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MODELLING OF ELECTRODYNAMIC PROPERTIES OF STRUCTURES WITH N-MULTIPLE PERIODICITY

Volodymyr Pavlysh, Viktor Hoblyk, Iryna Nychai, Oleksij Liske

Lviv Polytechnic National University

viktor_hoblyk@mail.ru

Abstract: In this paper, mathematical models of electromagnetic structures with N – multiple periodicity represented as branched continued fractions are described; the results of the investigation of the peculiarities of an electromagnetic field formed by a dielectric plate with the complex modulation of its dielectric permittivity are shown; the results of research of electrodynamic properties of modulated circular impedance cylinder are presented.

Key words: structures with N – multiple periodicity, branched continued fractions, mathematical models, the spatial field distribution.

1. Introduction

The construction of a wide range of waveguide and radiating components of infocommunication systems for various functional purposes is based on the electrodynamic properties of periodically nonuniform structures, which are used in all frequency ranges [1 – 4].

However, all possibilities of practical application of periodically nonunifom structures have not been studied yet. The development of the theory, methods and means of their investigation is behind the practical needs.

Therefore, issue of the day is the development of theory and the creation of effective methods of simulating electrodynamic properties of numerous periodic structures with the complex changing of their parameters.

In this paper, the theory describing electrodynamic structures with N – multiple periodicity of changing the parameters of their construction is proposed [5 – 10] for further developing software-based mathematical models.

This theory is based on the strict solutions of the problems concerning the electrodynamic stimulation of the structures, whose parameters are modulated by complex periodic functions.

The novelty of this work is the algorithm created and the results obtained while using the generalized mathematical model as a recurrent formula [6] for creating a wide range of the models of the electrodynamic structures with N – multiple periodicity. In particular, creating mathematical models and investigating two types of the electromagnetic fields of one-dimensional nonuniform dielectric structures is considered.

2. Generalized mathematical model

In [11, 12, 15] it has been proved that some problems of excitation of modulated electromagnetic structures (such as antenna arrays, photonic crystals, impedance and dielectric structures) come to the analysis of the solution obtained in the form of mathematical relations being described by branched continued fractions [13, 14] and allow the research using methods developed in [5 – 10].

The basic data of the algorithm for the creation of such mathematical model using branched continued fractions are set in a recurrence formula (1). This relation describes the general structural features of a wide range of mathematical models of radiative and waveguide structures with multiple periodicity. The formula (1) links the spectral density $\xi_n(\chi)$ of the spatial field distribution of one-dimensional modulated antenna arrays, impedance, dielectric and metal-dielectric structures in particular photonic crystals with a mathematical object, an unfolded form of which is a branched continued fraction [13], whose components contain the parameters of the construction of a periodically nonuniform structure:

$$\xi_N(\chi) \cong \xi_{N-1}(\chi) - \frac{A_N \sum\limits_{n_N=-\infty}^{\infty} \xi_{N-1}(\chi - n_N T_N) C_{n_N}}{\prod\limits_{m=0}^{N} D_{m,\Delta}(\chi)} \qquad (1)$$

where

$$D_{N,\Delta}(\chi) = 1 + A_N \sum_{n_N=-\infty}^{\infty} \frac{C_{n_N}}{\prod\limits_{m=1}^{N} D_{m-1,\Delta}(\chi - n_N T_N)};$$

A_N – coefficients describing the modulation parameters of antenna arrays, impedance and dielectric structures (in particular, photonic crystals), namely: the amplitude of the modulation of a load resistance, the period of nonuniformities, and linear dimensions;

C_{n_N} – coefficients describing the shape of the curve which modulates the parameters of the structure. As periodic functions modulating the parameters of a structure the superimposed multiple periodic sequences of Gaussian functions, rectangular, triangular and other impulse functions, including the δ-functions can be used;

$T_N=2\pi/d_N$, d_N is the period of the spatial modulation of the structure;

N is the number of superimposed multiple periodic sequences of functions;

χ is a generalized wave number (spatial frequency) described by the relation $\chi=2\pi/\lambda$, where λ is a wavelength, or the period of the spatial harmonic of a field.

