{T_{11}(x,\varphi)}{d} n_{11}(x,z) - \frac{12G_{11}(x,\varphi)}{d^3} m_{11}(x,z)\right]\left(1+\frac{z}{R}\right)^{-1}, \quad (14)$$

where $n_{11}(x,z)$, $m_{11}(x,z)$ are correction functions, representing the influence of the cellular structure on the stress state in the wall of the construction. Functions $n_{11}(x,z)$, $m_{11}(x,z)$ are set numerically using the finite element method. For this purpose let us consider a cellular plate element subjected to the plane strain conditions (Fig. 7) and loading by forces T_{11} and moment G_{11}. Using the finite-element program determine the stress state of the cellular specimen.

From the values of stress components $\sigma_{11}^T(x,z)$ and $\sigma_{11}^G(x,z)$ the functions are tabulated

$$n_{11}(x,z) = d\sigma_{11}^T(x,z), \quad m_{11}(x,z) = -d^3\sigma_{11}^G(x,z)/12,$$

Their values are presented in Fig. 9.

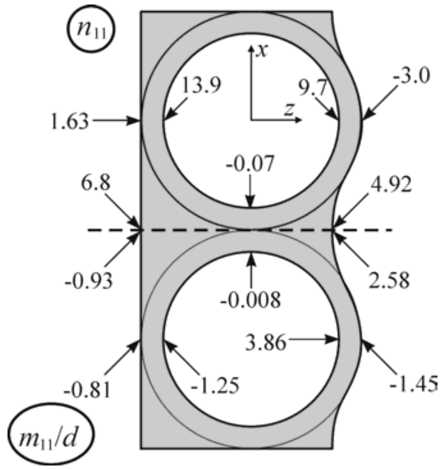

Fig. 9. Functions n_{11}, m_{11}/d in the cell wall

The stress component σ_{22} in the cellular cylindrical construction wall is estimated by the relation:

$$\sigma_{22}(x,\varphi,z) = \left[\frac{T_{22}^*(x,\varphi)}{d} n_{22} - \frac{12G_{22}^*(x,\varphi)(z-z_c)}{d^3} m_{22}\right], \quad (15)$$

where $n_{22} = 1/f_{22}$, $m_{22} = 1/p_{22}$.

Similarly, the formula for calculating the stress components σ_{12} is written. Other stress components are negligible small in comparison with the estimated ones. Therefore, in further calculations they are neglected.

Recommendations. Based on the proposed method for the stress-strain state evaluation of a polyethylene pipe with a tubular profile wall the design recommendations for a long pipe, placed in the soil (Fig. 1a) and a reservoir set in the soil (Fig. 1b) were developed.

For a long HDPE pipe, condition (1) holds at

$$\frac{\Delta}{D}100\% \le 5\%, \quad (16)$$

where Δ/D is relative deflection of a pipe evaluated from formula [1]

$$\frac{\Delta}{D} = \frac{0,11H\gamma}{8S_R + 0,061E'}. \quad (17)$$

Here D is HDPE pipe diameter, γ is specific weight of the soil, H is depth of soil filling above the pipe; E' is modulus of soil reaction; $S_R=EI/D^3$ is specific stiffness of the pipe [1]. Here I is moment of inertia of cross section of the pipe wall per unit length (Fig. 2).

For design of a HDPE tank with a tubular profile wall (Fig. 1b) vertically set in the soil, using conditions (1), the recommendation takes the form

$$\max\left\{\left|\frac{H\gamma D}{d}\left(-3{,}85+\frac{D}{d}\right)\right|,\left|-\frac{H\gamma D}{d}\left(3+0{,}32\frac{D}{d}\right)\right|\right\}\le MRS\,. \quad (18)$$

Here H is the distance from the ground to the top bottom of the reservoir.

Conclusions. Based on the theory of shells a system of equations to estimate the stress-strain state of the thin-walled structures with a tubular profile wall was built. Modified Poisson's ratios, flexural rigidities and extensional rigidities were determined numerically using the finite element method. The modified Poisson's ratios along the mutually perpendicular directions are found to be different, and in the direction Ox the Poisson's ratio ν_1 decreases and depends on the parameter d/t, while in the direction Oy it is constant with respect to the Poisson's ratio of polyethylene ν. It is shown that the modified rigidities (B_{11}^*, D_{11}^*) of an orthotropic shell in the axial direction are smaller compared with the same (B_{22}^*, D_{22}^*) in the direction along its edge. It turns out that rigidities B_{11}^*, D_{11}^* for d/t=10 are about three times smaller than the corresponding rigidities B_{22}^*, D_{22}^*. A correlation to evaluate the stress components in the wall of thin-walled structures with a tubular profile was proposed. From the obtained results, the recommendations for ensuring a durable and reliable operation of the buried HDPE pipes and tanks with a tubular profile wall were developed.

1. Moser A.P., Folkman S.L. Buried pipe design. – New York: McGraw-Hill, 2008. – 601 p. 2. Bulson P.S. Buried structures: static and dynamic strength. – London, New York: Chapman and Holl, 1985. – 227 p. 3. BS EN ISO 9080. Plastics piping and ducting systems-determination of the long-term hydrostatic strength of thermoplastics materials in pipe form by extrapolation. – 2003. 4. Krishnaswamy R.K. Analysis of ductile and brittle failures from creep rupture testing of high-density polyethylene (HDPE) pipes // Polymer. – 2005. – V. 46. – P. 11664–11672. 5. Zhou W., Chen D., Shulkin Y., Chudnovsky A. Ductile failure and delayed necking in polyethylene // Plastics Failure Analysis and Prevention. – 2001. – V. 23. – P. 343–351. 6. Jarvenkyla J.J. Thin-wailed thermoplastic pipes // Construction & Building Baterials. – 1989. – V. 3. – P. 191–200. 7. McGrath T.J., Moore I.D., Hsuan G.Y. NCHRP Report 631: Updated test and design dethods for thermoplastic drainage pipe. – Washington: Transportation Research Board. D.C., 2009. – 310 p. 8. Corporation "Energoresusrs-invest" presents - Start new technology for production of large-scale polymer products with honeycomb structure of a wall // Polymer Pipes. – 2007, № 2. – P. 6–7. 9. Stashchuk M.H., Maksymuk O.V., Dorosh M.I. Calculation of the deflection of a composite pipeline made by winding tubes // Mechanics of Composite Materials. – 2011. – V.47. – P.395–404. 10. Moore I.D., Hu F. Linear viscoelastic modelling of profiled high density polyethylene pipe // Canadian Journal of Civil Engineering. – 1996. – V. 23. – P.342–350. 11. Plastics Pipe Institute (PPI). Design service life of corrugated HDPE pipe. – TX, Irving: Technical report TR-43, 2003. 12. Ambartsumyan S.A. General theory of anisotropic shells. – Moscow: Nauka, 1974. – 448 p. 13. Chawla U.S. Orthotropic cylindrical shells under line load along a generator // Computers & Structures. – 1983. – V. 17. – P. 211–216. 14. Zienkiewicz O.C. The finite element method in engineering science. – London: McGraw-Hill1, 1971. – 521 p. 15. Rychkov S.P. MSC.visualNASTRAN for Windows. – Moscow: Press, 2004. – 552 p. 16. Timoshenko S.P., Goodier, J.N. Theory of elasticity. – New York: McGraw-Hill, 1970. – 567p. 17. Feodosev V.I. Resistance of materials. – Moscow: Nauka, 1972. – 596 p.

.