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MODELLING OF TEMPERATURE CONDITIONS IN ELECTRICAL DEVICES OF INHOMOGENEOUS STRUCTURE

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Abstract: Mathematical modelling of heat exchange processes for an electric device of inhomogeneous structure and the analytical numerical estimate of the relevant boundary problem of heat conduction have been considered.

Key words: temperature, heat conduction, steady-state, isotropic, heat dissipation, ideal thermal contact, foreign reach-through thermo-active inclusion.

1. Introduction

The problems of heating or cooling processes of different systems with internal heat sources are of great importance in power industry. Reliability of parts, components, elements, and in some cases of the whole construction cannot be guaranteed without compliance with the thermal conditions. Heat and temperature conditions limit the operating parameters of installation, affect the choice of structural materials, worsen the dynamic abilities of the device as an object of regulation and management, determine the technical and economic parameters, overall-weight characteristics etc. Therefore, the requirement to ensure optimum performance of devices in thermal transients and basic modes is clearly expressed. For example, microelectronic devices are used in different electric equipment for controlling the heat source. The power used by them is converted into electromagnetic and mechanical form of energy, and about 90% of it turns into heat. It comprises Joule losses in conductors, eddy current losses in transformers, losses in capacitors' dielectrics, in thyristors etc. Only 5-10% of used power is converted into useful signals. One of the major problems in design of an electric device is determining experimentally or by calculations the conditions under which the temperature of particular parts in service does not exceed edge values. Thus the design of specific items with given reliability is provided. Ensuring proper thermal conditions is one of the main problems solved while designing heat-emitting elements (thermal elements of nuclear reactors, transformer magnet cores etc.). Since the experimental study is impossible because of the high temperature and sealing material properties of heat-dissipating systems, the information on temperature conditions can only be provided by calculations, which in turn require solving complex boundary value problems of heat transfer, and their mathematical models would reflect the most significant aspects of thermo-physical processes.

Some studies of the thermal state of separate elements and units of microelectronic devices were conducted previously [1-8].

Hereinafter the modelling of the thermal transfer process for an electric device under heating has been considered, provided that it is described by isotropic (in the sense of thermo-physical characteristics) layer with thin reach-through thermo-active foreign parallelepiped inclusion [9, 10].

2. Problem statement

Let us consider an isotropic layer containing a parallelepiped inclusion of the volume $V_0 = 8hbd$, in which area $\Omega_0 = \{(x, y, z) : |x| \le h,\ y \le b, |z| \le d\}$ uniformly distributed internal heat sources of the power q_0 act. The body under consideration is referred to a Cartesian coordinate system $(Oxyz)$ with the beginning in the center of the inclusion. The conditions of an ideal thermal contact are fulfilled at the inclusion's boundary surface emitting heat; and on the layer's boundary surfaces

$$K_v = \{(x, y, d) : |x| < \infty, |y| < \infty\},$$
$$K_n = \{(x, y, -d) : |x| < \infty, |y| < \infty\}$$

the conditions of convective heat exchange with the environment having constant temperature t_c (Fig. 1) are given.

3. Mathematical model of the problem

Let's assume that the foreign inclusion is thin. To determine the stationary temperature field $t(x, y, z)$ in the considered system we use the heat conductivity equation [9, 10]

$$div[\lambda(x, y) \cdot grad\theta] = -Q(x, y), \qquad (1)$$

where

$$\lambda(x, y) = \lambda_1 + \Lambda_0 \cdot \delta(x, y) - \qquad (2)$$

is the thermal conductivity coefficient of an inhomogeneous layer; $\Lambda_0 = 4hb \cdot \lambda_0 -$ the reduced thermal conductivity coefficient of the inclusion; $\lambda_0, \lambda_1 -$ thermal conductivity of the inclusion and the layer materials, respectively; $\theta = t - t_c$; $\delta(x, y) -$ Dirac delta function; $Q(x, y) = Q_0 \cdot \delta(x, y)$; $Q_0 = 4hb \cdot q_0 -$ the reduced capacity of internal heat sources.

