New Method of Extremal Surfaces for Most Efficient Ap-plication of Crystalline Materials in Electro-Optic Devices

Oleg Buryy, Serhij Ubizskii, Anatoliy Andrushchak

Abstract – **New method for geometry determination of most efficient application of crystalline materials as active elements of electro-optic devices is proposed. This method is based on the analysis of properties of the extremal surfaces obtained by optimization procedure and consists in the determination of such directions of the electrical field and the wave normal that maximize the path difference for two orthogonally polarized waves, which can propagate along arbitrary direction in investigated crystal. The method of extremal surfaces was approbated in example of $LiNbO_3$ crystal.**

Keywords – **Electrooptics, Lithium Niobate Crystal, Extremal Surface.**

I. Introduction

The electrooptical effect, i.e. the change of the optical indicatrix under the influence of the electrical field is widely used in modulators, deflectors, converters, filters [1]. The effectiveness of these devices depends on the mutual orientation of the vectors of electromagnetic wave propagation and the electrical field. The optimization of the electro-optic effect geometry consists in determination of such directions of these vectors that maximize the absolute value of the path difference for orthogonally polarized waves referred to the absolute value of the electrical field and the crystal length. The aim of this paper is the solution of this problem by the extremal surfaces method proposed in [2]. We restrict our consideration only by the linear electro-optic effect because the analysis in the case of the quadratic effect is analogous. All calculations as well as in [2] are perfomed for $LiNbO_3$ crystals as a model crystal for linear electro-optic effect.

II. The main expressions

The linear electro-optic effect consists in changing of the tensor components of dielectrical non-permittivity $\hat{\eta}$ that is proportional to the electrical field $\vec{E}$ [3]:

$$\Delta\eta_{ij} \equiv \Delta\eta_{\lambda} = r_{ijl}E_l \equiv r_{\lambda l}E_l, \qquad (1)$$

where r_{ijl} are the tensor components of the electro-optic coefficients that can be written in two-index designations in accordance with the rule: $r_{\lambda l} \equiv r_{ijl}$ and $h_l \equiv h_{ij}$ $(j \leftrightarrow l = 1,...,6)$. For $LiNbO_3$ crystal that belongs to 3m symmetry class the non-zero values of $r_{\lambda l}$ are equal to [4]: $r_{22} = -r_{12} = -r_{61} = 6.79$, $r_{13} = r_{23} = 10.1$, $r_{33} = 33.2$, $r_{51} = r_{42} = 31.1$ (all values are in 10^{12} m/V).

Oleg Buryy - Lviv Polytechnic National University, S. Bandery Str., 12, Lviv, 79013, UKRAINE.
E-mail: crystal@polynet.lviv.ua

The normalized path difference $\delta\tilde{\Delta}_k$ for two orthogonally polarized waves, i.e. the absolute value of the path difference divided on the values of the electrical field $E_l \equiv |\vec{E}|$ and the crystal length d_k is:

$$d\tilde{\Delta}_k \equiv \frac{d\Delta_k}{E_l d_k} = E_l^{-1}\left|\left(n_i - n_{jn} - \left(n_o - n_{ej}\right)\right) + \left(n_i - n_j\right)\frac{\Delta d_k}{d_k}\right|, \qquad (2)$$

where n_i, n_j are the refractive indexes of the orthogonally polarized waves with the wave vector $\vec{k}$ in the presence of the electrical field $\vec{E}$, n_o, n_{ej} are the refractive indexes of the ordinary and extraordinary waves for uniaxial crystals in the same direction in the absence of the electrical field. The last term in (2) describes changing of the crystal length due to inverse piezoelectrical effect [3]:

$$\frac{\Delta d_k}{d_k} = \vec{k}_0\hat{\varepsilon}\vec{k}_0 = \vec{k}_0\vec{E}\hat{d}\vec{k}_0, \qquad (3)$$

where $\vec{k}_0 = \vec{k}/|\vec{k}|$ is the wave normal, $\hat{\varepsilon}$ is the tensor of deformations caused by electrical field $\vec{E}$, $\hat{d}$ is the tensor of the piezoelectrical modules with such non-zero components for $LiNbO_3$ crystal [5]: $d_{22} = -0.5d_{16} = -d_{21} = 20.1$, $d_{31} = -0.57$, $d_{33} = d_{32} = 6.9$, $d_{15} = d_{24} = 66.6$ (all values are in 10^{12} C/N).

