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DISTRIBUTION OF CURRENT DENSITY ON THE PERIMETER OF A CYLINDRICAL CONDUCTOR OF A WAVEGUIDE'S VIBROEXCITER

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Abstract: On the basis of electrodynamic analysis a considerable nonuniformity of the current density distribution on the perimeter of a cylindrical conductor of a waveguide exciter has been detected. It has been proved that the above mentioned nonuniformity mostly affects the input reactance of the exciter. Such effect is minimum at central location of the exciter in the waveguide. It rises sharply when the exciter is diplaced. Therefore, we come to conclusion that it is necessary to take into consideration the nonuniformity of the current density distribution when analyzing a vibroexciter.

Key words: waveguide vibroexciter, the connection: line – waveguide.

1. Introduction

Modern programs for analyzing VHF devices are intended to ensure adequate accuracy of specific calculations. Such a necessity requires that all the possible impacting factors be taken into account. The given paper considers the nature of nonuniformity of the current density distribution on the surface of a cylindrical vibrator, and the influence of this nonuniformity on the input impedance of the vibrator as an exciter of a rectangular waveguide.

The current density distribution over the perimeter of a thin vibrator ($\frac{a}{A} \le 0{,}03$; $a-$is the radius of the vibrator's conductor; $A-$ is the crosscut width of the rectangular waveguide) is practically uniform. The influence of a nonuniformity manifests itself in vibrators of enlarged diameter ($0{,}03 \le \frac{a}{A} \le 0{,}07$), as well as of large diameter ($\frac{a}{A} \ge 0{,}07$). It is obvious that inaccuracy of the input impedance's calculation leads to inaccuracy in the determination of all other parameters of the exciter [6].

The current density distribution over the surface of a passive inductive vibrator in a rectangular waveguide was described in some scientific works [2 - 5]. It was established that in the vibrator of this type, the distribution of the surface current density is unsymmetrical with respect to the transverse coordinate of the waveguide. The given work considers an active vibroexiter when the symmetry of the structure ensures symmetrical distribution of the current density with respect to the coordinate mentioned. The reciprocity theorem has been applied to avoid singularity of the Green's functions and to simplify the analysis.

2. Application of the reciprocity theorem

It has been assumed that the waveguide walls and the vibrator's conductor are ideal conductors. In such case we may consider the vibrator's current as a surface current with the density $\eta(x,\theta)$, where $x-$ is the coordinate of the vibrator's height ($x=0$ on the wide wall of the waveguide), and the angle $\theta-$ is the angular coordinate of a point on the perimeter of the cylindrical vibrator, measured in the polar coordinate system centered on the vibrator's axis. The elementary longitudal current $dI_x = \eta(x,\theta) a d\theta$ flows thruogh the elementary arc $ad\theta$. The volume density of the current can be represented as threads of the current with the help of the following delta functions:

$$\delta_x(x,\theta) = dI_x \delta(y - y_i)\delta(z - z_i) \qquad (1)$$

where $y-$ is the transverse coordinate of the waveguide's width ($y=0$ on the narrow wall of the waveguide); z is the coordinate of the waveguide's length ($z=0$ on the vibrator's axis); (y_i, z_i) are the coordinates of the current threads' location on the perimeter of the vibrator.

It is known that the reciprocity theorem may be written in the form [1]:

$$\int_{V_1} E_{x2}\delta_{x1}(x,\theta)dV_1 = \int_{V_2} E_{x1}\delta_{x2}(x,\theta)dV_2 . \qquad (2)$$