The boundary conditions can be expressed as

$$\left.\frac{\partial\theta}{\partial z}\right|_{|z|=d} = 0, \ \left.\theta\right|_{|x|\to\infty} = \left.\theta\right|_{|y|\to\infty} = 0,$$
$$\left.\frac{\partial\theta}{\partial x}\right|_{|x|\to\infty} = \left.\frac{\partial\theta}{\partial y}\right|_{|y|\to\infty} = 0. \tag{3}$$

Let us introduce the function

$$T = \lambda(x, y) \cdot \theta \tag{4}$$

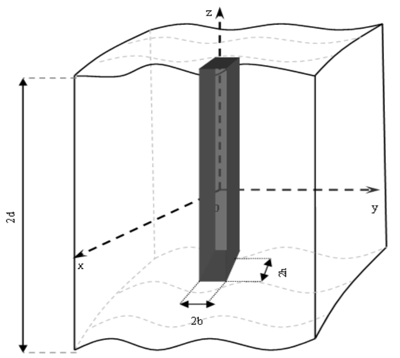

Fig. 1. Isotropic layer with a thin reach-through thermo-active foreign parallelepiped inclusion

and differentiate it with respect to variables x, y, z taking into account the description of the thermal conductivity coefficient $\lambda(x, y)$ (2). As a result we obtain:

$$\lambda(x, y)\frac{\partial\theta}{\partial x} = \frac{\partial T}{\partial x} - \Lambda_0\theta(0,0,z) \cdot \delta'_x(x, y),$$
$$\lambda(x, y)\frac{\partial\theta}{\partial y} = \frac{\partial T}{\partial y} - \Lambda_0\theta(0,0,z) \cdot \delta'_y(x, y), \tag{5}$$
$$\lambda(x, y)\frac{\partial\theta}{\partial z} = \frac{\partial T}{\partial z}.$$

By substituting expressions (5) in equation (1), we come to differential equation with partial derivatives having singular coefficients:

$$\Delta T - \Lambda_0\, \theta(0,0,z) \cdot [\delta''_x(x, y) + $$
$$+\delta''_y((x, y)] = -Q(x, y), \tag{6}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} -$ is Laplace operator in Cartesian coordinate system.

We approximate function $\theta(0,0,z)$ as (Fig. 2)

$$\theta(0,0,z) = \theta_1 + \sum_{j=1}^{n-1}(\theta_{j+1} - \theta_j) \cdot S_-(z - z_j). \tag{7}$$

Here

$z_j \in]-d; d[$, $z_1 \le z_2 \le \ldots \le z_{n-1}$; $\theta_j (j = \overline{1,n}) -$ unknown approximating temperature values;

$$S_-(\zeta) = \begin{cases} 1, & \zeta \ge 0, \\ 0, & \zeta < 0 \end{cases} -$$

an asymmetric unit function [11].

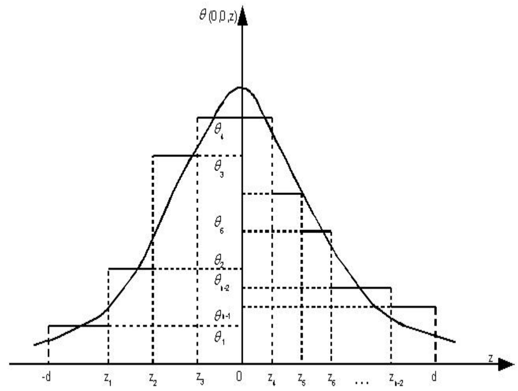

Fig. 2. Approximation of function θ(0,0,z)

By substituting expression (7) in equation (6), we obtain:

$$\Delta T = \Lambda_0 \cdot [\theta_1 + \sum_{j=1}^{n-1}(\theta_{j+1} - \theta_j) S_-(z -$$
$$-z_j)] \cdot [\delta''_x((x, y) + \delta''_y((x, y)] - Q(x, y). \tag{8}$$

