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INVESTIGATION OF THE INFLUENCE OF CAPACITOR IN THE EXCITATION WINDING CIRCUIT OF A SYNCHRONOUS MOTOR ON ITS STARTING TORQUE

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Abstract: The problem of mathematical modelling the asynchronous modes of synchronous motors with a capacitor in the excitation winding circuit is discussed. A technique for studying the effect of capacitance value on the starting torque of the motor has been proposed. The task has been solved as a boundary value problem for a non-linear system of power balance equations, in which the magnetic flux linkage is calculated on the basis of the magnetic field computation using the magnetic circuit theory.

Key words: synchronous motor, starting torque, capacitor, boundary value problem.

1. Introduction

High-power electric drives come with salient-pole synchronous motors (SM), which, as compared to asynchronous motors, offer a range of advantages since they generate reactive power. They are used in mining, woodworking industry, and metallurgy for cyclic load drives. The main problem of their operation in these drives is difficult starting conditions, which require an increased starting torque. The problem of reliable starting is usually solved by overpowering the driving motor, or, if possible, by using an additional low-power asynchronous motor [1]. The search for other simpler ways of solving this problem resulted in the development of methods for increasing the starting torque by using an excitation winding which is short-circuited on an active resistance at the starting moment.

The major way of starting SM is asynchronous running-up, which is the simplest one, too, but the increase ratio for the starting torque is not sufficient. Therefore, various methods are applied to raise the starting torque. It is known that the excitation winding has a high inductance. In addition, the current running through it in the starting mode is of inductive nature and thus cannot ensure a significant additional starting torque.

In recent years it was suggested that capacitors should be used in the excitation winding instead of an active resistance [2]. This results in an increased starting torque and power factor due to an increased current value in the excitation winding, which operates in the resonance mode or a mode close to it [3, 4]. However, studies of the problems occurring for such starting modes were conducted using a simplified mathematical model. Therefore, it would be necessary to verify the obtained results experimentally. Capacitors in excitation windings can cause resonance phenomena and lead to voltage surges dangerous for the excitation winding [3]. Study of the processes in synchronous motors with starting capacitors and the phenomena they are accompanied with is of a big practical value. Therefore, the issue of devising and developing reliable and effective mathematical models ensuring a high accuracy and reliability of the outcomes of the mathematical experiment is topical, since experiments on high-power synchronous motors are very expensive and sometimes altogether impossible.

2. Asynchronous mode equation

If the starting winding of a salient-pole synchronous motor is represented by two circuits in the orthogonal axes d, q, the system of equations describing the phenomena in the synchronous motor with capacitors in the excitation winding circuit can be written in the vector form as follows:

$$\frac{d\vec{\Psi}_{dq}}{dt} = (1-s)\Omega_{dq}\vec{\Psi}_{dq} - R_{dq}\vec{i}_{dq} + \vec{U}_{dq}, \qquad (1)$$

where $\vec{y} = \begin{vmatrix} \Psi_d \\ \Psi_q \\ \Psi_D \\ \Psi_Q \\ \Psi_f \end{vmatrix}$, $\vec{i} = \begin{vmatrix} i_d \\ i_q \\ i_D \\ i_Q \\ i_f \end{vmatrix}$, $\vec{u}_{dq} = \begin{vmatrix} u_d \\ u_q \\ 0 \\ 0 \\ u_c \end{vmatrix}$,

– the vectors of magnetic flux linkage, the current and the voltage in the stator circuit (d, q) and the rotor circuit (D, Q, f) respectively; $s = (\omega_c - \omega)/\omega_c$ – the rotor slip; ω_c, ω – the frequency of a supply power and of the rotor rotation measured in rad/s);

$$R_{dq} = \begin{bmatrix} r & & & & \\ & r & & & \\ & & r_D & & \\ & & & r_Q & \\ & & & & r_f \end{bmatrix}$$

– the diagonal matrix of active resistances of the circuits;

$$\Omega_{dq} = \begin{bmatrix} & -\omega & & & \\ \omega & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}.$$

