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A TWO-DIMENSIONAL MODEL OF AN ELECTROMAGNETIC ACCELERATOR

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Abstract: The mathematical model of an accelerator's magnetic field, which takes into account its symmetry, the nonlinearity of an armature characteristic B(H), and eddy currents, has been investigated. The model has been developed on the basis of the finite elements method. The armature that moves under the applied Ampere's force has been represented as a movable distribution of magnetic permeance.

Key words: coil, eddy currents, electromagnetic accelerator, ferromagnetic core (armature).

1. Introduction

Electromagnetic accelerators have been widely researched by scientists in recent years, as being used for military and space aims. Many authors of the research use the method of finite elements for calculating the magnetic field of a solenoid coil with a ferromagnetic armature inside [1] – [4]. The force applied to this armature is calculated on the basis of the power description of the magnetic field. The authors use the method of moving boundaries or the hybrid finite-element/boundary-element method (FE-BE) for the solution of this task taking into account the dynamics of the ferromagnetic armature's motion. The mathematical model of the accelerator must take into account the nonlinearity of environment, eddy currents, and change in the distribution of magnetic permeability of environment during the ferromagnetic core movement. Among the models of ferromagnetic mass accelerators the article by G. William Slade [5] can be distinguished. One of the flaws of this model is the representation of the ferromagnetic core in the form of a separate domain. Therefore, it is necessary to generate a new FEM mesh during the core movement taking into account a new core position.

2. Mathematical formulation of the task

The research object comprises the following main components: a pulse voltage source, a stationary inductance coil generating a magnetic field, and a movable ferromagnetic core (Fig.1). The magnetic field of the coil has an axial symmetry and, therefore, the mathematical model of the accelerator is two-dimensional in the cylindrical coordinate system. This model takes into account the effect of the core movement, the nonlinearity of its characteristics *B(H)*, and eddy currents in the magnetic core material.

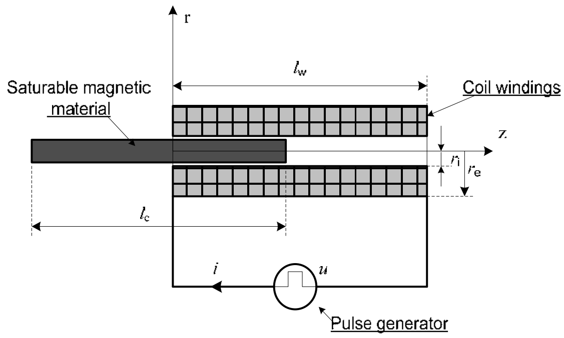

Fig. 1. Illustration of axial symmetry of the accelerator

As the ferromagnetic core moves, for the optimization of the numeral task solution, it is not modeled as a separate domain, but appears as one domain including the core and the environment around the coil windings. This domain is heterogeneous in relation to magnetic permeance and electric conductivity. These heterogeneities, changeable during the core motion, are represented as movable distributions of magnetic permeance and electric conductivity, that is interpolated on a fixed mesh (Fig.2). The research area is divided into two domains: $R1$ – the core and environment, and $R2$ – the coil windings.

The distributions of magnetic permeance and specific electric conductivity in the domain $R1$ are described by the expressions correspondingly:

$$\mu_r = 1 + \mu_{ra} * \exp(-k_p * (abs(r - r_0)\rangle(wr/2)) * (abs(r - r_0) - (wr/2))) * \exp(-k * (abs(z - z_0) \rangle(wz/2)) * (abs(z - z_0) - (wz/2))) \quad (1)$$

$$\gamma_c = \gamma_F * \exp(-k_c * (abs(r - r_0)\rangle(wr/2)) * (abs(r - r_0) - (wr/2))) * \exp(-k * (abs(z - z_0) \rangle(wz/2)) * (abs(z - z_0) - (wz/2))) \quad (2)$$

where μ_{ra} is the magnetic permeability of the core material; γ_F is the specific electric conductivity of the core material; k_p, k_c are coefficients; r_0, z_0 are the coordinates of the centre of the ferromagnetic core; *wr*, *wz* are the radius and the length of the armature (core).

