COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 2, No. 1, 2012

SWITCHED RELUCTANCE MOTOR WITH BUFFER OF ENERGY AND ITS MATHEMATICAL MODEL

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Abstract: the constructive schemes of a switched reluctance motor and the circuit diagrams of a commutator with capacity buffers of energy have been suggested. The mathematical model of carrying out the investigation of electromechanical processes occurring in electric drives with a switched reluctance motor with buffers of energy has been presented.

Key words: switched reluctance motor, capacity buffer of energy, salient poles.

1. Introduction

Economical and rational use of material and manpower in the branch of electrical machine manufacturing is closely connected with future developments of electrical machines, i.e. improving their quality and longevity, as well as with increasing the adaptability to manufacturing processes and, therefore, dropping unit costs. Application of electronic switched motors is one of the perspective solutions to such problems.

Brushless DC motors are electrical machines where a brush-collector unit is replaced by a semiconductor commutator – an inverter controlled by signals f (θ) from a rotor position sensor (RPS) [1].

One of the simplest in construction, more practically feasible and reliable is a switched motor with a salient-pole stator, concentrated coils of its winding and a toothed passive rotor [2]. Such a motor is simpler, cheaper and practically more technical than the elementary of electrical machines – asynchronous. The switched reluctance motor (SRM) with salient poles on the stator and concentrated coils of its winding, and the toothed passive rotor is one of the elementary motors by construction, technology and reliability.

2. Constructions of electromechanical converters (EMC) of SRM

Of the whole variety of inductor electromechanical converters the most expedient for the use in SRM are constructions with salient poles on the stator enabling to provide magnetic isolation of separate sections. The most known of them is the construction given in Fig.1.

On the basis of classic EMC construction of SRM it is possible to build a stator allowing to use wasteless steel cutting, to reduce expenditure of copper and to better use the volume of active machine part (Fig. 2). Let's name such a construction a stator with T-similar elements.

Another construction completely using the axial length of the machine as its active part is given in Fig. 3. Let's name it EMC with C-similar elements or with an axial magnetic flux. It is expedient to use it when it is necessary to receive minimum axial length of the machine (for example, in a lift drive of windows in automobiles' doors), and full magnetic isolation of stator's winding sections).

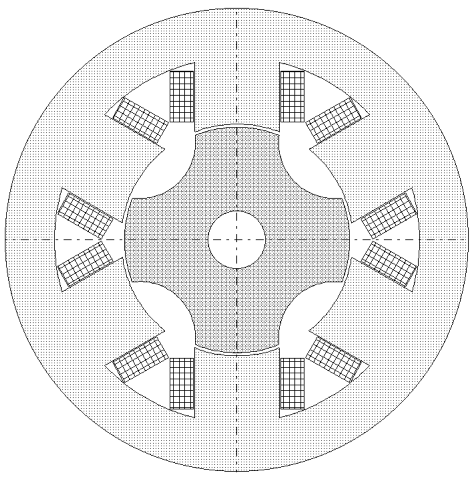

Fig. 1. Classical construction of a SRM

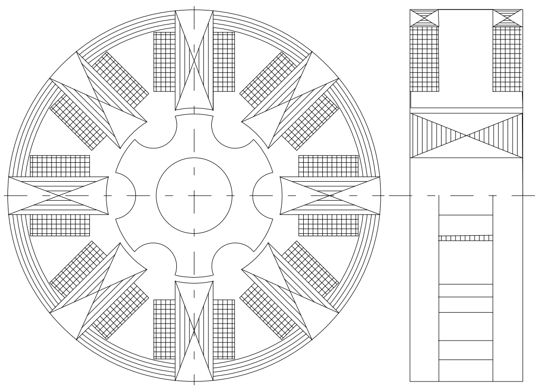

Fig. 2. EMC with T-similar stator elements

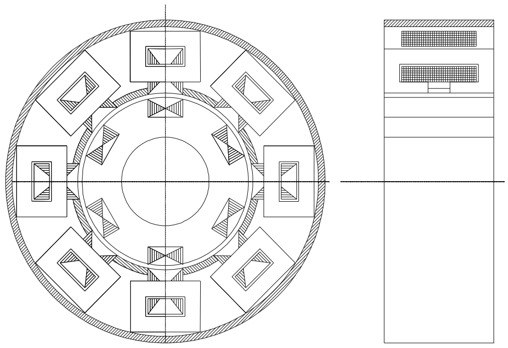

Fig. 3. EMC with an axial magnetic flux of stator

The construction of EMC with U-similar elements (Fig. 4) is suggested in [1] which, like the above mentioned construction with C-similar elements, ensures full magnetic isolation of the SRM sections.

