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MATHEMATICAL MODEL OF DESTRUCTION KINETICS DISLOCATIONS IN CUTTING PLASTOELASTIC WORKPIECE MATERIAL

Проналізовано реологічні моделі кінетики руйнування матеріалів у процесі їх різання на основі теорії дислокацій. Встановлені критерії пластичного та крихкого механізму руйнування матеріалу заготовки під час різання. Описано систему рівнянь, яка описує поведінку термопружнопластичної заготовки у процесі її механічного оброблення, що грунтується на основі класичного термодинамічного аналізу.

The article presents an analysis of rheological kinetics models of materials destruction in the process of cutting, based on the theory of dislocations. The criteria for ductile and brittle fracture mechanism of workpiece material during cutting are established. A system of equations, describing the behavior of termoelastoplastic workpiece in the process of its mechanical processing based on classical thermodynamic analysis, is given.

Statement of the problem. When cutting machining external mechanical action of box tool on the field blank causes corresponding manifestations of their mechanical properties. Specifically, distributed over the contact area surface forces cause solids in three-dimensional stress field. Fields stresses, that characterize the forces acting between atoms, created contacting bodies in the fields of deformations [5]. At the level of atoms or ions that form the structure of the material is expressed in the displacement of relatively established this equilibrium positions. There is an elastic deformation of the local fields of atoms that interact with each other, and ions of the body. These fields are somewhat distorted, overcoming the nuclear force of repulsion of ions or atoms in close proximity, as is appropriate compensation current stresses. At high values of stress is the destruction of existing atomic and molecular bonds, restructuring of the material and the formation of new connections that provide a new equilibrium state change material [13] Study and investigation of these phenomena is only possible with the provisions of physical kinetics [2]. Physical Kinetics – microscopic theory of nonequilibrium processes in the environment. In kinetics methods of classical statistical physics learning processes of energy and matter transfer in different physical systems (including solids) and the influence of external fields on them. Unlike thermodynamic equilibrium processes, kinetics based on the concept of molecular structure unstable environments. This allows us to calculate the kinetic coefficients of continuous media. If you know the distribution function of all particles in the system for their dynamic coordinates, we can compute all the characteristics of a nonequilibrium system. Calculate the total distribution function is practically impossible task, but to determine many properties of physical systems (e.g., the flow of energy or momentum), it is sufficient to know the distribution function of a small number of particles. This allows you to go from a complete description of the energy distribution function of all the coordinates to the reduced description using the distribution of any shares to its coordinates [6]. Thus, investigating the kinetics of energy and material flows in the area of destruction of the material of the workpiece, it is high enough reliability to analyze the stress-strain state of parts, chips and tools

Analyzed existing research and publications. As shown by experimental studies in recent years [11,13] at high strain rates $\left({}^{d\varepsilon}\!/_{d\tau} = 10^5 ... 10^6 \ c^{-1} \right)$ most materials used in mechanical engineering (steel, iron) are anomalous temperature dependence. This dependence is related to the restructuring of the mechanism of dislocation motion. Thermofluctuation mechanism change mechanism for background support. The dependence of resistance on temperature of the material is the opposite: with increasing temperature, increasing the strength of workpiece material! These effects can make problems in high-speed cutting. These problems in the literature not been studied well. Simulation of high-speed process requires the development of models that take into account the complex relationships viscoelastic material behavior and fracture criterion for calculating the background cracking and fragmentation of the material in the form of chips. To account for these effects requires not only sophisticated thermophysical model, but modern computational methods that will calculate the large deformation that does not allow the boundary Lagrangian grid distortion and discontinuity into account the influence of the material. These tasks require a large amount of calculations to simulate viscoelastic properties of internal connections. In addition, this problem is associated with a constant iterative reconstruction of mesh. Scientific paper is devoted to solving these problems.

The problem of research. The purpose of research is to analyze the rheological models plastic fracture and brittle materials in the process of cutting based on the theory of dislocations

Main material. The physical essence of the process of transformation properties of the material during cutting, as in all other known technological methods of processing, is the destruction of its current equilibrium. The only physical tool change is energy balance. Under the influence of this energy coming into effect of physical and chemical processes that try to maintain the equilibrium state of the material in new energy terms. It destroyed the old atomic-molecular bonds and form new ones. As a result, after the restoration of the parameters of the structure and materials properties that has undergone additional energy performance, the new settings are different from the original. Interaction mechanical fields followed by reaction at cutting temperature fields, chemical and diffusion processes, and many other processes and phenomena. The simultaneous occurrence of these processes provides a restructuring of material, its nanostructure defragmentation and destruction and formation of new atomic and molecular bonds [13].