For graphing the spatial field distribution as a function of spatial-angular coordinates it is necessary to substitute the equation $\chi=k\sin\theta°$ (where k is a wave number for vacuum: $k=2\pi/\lambda_0$; λ_0 is a wavelength in vacuum; $\theta°$ is an angle counted out from a normal to the plane of the structure) in (1) instead of the variable χ.

The function $D_{0,\Delta}(\chi)$ in (1) when N=0 is described by the relation:

$$D_{0,\Delta}(\hat{\chi}) = Z_0(a,\omega,\varepsilon_a) - P_0(\chi)B_0(a,\chi); \tag{2}$$

$Z_0(a, \omega, \varepsilon_a)$ is the constant component of the structure surface impedance determined by its geometrical and radiophysical parameters;

$$P_0(\chi) = -i\sqrt{\chi^2 - k^2}; \tag{3}$$

$B_0(a, \chi)$ is a multiplier that takes into account the transverse geometry of the structure;

$$\xi_0(\chi) = 2F(\chi)/D_{0,\Delta}(\chi) \tag{4}$$

$\xi_0(\chi)$ is the solution of the problem in the absence of a spatial modulation of the structure;

$F(\chi)$ is the spectral density of an extraneous field source (described by a correlation given in [7]).

The components of branched continued fractions (1) for calculating the radiation field created by the complex periodically nonuniform structures can be found in works [7, 15].

2.1. Algorithm of creating mathematical models of periodically nonuniform electromagnetic structures using branched continued fractions

To solve the problem of the external field excitation of a concrete modulated antenna array, photonic crystals, an impedance or dielectric structure using branched continued fractions it is necessary to take a few simple steps:

Step 1: to create an analytical form of the function $\xi_0(\chi)$ using formulae given in [7] and the formula (4) for a chosen law of the current distribution of an external source radiation field.

Step 2: to create the following components of branched continued fractions (1) by the formulae from [8 – 10] for the chosen structures of modulated antenna arrays, photonic crystals, impedance and dielectric structures, as well as modulation laws of parameters of the structures:

$$D_{0,\Delta}(\chi),\ A_N,\ C_{n_N};$$

Step 3: For N = 1 ("single periodicity" – the case of the modulation of structures by one periodic sequence of impulse functions), it is necessary to substitute the expressions of $\xi_0(\chi)$, and $D_{0,\Delta}(\chi)$ in the formula (1), having replaced their argument χ by $\chi-n_1T_1$.

When the structure is modulated by two periodic sequences of impulse functions (N=2, or "the double periodicity"), it is necessary to substitute the expressions of $\xi_2(\chi)$ and $D_{1,\Delta}(\chi)$ in the recurrence formula (1), having replaced the variable χ by $\chi-n_2T_2$, as it is shown above.

Further, the solution of the problem for structures with N-multiple periodicity can be found using this algorithm, and it will take the form of branched continued fractions with N – branches of complex components. The number N of branches is equal to the number of superimposed multiple sequences of periodic impulse functions.

2.2. The mathematical model of a periodically nonuniform dielectric plate

Let the dielectric plate be excited by a delta-source current

$$I_x^E(y) = I_{x0}^E\delta(y'-0).$$

The law of modulation of the plate dielectric permittivity $\varepsilon'(y)$ is described by the correlation (5). A mathematical model, created according to the formula (1) and the corresponding algorithm (section 2.1), is described by relations (6-10).