The disadvantages of the U-similar construction are its non-technology and low maintainability because separately made stator's elements need involving additional technological elements and fixing materials to be composed in a completed stator construction.

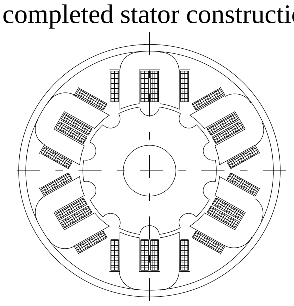

Fig. 4. U-similar constructive scheme of EMC of SRM

These disadvantages do not concern the construction of a stator given in Fig. 5. Let's name it pseudo-U-similar. In this construction the stator core is laminated in the axial direction from isolated plates of electrotechnical steel, then framed or unframed concentrated coils of the stator's winding are superimposed on the teeth, and fixed with the help of wedges between parallel or other teeth. On the basis of this stator it is possible to build EMC without a tank, the place for tightening bolts being between nonparallel teeth.

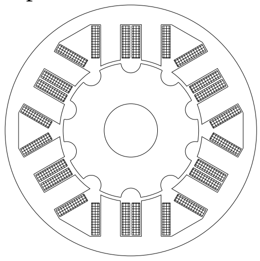

Fig. 5. Pseudo U-similar construction of EMC of SRM

The described construction is expedient to be used for designing a large diameter SRM with many teeth on a rotor, as it permits to considerably reduce, in comparison with a classical construction, the length of magnetic force lines as well as steel losses.

3. Transistor commutators with buffers of energy

The wide use of SRM is constrained by their low power indexes stipulated by the necessity of dissemination of energy, accumulated in the electromagnetic field during commutation of a current in sections by transistor keys, for the purpose of preventing them from overvoltage. This is especially displayed in SRM with one-semiperiodic commutator with a stabilitron receiver of magnetic field energy.

So, the task of creation of a switched drive on the basis of a simple, cheap and technological inductor machine is caused, to a large extent, by the necessity of creating new scheme solutions enabling to use the energy accumulated in the magnetic field of an armature winding section to execute efficient work. To solve this task, the Department of Electrical Machines has developed both structural and circuit diagrams of an electronic commutator which are characterized by the enhanced reliability and ensure reusable energy of a magnetic field in commutating sections. [2, 3, 4, 5, 6, 7].

One of the diagrams of transistor commutators with sequential capacity accumulators is given in Fig. 6 [8]. The principle of its operation is based on the following: if the rotor takes a position of a switching angle β (Fig.7), the signals *K1 – K4* RPS will open VT1 and VT4, which will remain in open state during the angle of commutation γ. The armature winding section of EMC *w1* is supplied with power from the source and series-connected capacitor *C1* charged in the preceding cycle through these keys. The diode *VD1* will be switched off by the application of reverse voltage of the capacitor *C1* to it. The section current *w1* dramatically increases, the capacitor *C1* is discharged, and when its voltage reaches zero, the diode *VD1* switches on, the section *w1* is connected to the source of energy in the circuit: the diode *VD1*, the transistor *VT1*. Under the influence of electromagnetic torque the rotor rotates, and when the value of the commutation angle reaches y, the transistors *VT1* and *VT4* are switched off.

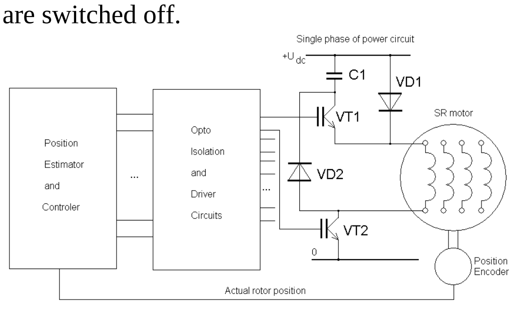

Fig. 6. Circuit diagram of a transistor commutator with sequential capacity accumulators in each section