For these reasons, we consider a mathematical model of dislocation and fracture mechanics at cutting the elastic-plastic material of the workpiece. Suppose that the crystal dislocation is an area of continuous medium whose dimensions are large enough. The nature of the stress state around a positive edge dislocation shown in Fig. 1

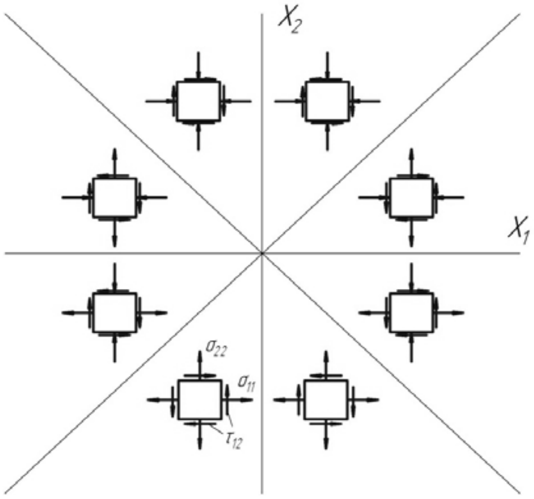


Fig. 1. Theoretical scheme of stresses distribution around the positive edge dislocation in orthogonal projection

The geometry of the outer boundaries of the crystal does not significantly affect the displacement near the center. Then, given the nature of the spiral screw dislocations, characteristic shear strain, you will notice that is different from zero only in the direction of displacement of the axis X3 (Fig. 1). Location planes do not change along the dislocation. It can be concluded that this shift will only function in a coordinate system X_1

and X_2. Location planes near the screw dislocation can be represented as a helix with a pitch equal to the interatomic distance b. The simplest formula for the displacement along the axis X_3 is:

$$u_3 = \frac{b}{2\pi} arctg \frac{x_2}{x_1}. \tag{1}$$

This shift will fit stress tensor, which has only two components are not zero (Fig.1 [5]):

$$\begin{aligned} \sigma_{13} &= -\frac{Gb}{2\pi} \cdot \frac{x_2}{x_1^2 + x_2^2}, \\ \sigma_{23} &= \frac{Gb}{2\pi} \cdot \frac{x_1}{x_1^2 + x_2^2}. \end{aligned} \tag{2}$$

This simple stress distribution satisfies the conditions of equilibrium. As the tension decreases to zero as far as the distance from the cutting area as a source of generating dislocations, it satisfies all the boundary conditions.

In this case, the displacement around the dislocation is independent of the coordinate along the axis X_3. Therefore, there is a conditions plane (orthogonal) strain. Then, the theory of plastic deformation [3] is nonzero only four remain stress tensor components - three normal (σ_{11}, σ_{22}, σ_{33}) and one tangent (τ_{12}) (Fig. 1):

$$\begin{aligned} \sigma_{11} &= -\frac{Gb}{2\pi(1-\nu)} \cdot \frac{x_2\left(3x_1^2 + x_2^2\right)}{\left(x_1^2 + x_2^2\right)^2}, \\ \sigma_{22} &= \frac{Gb}{2\pi(1-\nu)} \cdot \frac{x_2\left(x_1^2 - x_2^2\right)}{\left(x_1^2 + x_2^2\right)^2}, \\ \tau_{12} &= -\frac{Gb}{2\pi(1-\nu)} \cdot \frac{x_1\left(x_1^2 - x_2^2\right)}{\left(x_1^2 + x_2^2\right)^2} \end{aligned} \tag{3}$$

The formula for all components of stress edge and screw dislocations have a common factor Gb/$2\pi r$, which characterizing their dependence on the distance of dislocation. This factor should be used differentially for the adequacy and accuracy of the calculation of the stress-strain state of the workpiece and the tool caused by dislocation during the cutting process.

