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One property of the set of Δ-matrix

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Let $A(x)$ and $B(x)$ be two equivalent regular polynomial matrices over C[x] with the simple elementary divisors such that

$$P_A(x)A(x)Q_A(x) = P_B(x)B(x)Q_B(x) = diag(1,1,...,\Delta(x)) = S(x), \tag{1}$$

where $\Delta(x) = (x-\alpha_1)^{k_1}(x-\alpha_2)^{k_2}...(x-\alpha_t)^{k_t}$ and $\kappa_1 + \kappa_2 + ... + \kappa_t = \kappa$.

Let d(x) be an arbitrary polynomial over C[x]. We denote by d[Δ] a numerical $k\times k$ matrix of the form:

$$d[\Delta] = diag(d_{k_1}[\alpha_1], d_{k_2}[\alpha_2],..., d_{k_t}[\alpha_t]),$$

where $d_{k_i}[\alpha_i] = \begin{Vmatrix} C_0^0 d(\alpha_i) & 0 & ... & ... & 0 \\ C_1^0 d'(\alpha_i) & C_1^1 d(\alpha_i) & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ C_{k_i-1}^0 d^{(k_i-1)}(\alpha_i) & C_{k_i-1}^1 d(\alpha_i) & ... & ... & C_{k_i-1}^{k_i-1} d(\alpha_i) \end{Vmatrix}$, and $C_n^m = \begin{pmatrix} n \\ m \end{pmatrix}$.

Definition. A numerical matrix W is called the Δ-matrix, if there exists for this matrix a polynomial $d(x)$ such, that $W = d[\Delta]$.

Theorem. Matrix W is the Δ-matrix for the fixed polynomial Δ(x) if and only if there exists a polynomial $d(x)$ such that $W = d(\widetilde{J})$, where $\widetilde{J} = diag(\widetilde{J}_1, \widetilde{J}_2, ..., \widetilde{J}_t)$ and

$$\widetilde{J}_i = \begin{Vmatrix} \alpha_i & 0 & ... & 0 \\ \frac{1!}{0!} & \alpha_i & ... & 0 \\ 0 & \frac{2!}{1!} & ... & 0 \\ 0 & ... & \frac{(k_i-1)!}{(k_i-2)!} & \alpha_i \end{Vmatrix}.$$

$d(x)$, hence there exists a polynomial $d(x)$ such that $W = d[\Delta]$, and W is the Δ-мatrix.

1. Kazimirsky P.S., Bilonoga D.M. Semiscalar equivalency of the polynomial matrices with the relatively prime elementary divisors. Dopovidi AN USSR ,1990, #4, p.p. 8-9.
2. Kazimirsky P.S. Decomposition of matrix polynomials into factors. Kiev, Naukova Dumka, 1981, 224p.p.