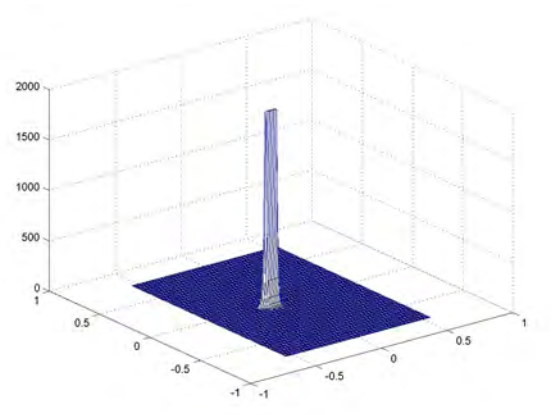

Fig. 2. Distribution of magnetic permeability in R1

The magnetic permeability of the core material μ_{ra} has been approximated by a polynomial of degree $n=5$ on the basis of the nonlinear characteristic $B = \alpha * arctg(\beta H) + \lambda H$

$$\mu_{ra} = a * B^5 + b * B^4 + c * B^3 + \\ + d * B^2 + e * B + f \quad ; \qquad (3)$$

where α=1,05536 ; β=0,008687; λ=9,1977*10^{-5}; *a*, *b*, *c*, *d*, *e*, *f* are the coefficients of the approximating polynomial.

In *R*1 domain, where the core moves, the magnetic field is described by the following equation

$$\gamma \frac{\partial \boldsymbol{A}}{\partial t} + \nabla \times \left(\mu^{-1} \nabla \times \boldsymbol{A} \right) + \gamma \left(-\boldsymbol{v} \times \nabla \times \boldsymbol{A} + \nabla \varphi \right) = 0 \quad (4)$$

where: $\boldsymbol{A}$ is the magnetic vector potential, φ is the electric potential, γ is the specific electric conductivity, μ is the magnetic permeability, v is the ferromagnetic core velocity.

In *R*2 domain (coil windings) the magnetic field equation has the form

$$\nabla \times \left(\mu^{-1} \nabla \times \boldsymbol{A} \right) = \frac{wi}{l_{\mathrm{w}} \left(r_{\mathrm{e}} - r_{\mathrm{i}} \right)} \boldsymbol{e}_{\varphi}; \qquad (5)$$

the electric field is described by the equation

$$\nabla \frac{\partial \left(\varepsilon_0 \varepsilon_r \nabla \varphi \right)}{\partial t} + \nabla \left(\gamma \nabla \varphi - \boldsymbol{J}^{\varphi} \right) = 0 \qquad (6)$$

where: w is the number of winding turns of the coil, i is the coil current, $l_{\mathrm{w}}(r_{\mathrm{e}}-r_{\mathrm{i}})$ is the area of the coil cut (Fig.1), $\boldsymbol{e}_{\varphi}$ is the azimuthal unit vector in circular cylindrical coordinates; $\frac{wi}{l_{\mathrm{w}} \left(r_{\mathrm{e}} - r_{\mathrm{i}} \right)} \boldsymbol{e}_{\varphi}$ is the vector of current density in the windings.

The electric circuit composed of a voltage source and the coil is described by the second law of Kirchhoff

$$\frac{\partial \psi}{\partial t} + Ri - u = \int_l \frac{\partial \boldsymbol{A}}{\partial t} d\,\boldsymbol{l} + Ri - u = 0 \qquad (7)$$

where: ψ is the linkage, $\Phi = \int_l \boldsymbol{A} d\boldsymbol{l}$ is the magnetic flux, R is the coil's electric resistance, i is the coil current, u is the voltage of the pulsed source.

The boundary magnetic condition on the external boundary of the environment is

$$\boldsymbol{n} \times \boldsymbol{A} = 0 \qquad (8)$$

The boundary magnetic conditions on the interior boundary is

$$\boldsymbol{n} \times \left(\boldsymbol{H}_1 - \boldsymbol{H}_2 \right) = 0 \qquad (9)$$

The Lorentz force accelerates the ferromagnetic core in the magnetic field. The core movement is described by the differential equation

$$m \frac{d^2 z_0}{dt^2} = F_{\mathrm{z}} \qquad (10)$$

where: m is the core mass, z_0 is the coordinate of the moving core, F_{z} is the z-component of the Lorentz force.

The Lorentz force is calculated by the expression

$$F = \int_V \left(\boldsymbol{J} \times \boldsymbol{B} \right) dV \qquad (11)$$

where: $\boldsymbol{J}$ is the vector of current density in the core, $\boldsymbol{B}$ is the vector of magnetic induction in the core, V is the core volume.