Power supplied to the stator circuit is calculated by the formula:

$$u_d = U_m \sin\theta \, , \; u_q = U_m \cos\theta \, ,$$

where θ is the rotor angle which continually changes in the asynchronous mode

$$\theta = \gamma - \omega_0 t = \gamma - \tau \, ,$$

and the angle γ changes according to the rule expressed by

$$\gamma = \int_{\tau_0}^{\tau} (1-s) d\tau + \gamma_0 = \int_{t_0}^{t} \omega_0 (1-s) dt + \gamma_0 \, ,$$

where γ_0 is the initial value of the angle γ. Therefore,

$$\theta = \gamma - \omega_0 t = \int \omega dt - \omega_0 t = -\omega_0 \int s dt + \gamma_0 \, .$$

Due to the salient pole construction of the rotor, the slip s is not constant during the rotor rotation: it varies with respect to a certain mean value s_c, so the angle θ changes nonlinearly. However, for most problems (for instance, in case of rotating inertia-type masses on a SM shaft or high values of a slip) it is considered to be constant and equal to the mean value.

The system (1) is complemented by dependencies $\vec{\Psi}_{dq} = \vec{\Psi}_{dq}\left(\vec{i}_{dq}\right)$ of the magnetic flux linkages on the currents determined by the mathematical model of electro-magnetic couples of SM in the axes d, q; the dependencies are non-linear owing to the saturation of the motor magnetic path.

The complete system of power balance equations in the asynchronous mode for a given constant rotor slip and on condition that the excitation winding is circuited on capacitors will contain the equation (1) and the equation given below:

$$\frac{du_c}{dt} = \frac{i_f}{C} \, . \qquad (2)$$

The system comprising equations (1) and (2) can be written as one vector equation of the $m = 6^{th}$ order:

$$\frac{d\vec{y}(\vec{x},t)}{dt} + \vec{z}(\vec{y},\vec{x},t) = 0 \, , \qquad (3)$$

where

$$\vec{y} = \begin{bmatrix} \vec{\Psi}_{dq} \\ u_c \end{bmatrix}; \; \vec{x} = \begin{bmatrix} \vec{i}_{dq} \\ u_c \end{bmatrix}; \; \vec{z} = \begin{bmatrix} -\omega\Psi_q + ri_d - u_d \\ \omega\Psi_d + ri_q - u_q \\ r_D i_D \\ r_Q i_Q \\ r_f i_f - u_c \\ i_f / C \end{bmatrix}.$$

For a constant slip s, the angle θ changes with the period $2\pi/s$, i.e. the asynchronous mode is periodic, and all the coordinates change with the time period $T = 2\pi/(s\omega_c)$. Therefore, the system (2) has the solution in the form of T-periodic dependencies of the vector components $\vec{x}(t) = \vec{x}(t+T)$, and the task of asynchronous mode computing is to solve a boundary value problem for the system (2) of first-order differential equations with periodic boundary conditions.

3. Problem solutving algorythm

The paper proposes a technique for studying the effect of the capacitance value in the excitation winding circuit by means of calculating stationary modes. This involves the application of a projection method developed on the basis of cubic splines [5], which allows not only to calculate a stationary asynchronous mode for a given slip, but also enables the study of the effect of the capacitance value on the starting torque of the synchronous motor. The main points are presented below.

By assigning a set of $n+1$ nodes on the period T, each differential equation of system (3) is approximated by the system of n algebraic equations, as described in [5]. This results in the system of algebraic equations having the following form:

$$S\vec{Y} + \vec{Z} = 0 \, , \qquad (4)$$

in which S is the square matrix of the transition from continuously changing coordinates to their nodal values for the variables approximated by cubic splines, the elements of which are determined only by an inter-nodal distance; $\vec{Y}, \vec{Z}$ are vectors with nodal values $\vec{y}_j$ and $\vec{z}_j$ as their components. The resulting system (4) of algebraic equations is a discrete counterpart of the non-linear system of differential equations (3). The solution of the system (4) is a vector $\vec{X}$ of nodal values of coordinates which can be determined by one of the iteration methods, for instance by the Newton method. However, due to non-linearity of the obtained system, there exists the problem of the iteration process convergence. Thus, in order to develop the algorithm of the problem solution, a differential method [6] is applied. For this purpose the vector $\vec{U}$ of supply voltages in the equation (4) is multiplied by an introduced parameter ε and later the equation (4) is differentiated with respect to the parameter. Taking into account that $\vec{Y} = \vec{Y}\left(\vec{X}, \varepsilon\right)$ and $\vec{Z} = \vec{Z}\left(\vec{Y}, \vec{X}, \vec{U}, \varepsilon\right)$, the following expression is obtained:

$$J \frac{\partial \vec{X}}{\partial \varepsilon} = \vec{U}\,, \qquad (5)$$

where $J = \left(S - \frac{\partial \vec{Z}}{\partial \vec{Y}} \right) \frac{\partial \vec{Y}}{\partial \vec{X}} - \frac{\partial \vec{Z}}{\partial \vec{X}}$ is a Jacobi matrix, in which $\frac{\partial \vec{Z}}{\partial \vec{Y}}$, $\frac{\partial \vec{Y}}{\partial \vec{X}}$, $\frac{\partial \vec{Z}}{\partial \vec{X}}$ are block-diagonal matrices, and each block is represented by the square matrices $\left.\frac{\partial \vec{z}}{\partial \vec{y}}\right|_j$, $\left.\frac{\partial \vec{y}}{\partial \vec{x}}\right|_j$, $\left.\frac{\partial \vec{z}}{\partial \vec{x}}\right|_j$, which are determined by the values of the coordinates in a j^{th} node and do not depend on their values in other nodes. The corresponding diagonal blocks have the following form:

$$\left.\frac{\partial \vec{z}}{\partial \vec{y}}\right|_j = \begin{array}{|c|c|c|c|c|c|} \hline & \omega_0(1-s) & & & & \\ \hline -\omega_0(1-s) & & & & & \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & & & -1 \\ \hline & & & & & \\ \hline \end{array}$$

$$\left.\frac{\partial \vec{y}}{\partial \vec{x}}\right|_j = \begin{array}{|c|c|c|c|c|c|} \hline x_{ddj} & x_{dqj} & x_{dDj} & x_{dQj} & x_{dfj} & \\ \hline x_{qdj} & x_{qqj} & x_{qDj} & x_{qQj} & x_{qfj} & \\ \hline x_{Ddj} & x_{Dqj} & x_{DDj} & x_{DQj} & x_{Dfj} & \\ \hline x_{Qdj} & x_{Qqj} & x_{QDj} & x_{QQj} & x_{Qfj} & \\ \hline x_{fdj} & x_{fqj} & x_{fDj} & x_{fQj} & x_{ffj} & \\ \hline & & & & & 1 \\ \hline \end{array}$$

$$\left.\frac{\partial \vec{z}}{\partial \vec{x}}\right|_j = \begin{array}{|c|c|c|c|c|c|} \hline -r & & & & & \\ \hline & -r & & & & \\ \hline & & -r_D & & & \\ \hline & & & -r_Q & & \\ \hline & & & & -r_f & -1 \\ \hline & & & & 1/C & \\ \hline \end{array}$$

The value of the vector $\vec{X}$ for the set values of the voltage, the slip and the capacitance is determined by integrating the system (5) of differential equations with respect to ε from $\varepsilon = 0$ to $\varepsilon = 1$ and then refined using the Newton method of iteration, according to which increments $\Delta\vec{X}^{(k)}$ at a *k*-th iteration step are determined from the equation

$$J \Delta\vec{X}^{(k)} = \vec{Q}\left(\vec{X}^{(k)}\right), \qquad (6)$$

where $\vec{Q}\left(\vec{X}^{(k)}\right)$ is the vector of residuals of the system for $\vec{X} = \vec{X}^{(k)}$.

The equation (4) allows us to investigate the effect of a varying capacitance value in any mode of the electric circuit operation, including the resonance mode. For this, the system (4) is differentiated and then integrated by the capacitance value C. The operations result in the differential equation

$$J \frac{d\vec{X}}{dC} = \frac{\partial \vec{Z}}{\partial C}\,, \qquad (7)$$

which differs from the equation (6) only in the vector of the right-hand parts, which allows one to solve the task of the asynchronous mode computation and study the effect of the change of the capacitance value, including possible resonance, using the same algorithm. In this case the characteristic can be ambiguous: it has points in which the differential $d\vec{X}/dC$ goes to infinity. While approaching them, one should switch to integration with respect to another coordinate that is a component of the vector $\vec{X}$, and while coming to a new special point, it is necessary to go back to the system of equations with argument ε.