$$\varepsilon'(y) = \varepsilon'_{a0} + \varepsilon'_{aM_1}\sum_{n_1=-\infty}^{n_1=\infty} rect\left(\frac{y-n_1d_1}{\Delta}\right) \tag{5}$$

$$F_1(\hat{\chi}) = -2i\frac{(\sqrt{\hat{\chi}^2-1})}{D_{0,\Delta}(\hat{\chi})D_{1,\Delta}(\hat{\chi})}, \tag{6}$$

where:

$$D_{0,\Delta}(\hat{\chi}) = \sqrt{\hat{\chi}^2-1} - Z_0; \tag{7}$$

$$D_{1,\Delta}(\hat{\chi}) = 1 + Z_1\sum_{n_1=-\infty}^{\infty}\frac{\mathrm{sinc}(n_1\pi\Delta/d_1)}{D_{0,\Delta}(\hat{\chi}-n_1T_1)} \tag{8}$$

$$Z_0 = \hat{b}\hat{\varepsilon}_{a0}, \qquad Z_1 = -\frac{\hat{\varepsilon}'_{aM1}\hat{b}\Delta}{d_1}, \tag{9}$$

$$\hat{\varepsilon}'_{a0} = \frac{\varepsilon'_{a0}}{\varepsilon_0},\ \hat{\varepsilon}'_{aM1} = \frac{\varepsilon'_{aM1}}{\varepsilon_0}. \tag{10}$$

The function $F_1(\hat{\chi})$ (6) describes the spectral density of a total field that is the result of imposing an external source field and the field formed by polarization currents induced in the periodically nonuniform dielectric plate.

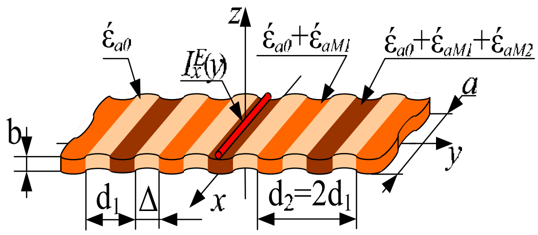


Fig. 1. A dielectric plate with a double periodicity

Let us show the functioning of the algorithm (section 2.1), while developing the model of a dielectric plate with a double periodicity (Fig. 1).

In this case, the dielectric permittivity can be described by a following mathematical correlation:

$$\varepsilon'(y) = \varepsilon'_{a0} + \varepsilon'_{aM_1} \sum_{n_1=-\infty}^{n_1=\infty} rect(\frac{y - n_1 d_1}{\Delta}) + \to$$

$$\to +\varepsilon'_{aM_2} \sum_{n_2=-\infty}^{n_2=\infty} rect(\frac{y - n_2 d_2}{\Delta}); \qquad (11)$$

Then, the mathematical model describing the spectral density $F_2(\widehat{\chi})$ of the total field is as follows:

$$F_2(\widehat{\chi}) = -2i \frac{(\sqrt{\widehat{\chi}^2 - 1})}{D_{0,\Delta}(\widehat{\chi}) D_{1,\Delta}(\widehat{\chi}) D_{2,\Delta}(\widehat{\chi})}, \qquad (12)$$

where:

$$D_{2,\Delta}(\widehat{\chi}) = 1 + Z_2 \sum_{n_2=-\infty}^{\infty} \frac{\mathrm{sinc}(n_2 \pi \Delta / d_2)}{D_{0,\Delta}(\widehat{\chi} - n_2 T_2) D_{1,\Delta}(\widehat{\chi} - n_2 T_2)}; \qquad (13)$$

$$Z_2 = -\frac{\widehat{\varepsilon}'_{aM\,2} \widehat{b} \Delta}{d_2}. \qquad (14)$$

Thereby, an additional multiplier appears when the additional modulation of the plate dielectric permittivity in the components of branched continued fractions takes place. This multiplier reflects the influence of the additional modulation of the parameters of the plate on the spectrum structure of spatial harmonics of the field. Similarly, we can develop mathematical models of such structures with a triple and higher multiplicity modulation of the parameters. The possibility of creating such models facilitates the development of the theory of branched continued fractions and confirms their practical significance.

The adequacy of the mathematical models is confirmed by full-scale experiments [16].

Computer programs based on the mathematical models (6, 11) of the periodically nonuniform dielectric plate have been created in the environment of MATLAB and used for testing the features of the set of the dielectric permittivity modulation parameters of a field structure (Fig. 2 – 7).

The analysis of the simulation of the field structure has shown that the period d_1 of the modulation of the dielectric permittivity $\varepsilon(y)$ the most strongly influences the formation of the field. The spatial field distribution takes the form of a directed radiation.