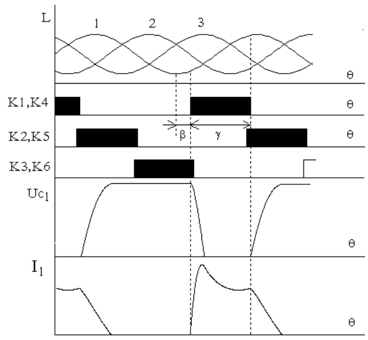

Fig. 7 Inductance and current of section, diagram of work of transistor and voltage on the storage capacity

Under the action of the EMF of self-inductions the section current w1 will begin to flow in the circuit: the diode VD4, the capacitor C1, the diode VD1. The capacitor C1 is charged, the section current decreases. For other sections this process will be repeated when the angle reaches the value of $2 \cdot \pi / m$, where m – the number of sections. Thus, the following three tasks are solved: (1) the utilization of the energy accumulated in the electromagnetic field of EMC armature; (2), the increase of the voltage on the collector-emitter passage of the commutator transistor of is limited to an acceptable level; (3), dynamic transistor switching losses considerably decrease due to almost instant interception of the deenergizing current of the transistor by the circle of the capacitor charging. Calculations and experimental investigations show that the application of schemes with CSE in SRM with a passive rotor improves its coefficient of response 1.7 – 1.8 times in comparison with those with a stabilitron that prevents commutator keys from overvoltage.

4. Mathematical Model of SRM

Tasks of creation of new types of electrical machines, such as salient pole switched reluctance motors with energy buffers, can be successfully solved only with the help of tools of exact computation of their static and dynamic characteristics.

At present many theories are being developed to calculate periodic processes in electromechanical devices, i.e. when the currents, flux linkages, voltages, linear and angular shifting are periodic functions of time [9].

The power method offers two modes for the evaluation of an electromagnetic force, which operates in the given direction on the chosen volume of a nonlinear system. With the first mode the electromagnetic force is expressed through an increase of the magnetic energy stipulated by small shifting of a chosen volume under the condition of flux constancy. With second – the electromagnetic force is expressed through the increase of magnetic co-energy that is caused by small shifting of the chosen volume under the condition of constancy of the excitation current circuit.

Results of desired precision can be obtained with the help of power approach only in those cases, when the increase of energy or co-energy in a nonlinear magnetic system is given analytically.

The energy of the magnetic field of an electromechanical converter with a passive rotor can be calculated as

$$W_m = \int_0^{\psi} i \cdot d\psi(\theta,i) \quad \text{if} \quad \psi = \Psi = Const, \qquad (1)$$

And co-energy as

$$W_k = \int_0^{I} \psi(\theta,i) \cdot di \quad \text{if} \quad i = I = Const. \qquad (2)$$

Flux linkage of the winding phase of an armature of EMC with a passive rotor is a unique nonlinear function, which can be approximated by an analytical expression. Thus, it is necessary that the mode of approximation ensure coincidence of real and approximated dependencies both as the function of rotor and stator mutual position angle and as the function of current, as well as possibility of integration and differentiation in analytical form, and do not require complicated and bulky evaluations of coefficients.

Equation (3) suits the mentioned conditions:

$$\psi(\theta,i) = [\psi_{10} - \psi_{1t} \sin(\theta/2)]i + \\ +\psi_y \sin(\theta/2)\left[1 - e^{-a \cdot i \cdot \sin(\theta/2)}\right] \qquad (3)$$

where θ is the electrical angle between the axis of a rotor slot and the axis of a stator tooth; i is the phase excitation current; Ψ_{10}, Ψ_{1t}, Ψ_y, a are coefficients.

The proposed analytical expression for the determination of the flux linkage of EMC with a passive rotor gives a possibility to write the equations of voltages for a magneto insulated phase of SRM and to calculate electromagnetic torque what can form the basis for development of mathematical models of SRM.

The electromagnetic torque of EMC with a passive rotor can be calculated with the help of the equation (4):

$$M = \frac{\Delta \int_0^I \psi(\theta,i) \cdot di}{\Delta\theta} = \left.\frac{\partial W_k}{\partial \theta}\right|_{I=Const}, \qquad (4)$$

For this it is necessary to calculate co-energy W_k.