The theorem (2) is true in the given structure because the surface integral of the electric field intensity on the waveguide walls is equal to zero, and two symmetric arms are considered to be infinitely long. The volume V_1 - is the volume of a current thread on the vibrator surface with the current density $\delta_{x1}(x,\theta)$; the volume V_2 - is the volume of a thread with the unknown current density $\delta_{x2}(x,\theta)$, located on the vibrator's axis. Obviously, these volumes are equal: $V_1 = V_2$. Later in the article, we substantiate that the electric field intensities are equal $(E_{x1} = E_{x2})$ if the first intensity, induced by the current on the vibrator's axis, is determined on the vibrator's surface, and the second one, induced by the elementary current on the vibrator's surface, is determined on the vibrator's axis. Thus, as the theorem (2) proves, the densities of elementary current threads both on the axis and on the surface of the vibrator are also equal: $\delta_{x2}(x,\theta) = \delta_{x1}(x,\theta)$. The result obtained will serve to model the current threads on the vibrator's perimeter.

The distribution of the current density on the vibrator's surface may be approximated by the sum of the current (I_o), distributed uniformly over the perimeter, and of two currents (I_1, I_2), distributed harmonically, what ensures the similar distribution at $\theta \geq 2\pi$. To satisfy symmetry condition the distribution function must be of the same sign for $z \leq 0, z \geq 0$, Otherwise, when analyzing, integration provides zero result. The harmonic functions of this kind can be $sin\left(\frac{\theta}{2}\right), sin\left(\frac{3\theta}{2}\right)$, etc., or $\left|cos\left(\frac{\theta}{2}\right)\right|$, etc. The longitudal current density distribution can be represented by the fundamental harmonic of the space distribution. So, the uniformly distributed component of the elementary current takes the following form:

$$dI_{ox} = \eta_o(x) a d\theta = I_o di_o(x,\theta), \quad (3)$$

where

$$di_o(x,\theta) = \frac{1}{2\pi} sin\left(\frac{\pi}{2h}(h-x)\right) d\theta. \quad (4)$$

Similarly, we obtain

$$di_1(x,\theta) = \frac{1}{2\pi} sin\left(\frac{\theta}{2}\right) sin\left(\frac{\pi}{2h}(h-x)\right) d\theta;$$

$$di_2(x,\theta) = \frac{1}{2\pi}\left|cos\frac{\theta}{2}\right| sin\left(\frac{\pi}{2h}(h-x)\right) d\theta, \quad (5)$$

where $h-$ is the vibrator's height. The surface current density now equals:

$$\eta(x,\theta) = \frac{I_x(x,\theta)}{2\pi} d\theta;$$

$$I_x(x,\theta) = I_o i_o(x,\theta) + I_1 i_1(x,\theta) + I_2 i_2(x,\theta). \quad (6)$$

The goal of our analysis the currents $I_o, I_1 I_2$. The necessary equations are obtained from the boundary condition on the vibrator's surface: $dE_x = 0$. The electric field intensity dE_x comprises not only field intensities induced by the current's components in the vibrator, but also that of the vibrator's power sourse. Let us utilize the structural model of the vibrator's power source, supplying the voltage U, in the form of a delta generator; then the field intensity is expressed by the delta function: $dE_{cx} = U\delta(x)\delta(\theta - \theta')d\theta$. The boundary condition looks like:

$$dE_{ox} + dE_{1x} + dE_{2x} + dE_{cx} = 0. \quad (7)$$

Given that the field intensities can be written as products $dE_{ox} = I_o \rho_o e_o(x,\theta) d\theta$, etc, where $\rho_o = 120\pi$ Ohm is the wave impedance of free space, their scalar products with the current $di_o(x,\theta)$ may be written for such normalized unknown values: $X_o = I_o \rho_o / U$; $X_1 = I_1 \rho_o / U$, etc. Thus, the equation (7) takes the following form:

$$\begin{aligned} X_o\left\langle e_o(x,\theta)d\theta, di_o(x,\theta)\right\rangle + X_1\left\langle e_1(x,\theta)d\theta, di_o(x,\theta)\right\rangle + \\ + X_2\left\langle e_2(x,\theta)d\theta, di_o(x,\theta)\right\rangle = \\ = -\left\langle \delta(x)\delta(\theta-\theta')d\theta, di_o(x,\theta)\right\rangle \end{aligned} \quad (8)$$