The second periodic sequence of rectangular functions causes the appearance of new components in the spectrum of spatial harmonics of the field. These components can be either slow or fast harmonics of the field. Latter can be considered to be an additional radiation of the field of the electromagnetic structures towards certain spatial angles, which considerably extends the functional properties of such structures.

Fig. 14 (a red line) shows the effect of the appearance of new spatial harmonics of the field: the introduction of nonuniformities with a period d_2 causes the directed radiation emitted by the structure.

When the plate has the same parameters, but without a double periodicity and radiation, the form of the field distribution coincides with the field of in-phase magnetic current source situated on the nonmodulated dielectric plate.

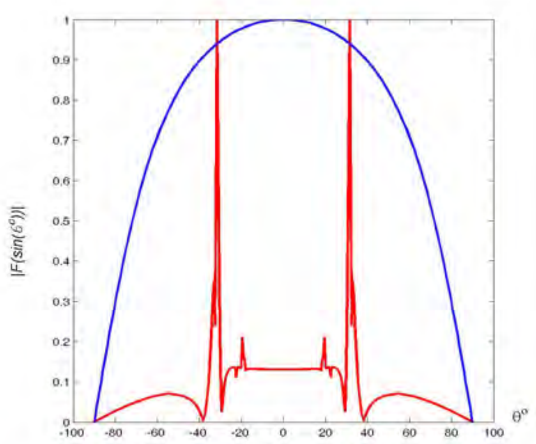


Fig. 14. The influence of the introduction of double modulation ε (y) on the spatial distribution of electromagnetic field (d_1=0, 28λ, d_2=2d_1)

The shape of the profile change of the dielectric permittivity of the plate [17] affects the spatial field distribution (Fig. 15 a, b). This process has been investigated using the proposed mathematical models. By the testing of the models it has been found out that the shape of nonuniformities (rectangular, triangular, Gaussian) determines the spectrum envelope of spatial harmonics fields, and their repetition cycle expresses the density of the discrete spectrum.

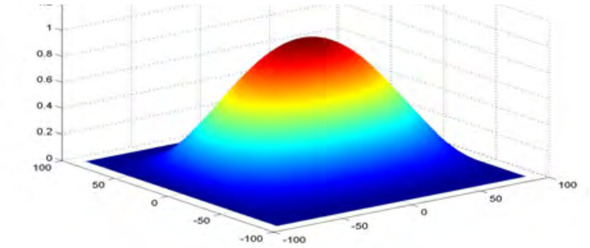


Fig. 2. Field distribution PNDP for $d_1=0.1\lambda$.

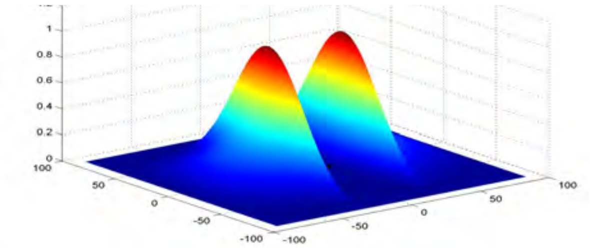


Fig. 3. Field distribution PNDP for $d_1=0.5\lambda$.

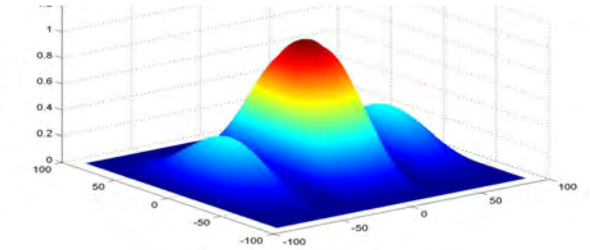


Fig. 4. Field distribution PNDP for $d_1=0.5\lambda$.