In the tensions field of screw dislocation in an isotropic material is not equal to zero only tangential components. Thus, at any point of the material surrounding the screw dislocation, there is no volume deformation. Edge dislocations, however, causes local changes in the volume of material. Such a change in volume can be determined using the McClintock formula [5]:

$$\frac{\Delta V}{V} = -\frac{b}{2\pi r}\left(\frac{1-2\nu}{1-\nu}\right) \cdot \sin\theta. \tag{4}$$

When tensile metals are relatively low ductility, i.e. destroyed with achievement of relatively low degree of deformation. However, when cutting metal ductility increases significantly, as in the transition zone of plastically deformed layer disturbed elementary volumes are in hydrostatic pressure (equilateral compression) and at the same time - under the influence of shear stresses (Fig. 2) [10]. In addition, increased plasticity contributes to temperature effect deformed volume when cutting metal

Analysis of the stress vector direction (Fig. 2), suggests that the local quantities of the processed half-plane decreases and the part machined half-plane increases. The total change in volume of material containing edge dislocations is zero. Thus, under the terms linear elastic deformation total volume of material containing screw or edge dislocations does not change, but redistributed, causing stress, plastic deformation of the workpiece and chip shrinkage in the area of deployment. In the core deployment of these conditions are not met. Therefore, there is an increase volume of material. This phenomenon is caused by the fact that the compressive force leads to a decrease in volume, which does not compensate for its increase caused by the action of stretching force. Full volume growth in the core of dislocations attributable to each atomic plane that passes through the dislocation cannot exceed the value of the crystal volume.

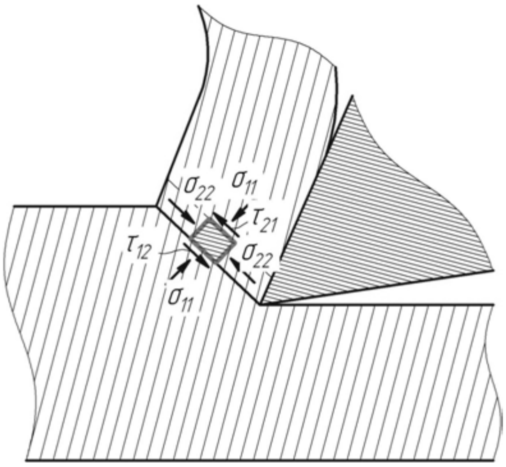


Fig. 2. Stress tensor in the transition zone of plastically deformed

Since the introduction of dislocations in the crystal does not cause a substantial change in the volume (with the exception of a small increase in volume in the cores of dislocations), then plastic deformation should not change quantities. Thus, in an isotropic plasticity Poisson's ratio can be taken equal to 0.5 [2].

Knowing the dislocation strain field, we can calculate the elastic energy per unit length of the dislocation (in the direction of X_3). This expression depending ask elastic energy density (from X_1 and X_2) and integrate it in volume within the size range dislocation core r_0. In this volume strain so great that the equivalent stress exceeds the yield strength σ_T, the value of the radius χ of crystal deformation. As a result, we obtain the specific energy of deformation per unit length of dislocation:

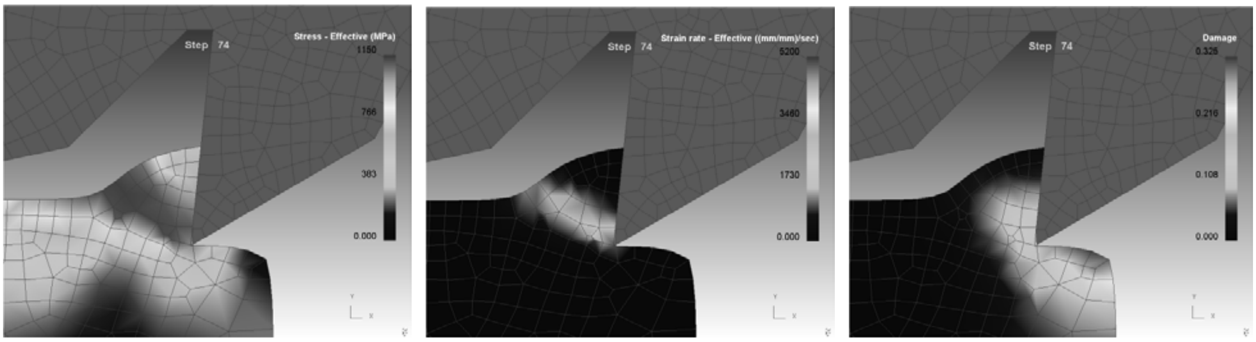
$$\mathrm{E} = \frac{Gb^2}{4\pi}\ln\frac{\chi}{r_0} \tag{5}$$

An important step in modeling the cutting process is the establishment of adequate fracture - brittle or ductile. The boundary between different types of destruction is still not defined. There is still no universally accepted definition of brittle fracture. There are some of them [3]:

1. Destruction is fragile, if it leaks and complete enough elastic energy shattered piece.
2. Brittle fracture is at which unstable crack growth in the area of the power of the cutting wedge occurs when the stress less than the yield strength $[\sigma_T]$.
3. Brittle fracture is no noticeable plastic deformation ε.
4. Qualitative difference between the cases of brittle and ductile fracture piece tied to the rate of crack propagation. In brittle fracture, this rate is very high and reaches 40 ... 50% of the speed of sound in the part material. In the case of viscous fracture, the crack extends with a relatively low speed, approximately equal to the intensity of material deformation in the chip formation zone - $\dot{\varepsilon}$.