The scalar products in the equation (8) are determined by the surface integral taken through the vibrator's surface $(\theta = 0...2\pi; x = 0...h)$. Such products can be considered as the sum of the scalar products of all elementary currents on the vibrator's perimeter and the elementary electric field intensity for the x-coordinate only. The iterated integration of the equation (8) over the vibrator's perimeter is the very sum of scalar products of the elementary intensities induced by all elementary currents. As a result the equation (8) is expressed as:

$$X_o A_{1,1} + X_1 A_{1,2} + X_2 A_{1,3} = -1 \quad (9)$$

Equations similar to (9) can be written for all the three vibrator's currents, and their solutions provide sought unknown values. The input impedance of the vibrator is determined as the ratio of the voltage U to the mean perimeter value of the current.

3. Uniform distribution of current density

Let us show the application of the above-mentioned algorithm on the example of uniform distribution of the current density (3), (4) over the vibrator perimeter. We count the angular coordinate θ off from the transversal axis of the waveguide (y). We choose an elementary current (a thread) at the point y_i, z_i on the vibrator's perimeter where

$$y_i = d + a\cos\theta;\ z_i = a\sin\theta\,. \qquad (10)$$

where d is the distance between the vibrator's axis and the narrow waveguide's wall (i.e. the origin of the coordinate system). We express the volume current density on the vibrator's axis according to the formula (4) by using delta functions:

$$\delta_{2x}(x,\theta) = \frac{I_o}{2\pi}\sin\left(\frac{\pi}{2h}(h-x)\right)d\theta\,\delta(y-d)\,\delta(z) \quad (11)$$

Both the magnetic vector potential and the electric field intensity may be found by using the known formulae [1]:

$$A_x = -\mu_o \int_{V'} G_x(\bar r \mid \bar r')\delta_x(\bar r')dV';$$

$$E_x = \frac{1}{j\omega\mu_o\varepsilon_o}\left(\frac{\partial^2 A_x}{\partial x^2} + k^2 A_x\right), \qquad (12)$$

where $\omega -$ is the circular frequency of the current; $\varepsilon_o, \mu_o -$ are the permittivity and permeability of free space; $k^2 = \omega\varepsilon_o\mu_o$; $G_x(\bar r \mid \bar r')-$ is the component of the tensor Green's function for a rectangular waveguide [1]:

$$G_x(\bar r \mid \bar r') = -\frac{2}{AB}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\gamma}\sin\left(\frac{m\pi y'}{A}\right)\times \times\sin\left(\frac{m\pi y}{A}\right)\cos\left(\frac{n\pi x'}{B}\right)\cos\left(\frac{n\pi x}{B}\right)\exp(-\gamma|z-z'|) \quad ,(13)$$

where $B-$ is the height of the waveguide wall $(B < A)$; $\varepsilon_n = 1/2$ for $n = 0$, $\varepsilon_n = 1$ for $n > 0$; $\bar r(x,y,z)$ are the coordinates of a point of the field, $\bar r'(x',y',z')$ are the coordinates of a point of the power source; $V'(x',y',z')-$ is the volume taken by the source currents; $\gamma -$ is the propagation constant of the wave in the waveguide:

$$\gamma = \sqrt{\left(\frac{m\pi}{A}\right)^2 + \left(\frac{n\pi}{B}\right)^2 - k^2}\,. \qquad (14)$$