When the equation (3**)** is substituted in (2), we get an expression for co-energy determination:

$$W_k = [\psi_{10} - \psi_{1t} \cdot Sin(\theta/2)] \cdot \frac{I^2}{2} + \psi_y \cdot Sin(\theta/2) \cdot \left\{1 + \frac{e^{[-a \cdot I \cdot Sin(\theta/2)]}}{a \cdot Sin(\theta/2)}\right\} \qquad (5)$$

Taking into account (4) and (5), the electromagnetic torque can be written as:

$$M(\theta,i) = \frac{\partial W_k}{\partial \theta} = \frac{Z_r \cdot \psi_y}{2} I \cdot cos(\theta/2) \times \\ \times \left\{I - e^{[-aI\sin(\theta/2)]} - \frac{\psi_{1t} I}{2}\right\} \qquad (6)$$

where $\theta_{м} = \theta / Z_r$ is the geometric angle of the rotor position; Z_r is the number of rotor teeth.

The equation (6) enables us to calculate the electromagnetic torque as a function of the current and the angle of the mutual position of the rotor and the stator.

To develop a mathematical model of this switched drive we shall accept the following assumptions:

the drive is connected to a power supply with internal resistance being equal to zero;

magnetic connections between sections are absent (absolutely fairly for T-, C – and U – similar constructions of stator and does not bring in significant errors for "classical " constructions);

power keys of a commutator are electronic inertialess keys for which it is possible to assume that, firstly, a transient process as well as the section commutation of EMC of SRM occur practically instantly; secondly, the inverse resistance of the closed key equals infinity;

diodes in conducting state are represented by a mathematical model in which the diode’s resistance in the closed condition is equal to infinity;

magnetic performance of the magnetic circuit of a magnetic isolated section is represented by the equation (3);

stator winding’s parameters are lumped.

Modelling of power transistor keys is realized by assuming that the transients process of their turning on and turning off happen instantly, the resistance of a closed key is equal to infinity, the volt-ampere characteristic of a saturated key is described with the following equation

$$\Delta U_T = U_{KE} = U_{KE.0} + R_{KE.HAC} \cdot i, \tag{7}$$

where $\Delta U_{KE.O}$ & $R_{KE.HAC}$ is given in the passport of the transistor.

The state of EC transistor keys is conditioned by a mutual position of the rotor and the stator that is they are controlled by signals of rotor position sensors**.** According to the state of these keys, let us introduce formal coefficients K_j, which get the value of "1" if the key is open, and "0" – if the key is closed; j is the key number.

The value of the formal coefficients K_j depends on the rotor position and is calculated according to the following switching function:

$$K_j = 1 \quad if \quad \beta + 2(N_j - 1)\pi + (j+1)2\frac{\pi}{m} < \theta \le \beta + \\ + 2(N_j - 1)\pi + (j+1)2\frac{\pi}{m} + \gamma \tag{8}$$

K_j = 0 for all other values,

where $N_j = \dfrac{\theta + \pi + (j-1)\cdot 2 \cdot \frac{\pi}{m}}{2 \cdot \pi} + 1$ is the period number for an appropriate section.

The scheme of EC contains a nonlinear element – a diode, whose volt-ampere characteristic can be approximated by the following expression

$$i_{\partial} = I_0 \left(e^{b \cdot \Delta U_{\partial}} - 1 \right), \tag{9}$$

and then the voltage drop on the diode can be written as:

$$\Delta U_{\partial} = ln \frac{i_{\partial} + I_0}{I_0} / b, \tag{10}$$

where I_0 & b are the diode current and the temperature potential coefficient.

Steel losses are known to be hysteresis losses and losses of Foucault. In SRM with a passive rotor the basic magnetic flux does not change the direction; its magnititude oscillates only in separate parts of a magnetic circuit from the maximum value to the minimum one without polarity change. Such magnetization is equivalent to a normal magnetization caused by an oscillating magnetomotive force in a solid already magnetized up to some magnetic condition with constant magnetomotive force; additionally, the hysteresis loop is narrow enough, as experience shows, to cause insignificant magnitude of Joule losses resulting from a magnetic aftereffect. This means that total steel losses consist mainly of the losses from eddy currents, and hysteresis losses can be neglected [10].

The alternating magnetic flux, when closing through the steel core, generates an electromotive force in it. The electromotive force induces Foucault currents in the steel that in turn cause losses in it. The intersection of magnetic circuit can be considered as such, that consists of elementary closed loops, which make short-circuits penetrated by the alternative magnetic flux. Let's represent them in the circuit of section of SRM as branches containing L_σ and R_σ, which are connected in parallel to a magnetization branch.