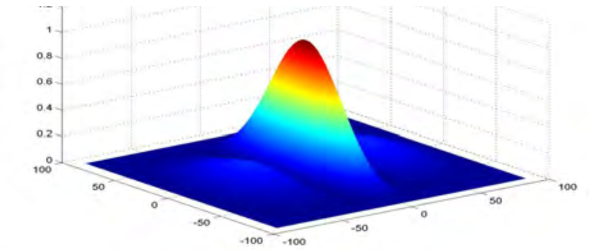


Fig. 5. Field distribution PNDP for $d_1=0.75\lambda$.

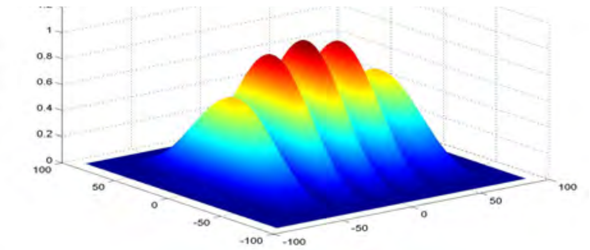


Fig.6. Field distribution PNDP for $d_1=0.95\lambda$.

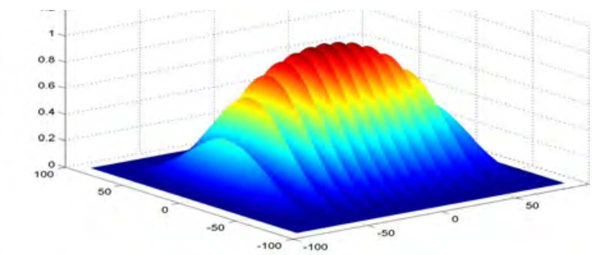


Fig.7. Field distribution PNDP for $d_1=4\lambda$.