Modeling of fracture based on the notion of destruction as the loss of the ability of the material to resist deformation due to violations of internal connections with increasing concentration of microcracks. It should also be noted that the destruction of the ideal conditions do not exist in nature [6]. Any material can only be attributed to the assumption of certain brittle or ductile materials. For these groups of materials provided by respective processing and mathematical formalism that describes or imitates them. In engineering practice is accepted that the strength of an ideal brittle material in compression is eight times greater than its tensile strength [2]. And, for brittle materials is that the destruction is instantaneous with achievement test failure. These models are discussed in the framework of brittle fracture mechanics by explicit allocation fracture surfaces as the main type of contact discontinuities. Mathematical description of the processes of brittle fracture faces considerable difficulties in describing the origin and development of cracks. Therefore, the mechanics of deformable media cases to calculate cutting brittle materials using fracture continuum description [11]. This approach provides a description of destruction as a process-based constitutive equations, written in a unified form for the intact and damaged condition of workpiece material. Continuum approach describes the emergence and development of surfaces and fracture zones without explicit selection of corresponding methods of calculating the pass-through [6]. This model is best described by the theory of finite elements (FEM).

When machining brittle materials fracture kinetics is not considered, i.e. when a fracture criterion mode of workpiece deformation and the tool in a small volume of mobile Lagrangian computational mesh is hopping. On the other hand, decreasing stress in the workpiece material due to loss of the ability to resist deformation is subject to permanent deformation [3]. This indicates that the continuum description of the processes of deformation and fracture of cutting tool blanks should be treated as independent. Development of destruction should not characterize the deformation and deformation own criteria (damage criteria), which depends primarily on the intensity of deformation (Fig. 3).



a *b* *c*

Fig.3. Example of simulation and calculation of equivalent stress (a), the deformation rate (b) and fracture criterion (c) during the milling of the workpiece (material - steel 35, S = 60 mm/min; N = 100 min^{-1}; V = 88 m/min.)

Brittle fracture is accompanied by minimal energy absorption. When the rate of destruction, close to the speed of sound submicroscopic cracks where stress concentration exceeds the body's ability to resist the tensions [6]. Conditions such destruction is that the elliptical crack with the main axle *c*, which is located directly perpendicularly to uniaxial tensile stress σ_0, and is in the details of the elastic modulus E, may reach their limit of instability. This corresponds to an increase in surface energy required for crack growth can be obtained by the energy of elastic deformation of the hard parts, which are distributed through the fracture. Nominal strain at the point of fracture is:

$$\sigma_0 = \sqrt{\frac{2\alpha E}{\pi c}}\,, \tag{6}$$

α – specific surface workpiece energy.

Nominal stress σ_0 for the destruction of machining can be obtained based on the stress concentration around an elliptical crack length 2*c* with a radius of curvature *a*, if we assume that the concentration of stress at the crack tip reached the ideal strength $\sigma_c = 2\pi\alpha/a = E/2\pi$ rated stress to σ_0:

$$\sigma_0 \approx \frac{\sigma_c}{2}\sqrt{\frac{a}{c}} \tag{7}$$

Conditions of elliptical cracks closing under compressive forces and related transmission and tangential component of stress due to surface cracks leads to best match the theory and experimental results in the treatment of fragile parts or semi-brittle materials - iron, silumins etc.

Brittle fracture and rupture are marginal forms of destruction. Between them lies the ductile fracture, defined as a separate body in the presence of some plastic deformation, but with a distinct fracture surface and at a lower strain than in the case of rupture. Sometimes, the plastic deformation is limited deformation at the shot boundaries, which is required for fusion brittle shear cracks in some crystals.