On the basis of the expressions (10) - (14) we obtain the electric field intensity at the point $(y_i.z_i)$ of the vibrator's perimeter, induced by the electric current on the vibrator's axis:

$$dE_{ox} = \frac{j}{\omega\varepsilon_o}\frac{2I_o}{2\pi A}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\gamma}F(n)\left(\left(\frac{n\pi}{B}\right)^2 - k^2\right)\sin\left(\frac{m\pi d}{A}\right)\times \times\cos\left(\frac{n\pi x}{B}\right)\sin\left(\frac{m\pi y_i}{A}\right)\exp(-\gamma|z_i|)\,d\theta \quad (15)$$

where

$$BF(n) = B\int_0^{h/B}\cos\left(\frac{n\pi x'}{B}\right)\sin\left(\frac{\pi}{2h}(h-x')\right)d\left(\frac{x'}{B}\right) \quad (16)$$

The expression (15) also is true in the case of determination of the electric field intensity on the vibrator's axis. In such case the vector potential is determined at $y_i = d, z_i = 0$, and the coordinates of the field point (y,z) are given by the formulae (10). This has proved the appropriateness of the reciprocity theorem application.

The scalar product in the equation (8) can be obtained by integrating the products of the expressions (15) and (4) through the vibrator's surface ($x = 0...h;\ \theta = 0...2\pi$):

$$\langle e_o(x,\theta)d\theta, di_o(x,\theta)\rangle = = \frac{j2\lambda B}{(2\pi)^2 A}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\gamma}F^2(n)\left(\left(\frac{n\pi}{B}\right)^2 - k^2\right)\sin\left(\frac{m\pi d}{A}\right)\times \times F_o(m,n)\,d\theta \quad ,(17)$$

where

$$F_o(m,n) = \int_0^{2\pi}\sin\left(\frac{m\pi y_i}{A}\right)\exp(-\gamma|z_i|)\,d\theta\,. \quad (18)$$

The iterated integration over the angle θ gives us the coefficient $A_{1,1}$ that is present in the equation (9):

$$A_{1,1} = \frac{j2\lambda B}{2\pi A}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{\gamma}F^2(n)\left(\left(\frac{n\pi}{B}\right)^2 - k^2\right)\sin\left(\frac{m\pi d}{A}\right)F_o(m,n) \quad (19)$$

The right part of the equations (8) and (9) has the form:

$$-\langle\delta(x)\delta(\theta-\theta')d\theta, di_o(x,\theta)\rangle = -\frac{1}{2\pi}d\theta\,; \quad (20)$$

The solution of the equation (9), if $I_1 = I_2 = 0$, gives the ratio X_o, its inverse negative value is equal to the normalized input vibrator's impedance

$$(Z/\rho_o = -1/X_o = -A_{1,1})$$

if the current density is uniformly distributed. The obtained result characterizes the influence of a vibrator's conductor radius. By using the above given algorithm one can determine the input vibrator's impedance, whose conductor has a cross section other than circular.

4. Nonuniform distribution of current density

It has been suggested above to describe the nonuniform distribution of the current density by the sum of the current (3), (4), distributed uniformly, and two other currents (5), distributed harmonically.

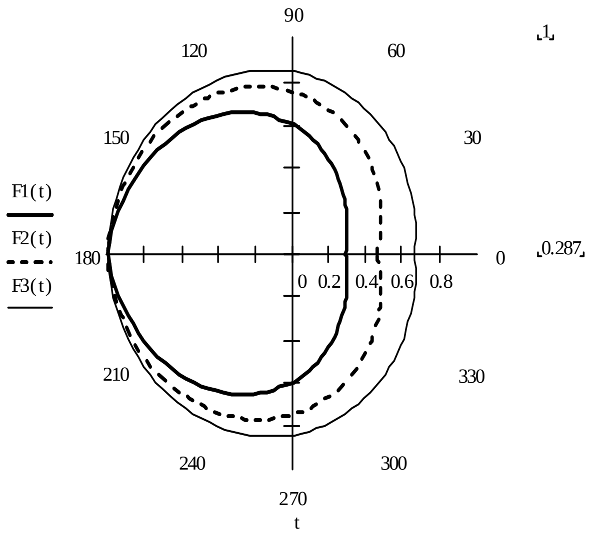

Fig. 1. The distribution of the current density $F(t)=\left|I_x(\theta)/I_{x\,max}\right|$ *over the vibrator's perimeter*

for $s=0{,}17$; $t=\theta^o$. $F1(t)$

for $q=0{,}6; e=0{,}3;$ $F2(t), F3(t)$

for $e=0{,}4$; $q=0{,}6;0{,}8$.