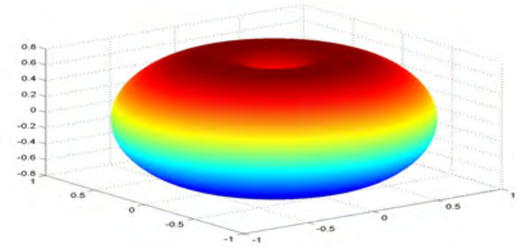


Fig. 8. Field distribution PNDC for $d_1=0.11.29\lambda$

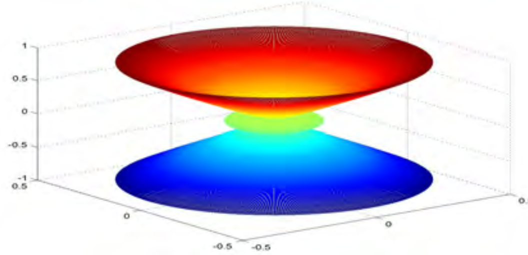


Fig. 9. Field distribution PNDC for $d_1=0.5\lambda$

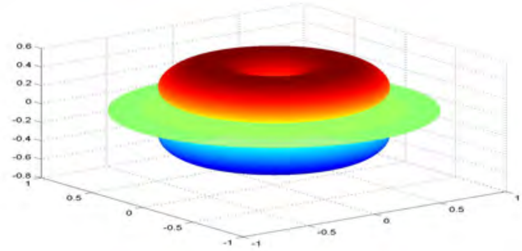


Fig. 10. Field distribution PNDC for $d_1=0.88\lambda$

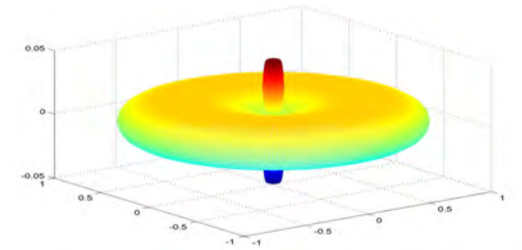


Fig. 11. Field distribution PNDC for $d_1=0.91\lambda$

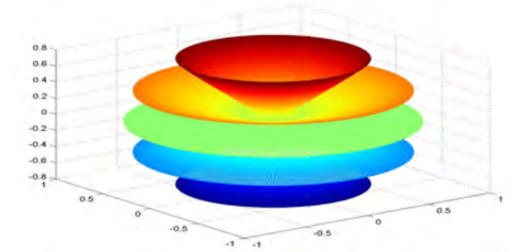


Fig. 12. Field distribution PNDC for $d_1=2.57\lambda$

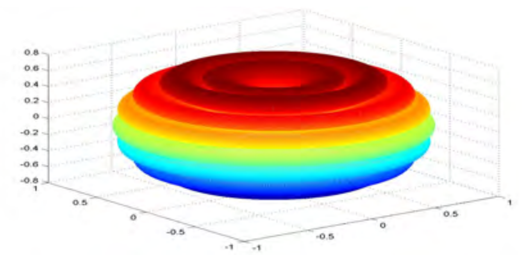
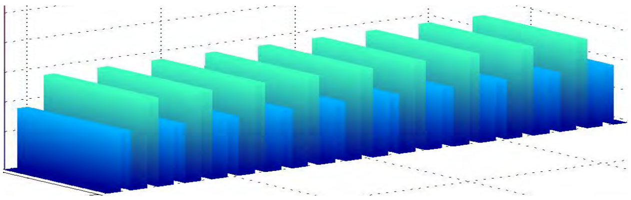
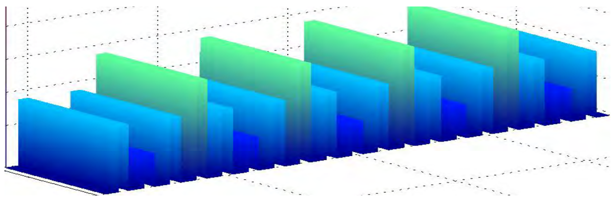


Fig. 13. Field distribution PNDC for $d_1=4.6\lambda$



a)



b)

Fig. 15. Variants of the the profile changes of dielectric permittivity of the plate, modulated by two periodic sequences of rectangular impulses

2.3. The mathematical model of a cylindrical structure, modulated by one periodic sequence of impulse functions

Let us apply the algorithm (2.1) for creating a mathematical model of a periodically nonuniform dielectric cylinder (fig. 16).

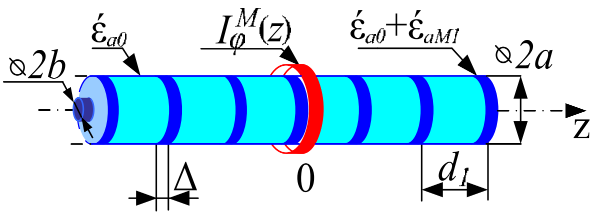


Fig.16. Periodically nonuniform dielectric cylinder

The modulation of dielectric permittivity ε´(y) on a cylinder along the z-axis is described by the function with a single periodicity as it is shown in (5), (Fig. 17).

$$\varepsilon'(y) = \varepsilon'_{a0} + \varepsilon'_{aM_1} \sum_{n_i=-\infty}^{n_i=\infty} rect(\frac{z - n_i d_i}{\Delta}); \qquad (15)$$

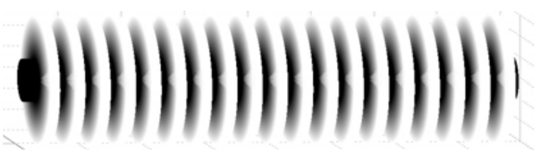


Fig. 17. The change of the dielectric permittivity of a cylinder, modulated by one periodic sequence of rectangular impulses

Let the structure be excited by a ring of an in-phase magnetic current:

$$I^M(z) = I_0^M \delta(z'-0)\delta(r'-a), \qquad (16)$$

where I_0^M is the amplitude of the magnetic current.