The steel resistance to Foucault currents can be expressed with sufficient for engineering practice precision by the following equation

$$R_s = \frac{E^2}{\Delta P_s}, \tag{11}$$

where $E = 4.44 \cdot f \cdot w_z \cdot B \cdot s$ and $\Delta P_s = P_0 \cdot \gamma_s \cdot s \cdot l_м \cdot B^2 \cdot (f / f_0)^2$.

When values E and ΔP_s are substituted in (11), we get the following equation to determine R_s of one section:

$$R_s = 4{,}9 \cdot 10^4 \cdot \frac{w_z^2 \cdot S \cdot q}{p_0 \cdot \gamma_s \cdot l_m}, \tag{12}$$

where $p_0, \gamma_s, S, l_m, w_z, q$ are the specific losses in steel, the density of magnetic circuit material, the length of a magnetic flux line, the number of windings on one tooth, the number of stator teeth in one section respectively.

The leakage inductance L_σ of circuits made by Foucault currents is insignificant and its magnitude could be neglected. However, for ensuring the stability of solving process of the differential equations while using numerical methods we propose that the value of L_σ be accepted as follows:

$$L_\sigma \cong \frac{\Delta t}{12} \cdot R_s\ , \qquad (13)$$

where Δt is the integration step of the differential equations.

According to the algorithm of SRM work and accepted assumptions, a structure of the connection circuit of a section and the equation of voltages will discretely vary with the change of the angle θ.

Taking the abovementioned into account, and also considering that SRM is fed from a source with a zero internal resistance, for m-sectional EMC with a passive rotor, magnetic insulated sections and EC with sequential CBE in each phase the expanded equations after transformations can be given in the form (14).

$$\begin{cases} \dfrac{di_j}{dt} = \left[u_j + R \cdot i_j + \dfrac{A_j}{L_\sigma}(u_j - R \cdot i_j + R_S \cdot i_{Sj}) - B_j \omega \right] / A_j \\ \dfrac{di_{Sj}}{dt} = -(R_s i_{Sj} + u_j - R \cdot i_j) / L_\sigma \\ \dfrac{du_{Cj}}{dt} = (1 - K_j - K_{j+m}) \dfrac{i_j}{C} \\ \dfrac{d\omega}{dt} = \left\{ \sum_{j=1}^{m} \left[\begin{array}{l} \dfrac{Z_r}{2}(i_{j+i_{Sj}}) \cos\theta_{pj} \cdot \psi_y \times \\ \times (1 - e^{-a(i_j + i_{Sj}) \sin\theta_{pj}} - \dfrac{\psi_{1t}(i_j + i_{Sj})}{2\psi_y}) \end{array} \right] - M_C \right\} \dfrac{Z_r}{J} \\ \dfrac{d\theta}{dt} = \omega \end{cases} \qquad (14)$$

where:

$$u_j = (U + \Delta U_\partial) \cdot (K_j - K_{j+m}) + u_C(K_j + K_{j+m} - 1) - \\ - \Delta U_T(K_j + K_{j+m}) - \Delta U_\partial (3 - K_j - K_{j+m})$$

$$A_j = \psi_{10} - \psi_{1t} \sin\theta_{pj} + \psi_y \cdot a \cdot \sin^2\theta_{pj} \cdot e^{-a(i_j + i_{Sj}) \sin\theta_{pj}}$$

$$B_j = \frac{\cos\theta_{pj}}{2} \left[\begin{array}{l} \psi_y - \psi_{1t}(i_j + i_{Sj}) - \psi_y \cdot e^{-a(i_j + i_{Sj}) \sin\theta_{pj}} \times \\ \times (1 - a(i_j + i_{Sj}) \sin\theta_{pj}) \end{array} \right]$$

$$\theta_{pj} = \frac{\theta}{2} - (j - 1)\frac{2\pi}{m}$$

$j = 1,2,...m;\ J$ – moment of inercia; M_C – moment of load resis tan ce

The reduced mathematical model (14) is the base for development of numerical models of electric drives with SRM with buffers of energy to do research in dynamic and quasi-steady state regimes of operation, to receive instantaneous values of currents, a torque, a rotation frequency, a voltage on an accumulator, as well as integrated values of these magnitudes and separate composite losses (in copper, in steel, on the power electronic elements of a commutator etc.). For the numerical solution of the differential equations' system the method of Runge – Kutta of the fourth order with a fixed integration step has been used.