In a linear system, the fields induced by each particular current can be considered separately. Let us apply the above suggested algorithm to determine the current I_1. As a result, the function $sin(\theta/2)$ of the current distribution over the perimeter appears in the expression for the electric field intensity (15). That is why the scalar product $\langle dE_{1x}, di_1(x,\theta)\rangle$ is complemented by the factor $F_1(m,n)$:

$$F_1(m,n)=\int_0^{2\pi}\left(sin\left(\frac{\theta}{2}\right)\right)^2 sin\left(\frac{m\pi y_i}{A}\right)exp\left(-\gamma|z_i|\right)d\theta\,. \quad (21)$$

Obviously, the scalar product $\langle dE_{ox}, di_1(x,\theta)\rangle$ includes the factor $F_{o1}(m,n)$ with the function $sin\left(\frac{\theta}{2}\right)$ raised to the first power, and the product $\langle dE_{2x}, di_1(x,\theta)\rangle$ includes the factor $F_{12}(m,n)$:

$$F_{12}(m,n)=\int_0^{2\pi} sin\frac{\theta}{2}\left|cos\frac{\theta}{2}\right| sin\frac{m\pi y_i}{A} exp\left(-\gamma|z_i|\right)d\theta\,. \quad (22)$$

For the extranal intensity we obtain:

$$\langle dE_{cx}, di_1(x,\theta)\rangle = U\, sin\left(\frac{\theta}{2}\right)\frac{d\theta}{2\pi}\,. \quad (23)$$

After the second integration, the right part of the expression (22) equals $U2/\pi$ both for the function $sin\frac{\theta}{2}$ and the function $\left|cos\frac{\theta}{2}\right|$.

The given recommendations allow us to formulate for each distribution function an equation like (9), and the solution of the simultaneous equations provides the sought currents I_o, I_1, I_2.

5. Numerical examples and conclusions

According to the above considered algorithm the distribution of the current density over the surface of a cylindrical vibrator located in a rectangular waveguide has been computed. The parameters of the structure are written in the normalized form: $\zeta=B/A=0.44$, where B is the height of the waveguide: $B<A$; $\nu=h/B=0.7$, where h is the height of the vibrator. The variable parameters are $e=d/A$, where d is the distance between the location of the vibrator's axis and the narrow waveguide's wall; $s=a/A$; $q=\lambda_o/2A$ is the normalized length of a transmitting wave.

As we have already mentioned, the structure of the vibrator power source has been represented by the delta-generator model. The edge effects have also been neglected, i.e the vibrator is assumed to be tubular one with thin walls. We do not take into consideration the effect of the transverse electric currents on the vibrator's surface. The distribution of the current density is approximated by the functions (4) and (5).

Fig. 1 illustrates the distribution of the current density module on the perimeter of the cylindrical vibrator. Maximum of the current density is located on the closer side of the waveguide's wall. Such a result is logical due to the proximity effect.

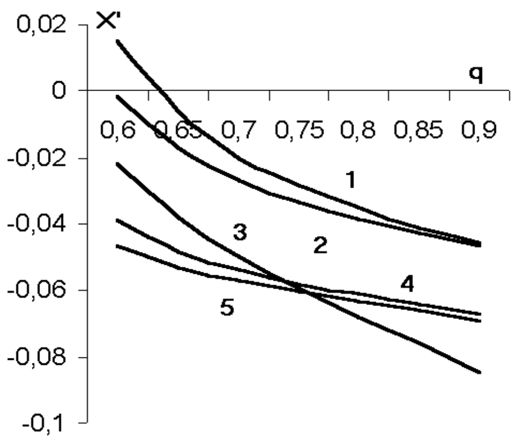

Fig. 2. Dependence of the input reactance ($X' = X / \rho_o$*)*
on λ_0 ($q = \lambda_o / 2A$).