Then, using the algorithm shown in section 2.1 of this paper and taking into account the results of [10], we can find a solution to the problem of exciting the structure (Fig. 16) by the ring of the in-phase magnetic current in the following form:

$$\xi_1(\chi) \cong \xi_0(\chi) - \frac{A_1 \sum\limits_{n_1=-\infty}^{\infty} \xi_0(\chi - n_1 T_1) C_{n1}}{D_{0,\Delta}(\chi) D_{1,\Delta}(\chi)} \qquad (17)$$

where:

$$\xi_0(\chi) = -\frac{4iF_1^M(\chi) J_1[ap_0(\chi)] p_0(\chi)}{D_{0,\Delta}(\chi)};$$

$$D_{0,\Delta}(\chi) = \omega^2 \mu_0 \varepsilon_{a0} B[ap_0(\chi)] H_1^{(2)}[ap_0(\chi)] - 4ip_0(\chi);$$

$$A_1 = -\omega^2 \mu_0 \varepsilon_{aM1} \frac{\Delta}{d_1};$$

$$D_{1,\Delta}(\chi) = 1 + A_1 \sum_{n_1=-\infty}^{\infty} \frac{C_{n1}}{D_{0,\Delta}(\chi - n_1 T_1)},$$

$$C_{n_1} = \mathrm{sinc}(n_1 \pi \frac{\Delta}{d_1}); \; B[ap_0(\chi)] = \int_{x=0}^{ap_0(\chi)} x J_1(\chi) d\chi;$$

$k = \omega\sqrt{\varepsilon_a \mu_a}$; J_1, $H_1^{(2)}$ - Bessel and Hankel functions;

$F_1^M(\chi) = -iI_0^M \chi / 4\pi$; $p_0(\chi) = -i\sqrt{\chi^2 - k^2}$.

The mathematical expression (17) interconnects the spectral density of spatial harmonics of the field $\xi_1(\chi)$ with such parameters of the periodically nonuniform dielectric cylinder, as its period *d*, its diameter *2a*, the width of nonuniformity Δ, the depth of the modulation of the dielectric permittivity $\hat{\varepsilon}_{aM_1}$.

Using the model (17) made with the application of MATLAB software [18], appropriate numerical results have been obtained while investigating the influence of the period d_1 of dielectric permittivity nonuniformities over a remote external source field (located at the distance being several times longer than the wavelength) (Fig. 8 – 13).

The analysis of the graphs shows that the radiation pattern of the periodically nonuniform dielectric cylinder excited by the ring of the in-phase magnetic current is mostly "funnel shaped". In some cases, when the period d_1 of dielectric permittivity uniformities is equal to or multiple of the wavelength of a zero (basic) spatial harmonic of the field propagating along the *z*-axis of the structure, the radiation is emitted along a normal to the *z* axis of the structure observed. The radiation pattern of the periodically nonuniform dielectric cylinder in such cases takes the shape of a disk (Fig. 11).

7. Conclusion

The solution of the electrodynamic problem of exciting the fields of impedance and dielectric structures with N – multiple periodicity of changing their parameters by an arbitrary external source has been obtained in a closed form. It allows the creation of new effective mathematical models of such structures in the form of branched continued fractions.

The mathematical models and results of investigating the electrodynamic properties of the structures with N – multiple periodicity changes have a great practical value in creating the components of infocommunication systems with improved parameters, namely, antenna arrays, photonic crystals, interferometers, spatial-frequency and time – frequency information filters, collimators, slow-wave structures and the transformers of spatial harmonics of a field, switches, and logic elements.

Reference

1. Gippius N.A., et al. Waveguide plasmon polaritons in metal-dielectric photonic crystal slabs // PSS. – 2005. – Vol. 47. № 1. – P. 145-149. [Rus]

2. Senyk T. Numerical modeling of electromagnetic waves’ diffraction on periodic structures // PhD thesis. – Lviv, 2005. – 17 p. [Ukr]

3. Dmitruk N., et all. Optical properties of gold 1D nanowires fabricated by holographic method on flat dielectric substrates // Physics and chemistry of solid state. – 2007. – Vol. 8, № 2. – P. 281 – 286. [Ukr]

4. Lerer A.M., et all. Theoretical modeling of enhanced optical transmission through doubly periodic metallic nanostructures. // Proc. of 1-st intern. congress “Metamaterials-2007”. – 2007.– P.750-753.