While computing, at each step of integration or iteration it is necessary to determine elements of matrix L_{dqj} of differential inductances of SM in the axes d, q in a j^{th} node, which is done by estimating the magnetic field in the air-gap of the motor. Such calculation is done

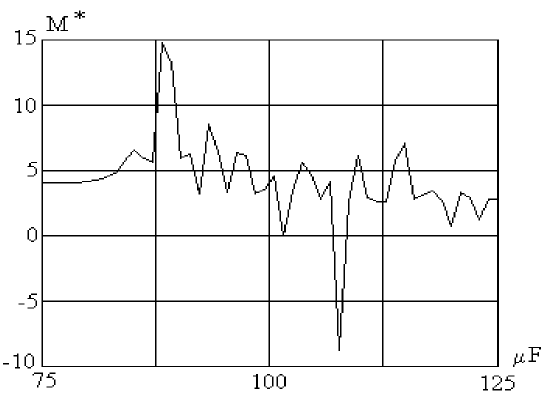

Fig. 1. The dependency of the electromagnetic torque on the capacitance value of the capacitor in the excitation winding

by means of representing the magnetic path by a branched equivalent circuit according to [7].

Fig.1 presents the dependency of the electromagnetic torque on the capacitance value of capacitors in the excitation winding of the synchronous motor

$$\left(P = 630kW; U = 6{,}3kV; I = 72{,}5A; I_f = 225A; p = 8\right)$$

The figure shows that in case of poor choice of the capacitance value, the electromagnetic torque can acquire negative values.

4. Conclusions

The developed method of the mathematical simulation of starting characteristics of synchronous motors allows us to study the effect of the value of capacitance of the capacitors in the excitation winding on the starting torque. The task is solved as a boundary value problem in a timeless zone for a non-linear system of power balance equations describing dynamic modes of the motor operation. It involves using a high-adequacy SM mathematical model, in which the values of the magnetic flux linkage of the circuits are determined on the basis of a model in which the magnetic system of the synchronous motor is represented by a branched equivalent circuit.

References

1. Boroday V.A. Estimation of dynamics of mining motor drive with heavy start-up // Gornaia Electromekhanika i Avtomatika Journal. – 2003. – Vol. 70 – P. 19–2. [Rus].

2. Velin N.V. Static exciter of synchronous motor with a permanently connected capacitor // Electrotechnika Journal. – 1973. – No.12. –P.35–36. [Rus].

3. Kyrychenko V.I., Boroday V.A., Yalanskyi A.A. Improvement of startic properties of large synchronous motors // Herald of East-Ukrainian Volodymyr Dal National University. – 2003. – No. 6[64] – P. 38–40. [Rus].

4. Pivniak G.G., Kyrychenko V.I., Boroday V.A. About new area of improvement of large synchronous motors // Technichna Electrodynamika Journal. Special issue Problems of Modern Electric Engineering. – 2002, Part. 2. – P. 62–65. (Rus).

5. Malyar V.S. Main propositions of spline method for computation of periodic modes of electric circuit operation //Electronika i Sviaz Journal– 1998. – No.5. – P.11–14. [Rus].

6. Yakovliev M.N. To the issue of solving systems of non-linear equations by means of differentiating by parameter // Zhurnal Vychislitelnoy Matematiki i Matematicheskoy Fiziiki Journal. – Vol.4. – 1964. – No. 1. – P. 146-149.[Rus].

7. Filts R.V., Liabuk N.N. Mathematical simulation of salient-pole synchronous motors. – Lviv: Svit Publishers. 1991. – 176 p. [Ukr].

ДОСЛІДЖЕННЯ ВПЛИВУ КОНДЕНСАТОРІВ В КОЛІ ОБМОТКИ ЗБУДЖЕННЯ СИНХРОННОГО ДВИГУНА НА ПУСКОВИЙ МОМЕНТ

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Розглядається проблема математичного моделювання асинхронних режимів синхронних двигунів з конденсаторами в обмотці збудження. Пропонується метод дослідження впливу величини ємності конденсаторів на пусковий момент двигуна. Задача розв’язується як крайова для нелінійної системи диференціальних рівнянь електричної рівноваги, в яких потокозчеплення обчислюються на основі розрахунку магнітного поля з використанням теорії магнітних кіл.

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