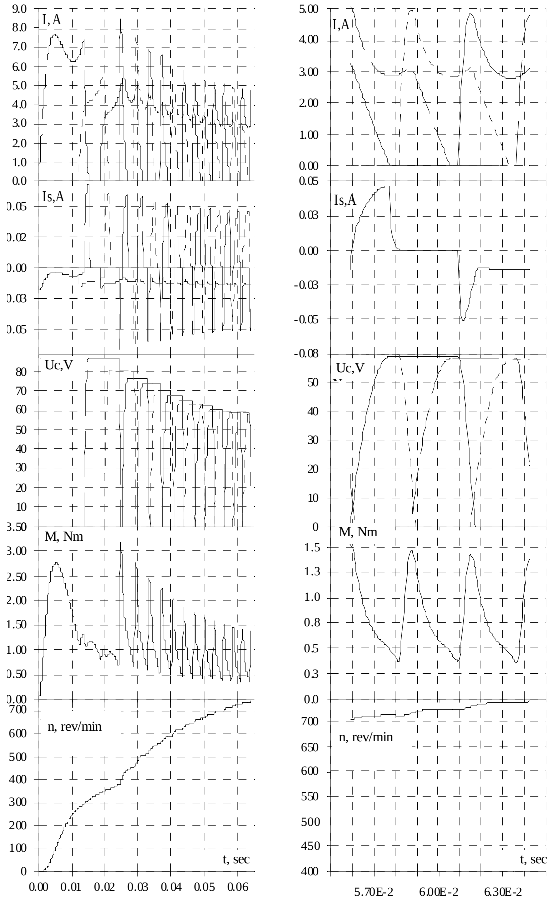

Fig. 8. Simulation result of starting and quasi-steady state regimes of operations of three-phase SRM with sequential CBE

The method of inverting the differential equations with regard to the rotation angle of the rotor, the section current or the voltage on the accumulative capacitor has been applied for the exact determination of instants of turning on (turning off) EC power keys, diminution of the current in any section to zero, full discharge of capacitors. Figure 8 gives an example of computing instantaneous values at a starting point, and in quasi-steady state regimes of operation of the electric drive on the basis of three-phase SRM with sequential CBE in each section – the section current, the current representing losses in steel, the voltage on accumulative capacitors, the electromagnetic torque, and the rotation frequency.

The comparison of simulation results with oscillograms of the real system, and also average values of a torque, a rotation speed and composite losses for examined SRM with nominal torque from 0.05 up to 20 Nm shows that the divergences do not exceed 5 %, what confirms the adequacy of the mathematical model to the physical sample.

5. Conclusions

The switched reluctance motor is simpler, more suitable for efficient manufacturing and cheaper than other known small power electrical motors. As the presented dynamic and static characteristics show, the proposed mathematical models for SRM provide a good basis for computation of its characteristics.

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ВЕНТИЛЬНИЙ РЕАКТИВНИЙ ДВИГУН З БУФЕРОМ ЕНЕРГІЇ ТА ЙОГО МАТЕМАТИЧНА МОДЕЛЬ

В. Ткачук

Вентильні реактивні двигуни (ВРД) є простішими та ефективнішими ніж інші електричні двигуни малої потужності. Широке застосування ВРД стримується тільки через їхні низькі енергетичні показники. В статті запропоновані нові конструктивні схеми електромеханічного перетворювача ВРД, принципові електричні схеми з буферами енергії, які не тільки захищають силові транзистори комутатора від перенапруг під час вимкнення секції, але також використовують запасену в магнітному полі енергію для форсування струму в наступній секції, тобто економиться енергія, а також математична модель для миттєвих значень цього типу двигуна, яка є базою для розрахунку його характеристик. Представлено приклади розрахунку динамічних та статичних характеристик, які показують, що запропоновані математичні моделі є доброю базою для розрахунку характеристик таких двигунів.

Vasyl Tkachuk – Head of Department of Electrical Machines and Devices, Institute of Power Engineering and Control System, Lviv Polytechnic National University, doctor of sciences, professor; research fields: electrical machines, machine drives, control systems, power electronics and electromechanotronics, electronic switched electric motors with permanent excitation, switched reluctance motors, mathematical modelling.