With the destruction of the material to plastic chip separation from the workpiece should be large residual deformation. Limiting case further split into pieces as a result of plastic deformation, which lasts as long as the cross-sectional area is reduced to zero, there is a breaking. If appreciable plastic deformation occurs, but formed surface cracking and deformation magnitude smaller than the breaking, a process called plastic destruction

Since the collapse begins at a certain combination of stress and deformation analysis of cases, based on consideration of the structure and motion of dislocations leads to destruction test approach which is much more dependent on the background strain and strain than those of plasticity. It also points to the strong dependence of the anisotropy of fracture inclusions, some of which are growing stress concentrators. At very high concentrations deformations that occur in the presence of fractures, ductile fracture may occur at the rated stress below the yield point [σ_T]. Then, even if the material and plastic construction destroyed fragile in the sense that the total value of plastic strain to fracture was. This process of destruction characteristic for machining of titanium alloys workpiece under adiabatic shear [8].

The main characteristic of the material by mechanical cutting is shear modulus G, which is characterized by the ratio of shear stress τ_{xy} to the shear strain γ_{xy}:

$$G=\frac{\tau_{xy}}{\gamma_{xy}}=\frac{F/A}{\Delta x/t}=\frac{Ft}{A\Delta x}, \qquad (8)$$

$\tau_{xy}=F/A$ – shear cutting stress; F – cutting force component directed towards the shift vector; A – area, which the force F runing; $\gamma_{xy}=\Delta x/t$ – shear deformation; t – depth of cut.

Consider the rheological model quasibrittle cutting elastoplastic materials. It attempts to overcome the above shortcomings in solving the problem of destruction that can follow the development of the main crack in the form of bands quasibrittle destruction, causing destruction and jumping outbreak strain. The model is valid for the case of limiting plastic deformation and takes into account the fact that the destruction is accompanied by a change in deformation mode from quasi-static to dynamic in the development of narrow zones of strain localization.

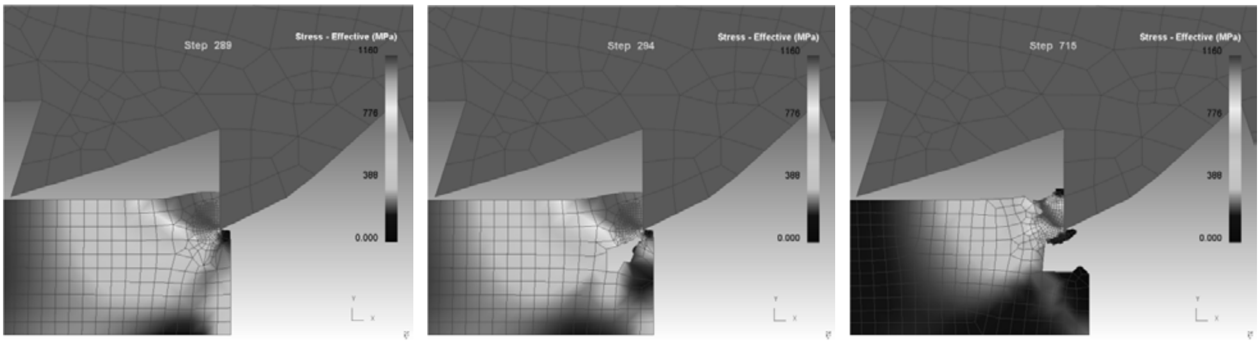


Fig. 4. Example of brittle fracture of the workpiece during milling in the dynamics (material – gray cast iron 30, S = 60 mm/min; N = 100 min^{-1}; V = 88 m/min)

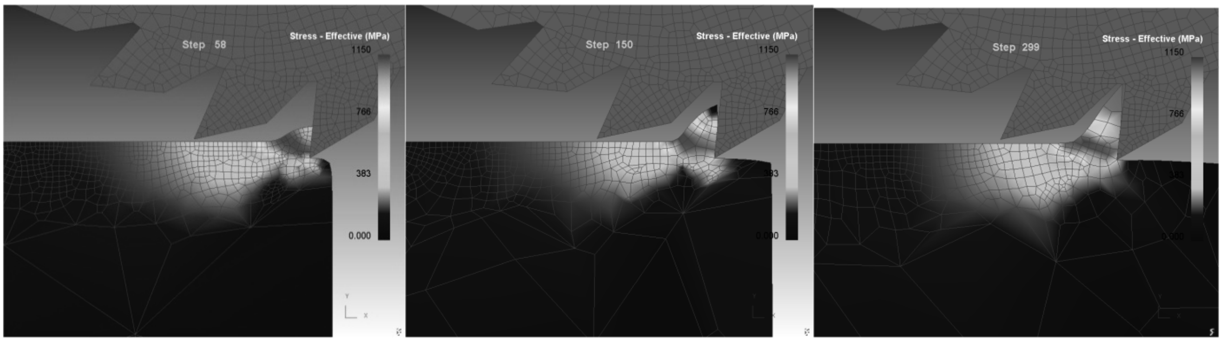


Fig. 5. Example of ductile fracture of the workpiece during milling in the dynamics (material - steel 35, S = 60 mm/min.; N = 100 min^{-1}; V = 88 m/min)