The nonuniformity of the current density distribution increases when the transmitting wave's length decreases, the vibrator's diameter increases, and when the distance between the vibrator and the wall decreases. The nonuniformity of current density distribution on the perimeter remains even when $s < 0.1$.

The input impedance of the vibrator is determined as the ratio of the voltage to the total current defined as the integral of the current density along the vibrator's perimeter. The nonuniformity of the current density distribution has a poor influence on the working component of the input impedance. With respect to the uniform distribution, this component decreases up to about 10%. However, the nonuniformity of the current density distribution considerably affects the input reactance of the vibrator. This impact increases with the increasing of the vibrator's diameter, with the descreasing of the transmitting wave's length, and with the decreasing of the distance between the vibrator's axis and the narrow waveguide's wall, as Fig. 2 shows. (In Fig. 2 the curves 1, 2, 4, 5 correspond to $s = 0.17;$ the curve 3 corresponds to $s = 0.1;$ the curves 1, 2 correspond to $e = 0.3;$ the curves 3, 4, 5 correspond to $e = 0.4;$ the curves 1, 3, 4 are valid for the nonuniform distribution, the curves 2, 5 are valid for the uniform distribution of the current density). Especially, with values of the parameter e getting smaller, the reactance value, determined by taking into consideration the nonuniformity of the current density distribution, can noticeably differ from its value under uniform distribution. The central location of the vibrator ($e = 0{,}5$) "drags" the linear dependences of the reactance on the vibrator's radius closer to each other (Fig. 3). (Fig. 3 depicts curves for $q = 0.6$: the curves 1, 2 correspond to $e = 0.3,$ the curves 3, 4 correspond to $e = 0.4;$ the curves 1, 3 are valid for the nonuniform distribution, the curves 2, 4 are valid for the uniform distribution of the current density).

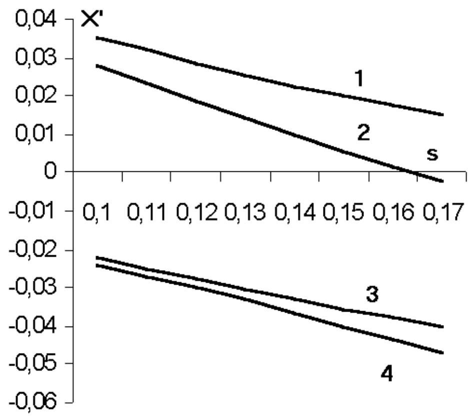

Fig. 3. Dependence of the input reactance on the vibrator's radius $s = a / A$.

The given examples of the analysis show that it is necessary to take into account the effect of nonuniformity of the current density distribution on the perimeter of a waveguide's vibrator, when $s > 0{,}07$. Assumption of solely uniform distribution leads to significant errors, primarily, in determining the input reactance of a vibrator.

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РОЗПОДІЛ ГУСТИНИ СТРУМУ НА ПЕРИМЕТРІ ЦИЛІНДРИЧНОГО ПРОВІДНИКА ВІБРАТОРНОГО ЗБУДЖУВАЧА ХВИЛЕВОДА

Й. Захарія

На основі електродинамічного аналізу виявлено значну нерівномірність розподілу густини струму по периметру циліндричного провідника збуджувача хвилевода.

Встановлено, що згадана нерівномірність розподілу найбільше впливає на вхідний реактанс збуджувача. Такий вплив є найменшим при центральному розташуванні збуджувача у хвилеводі. Він різко зростає при зміщенні збуджувача. Цим обгрунтовано висновок про необхідність враховування нерівноморності розподілу густини струму при аналізі вібраторного збуджувача.

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