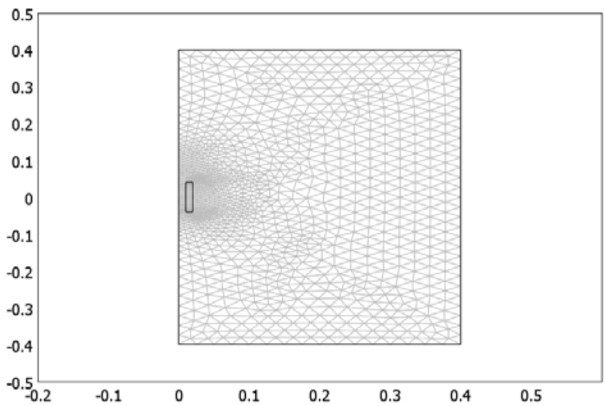

Fig. 3. The research area with an applied FEM mesh

3. Results of transient processes modelling

The set of equations describing the electromagnetic field, the electrical circuit, the and ferromagnetic core movement has been solved by the method of finite elements using the computer program Comsol FEMLAB. One irregular FEM mesh has been imposed on research domains R1 and R2 using elements of Lagrange quadratic type (Fig.3).

The backward differentiation formula method has been applied in the time domain.

The approbation of the model has been accomplished for the electromagnetic accelerator with such parameters

Table 1

Accelerator parameters

Dimension	Value	Unit
U	100	V
N	400	windings
R	0,6	Om
M	0,3	kg
l_w	0,08	m
l_c	0,08	m
r_e	0,01	m
r_i	0,005	m

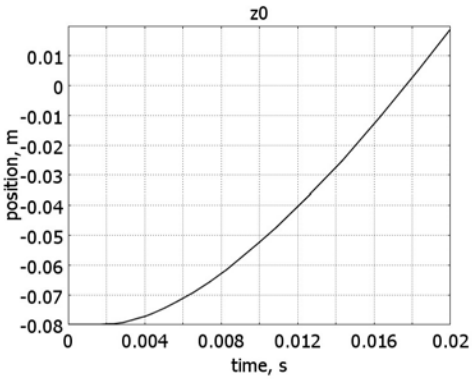

Fig. 4. The coordinate of the ferromagnetic core during its movement

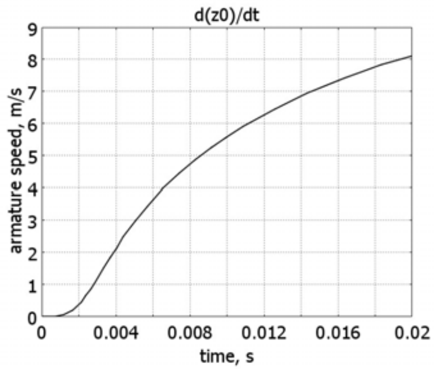

Fig. 5. The magnetic armature speed

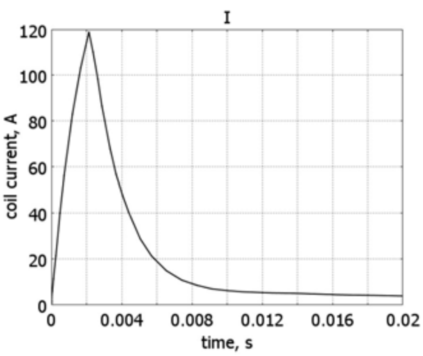

Fig. 6. The coil current of the electromagnetic accelerator

The voltage impulse with the amplitude of U=100 V and the duration of $t_{\text{ім}}$=0,0022sec has been applied to the electromagnetic accelerator. The calculation results are shown in form of time diagrams for the core coordinate $z0(t)$ (Fig.4), the core movement speed $v=d(z0)/dt$ (Fig.5), and the coil current $i(t)$ (Fig.6).

4. Conclusions

The developed mathematical model of an accelerator adequately describes electromagnetic processes in it. It is possible to research and optimize the parameters of devices of such types with the help of this model.

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ДВОВИМІРНА МОДЕЛЬ ЕЛЕКТРОМАГНІТНОГО ПРИСКОРОЮВАЧА

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Розглянуто математичну модель магнітного поля прискорювача, що враховує його симетрію, нелінійність характеристики B(H) арматури, вихрові струми. Модель створено на базі методу скінченних елементів. Арматуру, що рухається під дією сили Ампера, змодельовано як рухомий розподіл магнітної проникності.

Orest Hamola was born on October 21, 1957. In 1979 he graduated from Lviv Polytechnic Institute with the speciality "Electrical drive and automation of industrial units". Since 1979 he has been working at Lviv Polytechnic National University at the Department of Theoretical and General electrical engineering holding the post of an associate professor (senior lecturer),candidate of technical sciences (PhD). Research interest: modelling of electromagnetic fields of heterogeneous structures.

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