In a state of mechanical equilibrium applied outside force acts only on the surface of the workpiece. Each volume has on the neighboring with the same force and effect on neighboring himself (Newton’s third law). In this case we can write the equation of equilibrium, which determines the deformation of the body [1, 12]:

$$F_i = \sum_k \frac{d\sigma_{ik}}{dx_k} = 0 \tag{9}$$

Since the workpiece are deconcentrative (plane or volume) cutting force, we can write:

$$\sum_k \frac{d\sigma_{ik}}{dx_k} = \rho \mathrm{E} \tag{10}$$

де ρ – density of the workpiece; E – intensity of deformation tensor.

The system of equations describing the behavior termoelastoplastic workpiece in the process of cutting, developed based on classical thermodynamic analysis [4]. The system of equations reflects the conservation laws of energy, mass and momentum and the kinematic relation:

$$\begin{gathered}
\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\dot{\varepsilon} : \mathrm{I} = 0, \qquad \rho\frac{\mathrm{dS}}{\mathrm{d}t} + \nabla\cdot\sigma = 0, \qquad \rho\frac{\mathrm{dS}}{\mathrm{d}t} = \sigma : \dot{\varepsilon} + \nabla \mathrm{q} + \rho \mathrm{r} = 0, \\
\mathrm{F}^{-1} = \nabla x^{\circ}, \qquad \varepsilon = \frac{1}{2}\left(\mathrm{I} - \mathrm{F}^{-\mathrm{T}} \cdot \mathrm{F}^{-1}\right), \qquad \dot{\varepsilon} = \frac{1}{2}\left(\mathrm{L} + \mathrm{L}^{\mathrm{T}}\right), \\
\mathrm{E} = \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + \varepsilon\cdot\mathrm{L} + \mathrm{L}^{\mathrm{T}}\cdot\varepsilon, \qquad \mathrm{L} = \nabla \mathrm{S}, \qquad \frac{\mathrm{dx}}{\mathrm{d}t} = \mathrm{S}.
\end{gathered} \tag{11}$$

де S – feed; x – current position of the tool (Euler radius vector); x^o – original position of the tool (Lagrange radius vector); F – deformation gradient, L –cutting speed gradient; ε – strain tensor; σ - Cauchy stress tensor; U –internal energy per unit volume of the workpiece; q – heat flux vector.

Strain tensor – a mathematical object that describes the displacement of each point of the body during deformation. Strain tensor is defined by the formula [12]:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{d\mathrm{h}_\mathrm{i}}{dx_j} + \frac{d\mathrm{h}_\mathrm{j}}{dx_i}\right), \tag{12}$$

h – vector which describes the shift of body.

When considering the orthogonal cutting model, deformation tensor is symmetric, i.e. $\varepsilon = \varepsilon_{ij} = \varepsilon_{ji}$. The diagonal elements of the strain tensor described the linear deformation or compression diagonal - shear deformation.

Conclusions. On the basis of the above relationships, it can be concluded:

1. The component of velocity dissipation that characterizes the plastic deformation of the workpiece is first order homogeneous function of the rate of plastic deformation, which corresponds to the case of elastoplastic medium, regardless of changes in the time scale of the process of cutting. Obviously, the plastic deformation increases if the resistive load.

2. Resistance to the environment, characterized by elastic modulus (cutting - shear modulus G and yield point σ_T), but the temperature also depends on an additional parameter condition characterized by the destruction criterion. Kinetics of the destruction of the dependence of the rate of dissipation is determined by the rate of change criterion destruction.

3. Dislocations exhibit a dual effect on the material strength and resistance to cutting. Thus, increasing the number of dislocations to a certain limit leads to a sharp decrease in strength properties of the material, however, since a concentration of dislocation. interacting with each other and with

imperfections in the crystal lattice of other types [9] increase the strength of materials, which confirms the assumption made by prof. Podurayev [7]. Moreover, the maximum strength is determined by a certain density of dislocations - 10^7 - 10^8 to the 1 cm^2

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