4. Obtaining an analytical solution to the boundary problem

Having applied the Fourier integral transform by the coordinates x and y to the equation (8) and to the boundary conditions (3) and taking into account the relation (4), we obtain the ordinary differential equation with constant coefficients

$$\frac{d^2\overline{T}}{dz^2}-\gamma^2\overline{T}=-\frac{1}{2\pi}\left\{\Lambda_0\gamma^2\left[\theta_1+\right.\right.$$
$$\left.\left.+\sum_{j=1}^{n-1}(\theta_{j+1}-\theta_j)\cdot S_-(z-z_j)\right]+Q_0\right\} \quad (9)$$

and the boundary conditions

$$\left.\frac{d\overline{T}}{dz}\right|_{|z|=d}=0. \quad (10)$$

where $\overline{T}=\frac{1}{\sqrt{2\pi}}\int\int_{-\infty}^{\infty}e^{i(\xi x+\eta y)}Tdxdy-$ the is function's $T(x,y,z)$ transformant; $\xi,\eta-$ the parameters of the integral Fourier transform; $i=\sqrt{-1}-$ an imaginary unit; $\gamma^2=\xi^2+\eta^2$.

The general solution of the equation (9) is:

$$\overline{T}=C_1e^{\gamma z}+C_2e^{-\gamma z}+\frac{1}{2\pi}\{\Lambda_0[\theta_1+$$
$$+\sum_{j=1}^{n-1}(\theta_{j+1}-\theta_j)\cdot(1-ch\gamma(z- \quad (11)$$
$$\left.-z_j))S_-(z-z_j)]+\frac{Q_0}{\gamma^2}\right\},$$

where C_1, C_2- are constants of integration.

Having applied the boundary conditions (10) we obtain a partial solution of the problem (9), (10):

$$\overline{T}=\frac{1}{\pi}\left\{\Lambda_0\cdot\left[\theta_1\cdot\left(\frac{1}{2}+\frac{sh\gamma d}{sh2\gamma d}\cdot sh\gamma z\right)+\right.\right.$$
$$+\frac{1}{2}\left(\sum_{j=1}^{n-1}(\theta_{j+1}-\theta_j)\cdot\left((1-\right.\right.$$
$$-ch\gamma(z-z_j))S_-(z-z_j)+ \quad (12)$$
$$\left.\left.+\frac{sh\gamma(d-z_j)}{sh2\gamma d}\cdot ch\gamma z)\right)\right]+\frac{Q_0}{2\gamma^2}\right\}.$$

Applying the back transformation to the expression (12) to find its original, we obtain the expression for the sought temperature

$$T=\frac{2}{\pi}\int\int_0^{\infty}\cos\xi x\cdot\cos\eta y\,\overline{T}\,d\xi d\eta. \quad (13)$$

Therefore, the unknown approximating temperature values θ_i $(i=1,2,...,n)$ can be found by solving the system of n linear algebraic equations, derived from the expression (13).

Thus, the temperature field in the inhomogeneous layer under consideration is described by the formula (13) using which we can obtain the temperature value at an arbitrary point of the layer and of the foreign inclusion.

5. Conclusions

Using generalized functions and the piecewise linear approximation of the excess temperature $\theta(0,0,z)$ by the height of the foreign inclusion $z\in]-d;d[$ and the expression (7), the equation for the thermal conductivity (8) with a singular right-hand part has been developed. Using the Fourier integral transform, the analytical solution (13) of the boundary thermal conductivity problem (1), (3) has been found, which allows to calculate the temperature value at an arbitrary point using the newly developed algorithms and software tools, to predict operating modes of individual elements and blocks of microelectronic devices, to compute unknown parameters and to increase the heat resistance of the devices for increasing their lifetime.

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МОДЕЛЮВАННЯ ТЕМПЕРАТУРНИХ РЕЖИМІВ У ЕЛЕКТРИЧНИХ ПРИСТРОЯХ НЕОДНОРІДНОЇ СТРУКТУРИ

Д. Федасюк, В. Гавриш

Розглядається побудова математичної моделі процесу теплообміну для електричного пристрою неоднорідної структури та знаходження аналітико-числового розв’язку відповідної крайової задачі теплопровідності.

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