5. Chaplin A. Excitation of periodically nonuniform impedance structures // Proc. of the VIII Symp. "Waves and Diffraction." – Moscow, 1981. – Vol. 3. – P.73-76. [Rus]

6. Hoblyk V. Analysis of field over the impedance plane with periodic discrete nonuniformities by the method of A.F.Chaplin // Theoretical and experimental methods of research of the antennas and microwave devices. Dep. in UkrNIINTI 11.11.84. – Lviv., 1984.– №1874.– P. 27-70. [Rus]

7. Markov G., Chaplin A. Excitation of electromagnetic waves. – 1983 – 179 p. [Rus]

8. Hoblyk V. Thesis for a scientific degree of candidate of physical-mathematical sciences. – Kharkov State University, 1986. – 210 p. [Rus]

9. Chaplin A. F., Hoblyk V. V. A generalization of the solution of problems of excitation modulated impedance structures.// Rus. Dep. in UkrNIINTI. – Lviv, 1986. – № 813, UK-86. – 8 p. [Rus]

10. Hoblyk V. Theory of uniformly spaced arrays with parameters that change according to the periodic law // Bulletin of Lviv Polytechnic National University, series "Theory and design of semiconductor and electronic devices". – Lviv, 1998. – № 343- P. 49-53. [Ukr]

11. Hoblyk V., Hoblyk N. About solution of the Fredholm integrated equation in a branched continual fraction type // International School-Seminar “Continued Fraction, their Generalization and Application”. – Uzhhorod National University.– 2002.–P. 16 – 18.

12. Hoblyk V., Hoblyk N. The modeling antenna arrays by branched continual fractions’// Proc. 5th ICATT. – Kyiv, Ukraine, 2005, – p. 234-237.

13. Skorobogatko V. The theory of branching continued fractions and its application in computational mathematics. – Moscow: Nauka, 1983. – 312 p. [Rus]

14. Bodnar D.I. Branching continued fractions. – Kyiv: Nauk. Dumka.– 1986.-176 p. [Rus]

15. Hoblyk V., Pavlysh V., Nychai I. Modelling of photonic crystals by branched continual fraction. // "Radioelectronics and telecommunications", Lviv Polytechnic National University. – 2007. – № 595. – p. 78-86. [Ukr]

16. Hoblyk V., Nychai I., Liske O. M. Plasmon Antenna with Complex Profile of Dielectric Permittivity Change // Materials of 7th ICATT’2009. – Lviv, 2009. – p. 138-140.

17. Hoblyk V., Pavlysh V., Nychai I. Distribution of the field of dielectric plates with a complex profile of change of dielectric permittivity // Bulletin of Lviv Polytechnic National University, series "Electronics". – 2009. – № 646. – P. 51–56. [Ukr]

18. Hoblyk N., Hoblyk V. MATLAB in engineering calculations. Computer practicum:a tutorial. – Lviv: Publishing House of Lviv Polytechnic, 2010. – 132 p. [Ukr]

МОДЕЛЮВАННЯ ЕЛЕКТРОДИНАМІЧНИХ ВЛАСТИВОСТЕЙ СТРУКТУР З N-КРАТНОЮ ПЕРІОДИЧНІСТЮ

В. Павлиш, В. Гоблик, І. Ничай, О. Ліске

В даній статті наведено результати розробки у вигляді гіллястих ланцюгових дробів математичних моделей електродинамічних структур з N – кратною періодичністю, результати дослідження особливостей формування поля діелектричною пластиною зі складними законами модуляції її діелектричної проникності, а також результати дослідження електродинамічних властивостей модульо-ваного кругового імпедансного циліндра.

Volodymyr Pavlysh– PhD, professor, Vice-Rector of Lviv Polytechnic National University, Head of Department of Electronic Means of Computer Aided Technologies, Institute of Telecommunications, Radioelectronics and Electronic Engineering.

Viktor Hoblyk – PhD, associate-professor, Deputy-Head of Department of Electronic Means of Computer Aided Technologies, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.

Oleksij Liske – Assistant, Department of Electronic Means of Computer Aided Technologies, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.

Iryna Nychai – Assistant, Department of Electronic Means of Computer Aided Technologies, Institute of Telecommunications, Radioelectronics and Electronic Engineering, Lviv Polytechnic National University.