In general the refractive indexes n for two orthogonally polarized waves with the wave normal $\vec{k}_0$ can be determined from the equation [3]:

$$\begin{vmatrix} h_{11}\text{-}n^{-0.5} & h_{12} & h_{13} & k_{0_1} \\ h_{21} & h_{22}\text{-}n^{-0.5} & h_{23} & k_{0_2} \\ h_{31} & h_{32} & h_{33}\text{-}n^{-0.5} & k_{0_3} \\ k_{0_1} & k_{0_2} & k_{0_3} & 0 \end{vmatrix} = 0, \qquad (4)$$

where k_{0_α} (α = 1, 2, 3) are the components of $\vec{k}_0$.

III. The optimization procedure

In accordance with [4] we consider the normalized path difference $\delta\tilde{\Delta}_k$ as the optimization target function. The maximum of $\delta\tilde{\Delta}_k$ can be determined from the analysis of the extremal surface that is constructed in the analogous way as in the case of the acousto-optic effect in [2]. For electro-optic effect this surface can be obtained by two equivalent ways depending on the type of the vector components ($\vec{i}_0$ or $\vec{E}$) that are varied during the optimization procedure. Correspondingly, further we use the term 'the surface of the wave normal' for the case when the angles of the spherical coordinate system θ, φ determining the direction of the radius vector from the origin of coordinates to the points on the extremal surface are the angles θ_i, φ_i determine the direction of the normal to the wave front and the parameters of the optimization θ_p, φ_p are the angles that determine the direction of the electrical field θ_E, φ_E. Conversely, when $\theta = \theta_E$, $\varphi = \varphi_E$ and $\theta_p = \theta_i$, $\varphi_p = \varphi_i$ we use the term 'the surface of the electrical field' for the corresponding extremal surface. Obviously, in both cases the analysis should give the same values of the maximal value of the normalized path difference.

We use the Levenberg-Marquardt method [6] for optimization that joins the advantages of the gradient and Gauss methods. At that the extremal surface can be constructed in accordance with the following algorithm:

1) the electrical field strength as well as all necessary optical, electro-optic, piezoelectrical parameters of the crystal are determined at the given value of the wavelength λ;
2) the set of the angles θ, φ that are used for construction of the extremal surface are determined; in the case of the surface of the wave normal $\theta = \theta_i$, $\varphi = \varphi_i$, in the case of the surface of the electrical field $\theta = \theta_E$, $\varphi = \varphi_E$;
3) for each pair of the angles θ, φ determined in (2) the optimization procedure with the target function $\delta\tilde{\Delta}_k$ and the optimization parameters θ_p, φ_p are realized; in the case of the surface of normal $\theta_p = \theta_E$, $\varphi_p = \varphi_E$, in the case of the surface of the electrical field $\theta_p = \theta_i$, $\varphi_p = \varphi_i$;
4) in accordance with the values of the normalized path differences $\delta\tilde{\Delta}_k$ obtained in (3) the extremal surface is constructed.

IV. The results of optimization

For example, the extremal surfaces of the wave normal (*a*) and the electrical field (*b*) for $LiNbO_3$ crystal obtained at the wavelength of 633 nm are shown in Fig. 1. The maximal values of $\delta\tilde{\Delta}_k$ are obtained at the points located on the maximal distances from the origin of coordinates and are equal to $244\cdot10^{-12}$ m·V^{-1} at the angles $\theta_i = 42^o$, $\varphi_i = 30^o$, $\theta_E = 103.2^o$, $\varphi_E = 30^o$ and the ones connected with them by the elements of symmetry class $\bar{3}m$. So, the angles between the directions of the wave normal and the electrical field $\Delta\theta = 61.2^o$, at that the disorientation of the vectors of the wave normal and electrical field are ensured due to the difference in the polar angles only whereas the azimuth angles are the same for both vectors.

The comparison of these results with the ones obtained in [7], where some particular cases are analyzed, shows that the maximal value of $\delta\tilde{\Delta}_k$ obtained by extremal surfaces method are 13 % higher than one determined in [7].

It should be noted that although the calculation of the normalized path difference $\delta\tilde{\Delta}_k$ in accordance with (2) needs the absolute value of electrical field *E*, the dependence $\delta\tilde{\Delta}_k(E)$ is very weak at the values of electrical field that are used in the electro-optic devices based on $LiNbO_3$ crystal. Thus the value of the electrical field practically has not an influence on the results of the optimization and may be considered as its formal parameter.

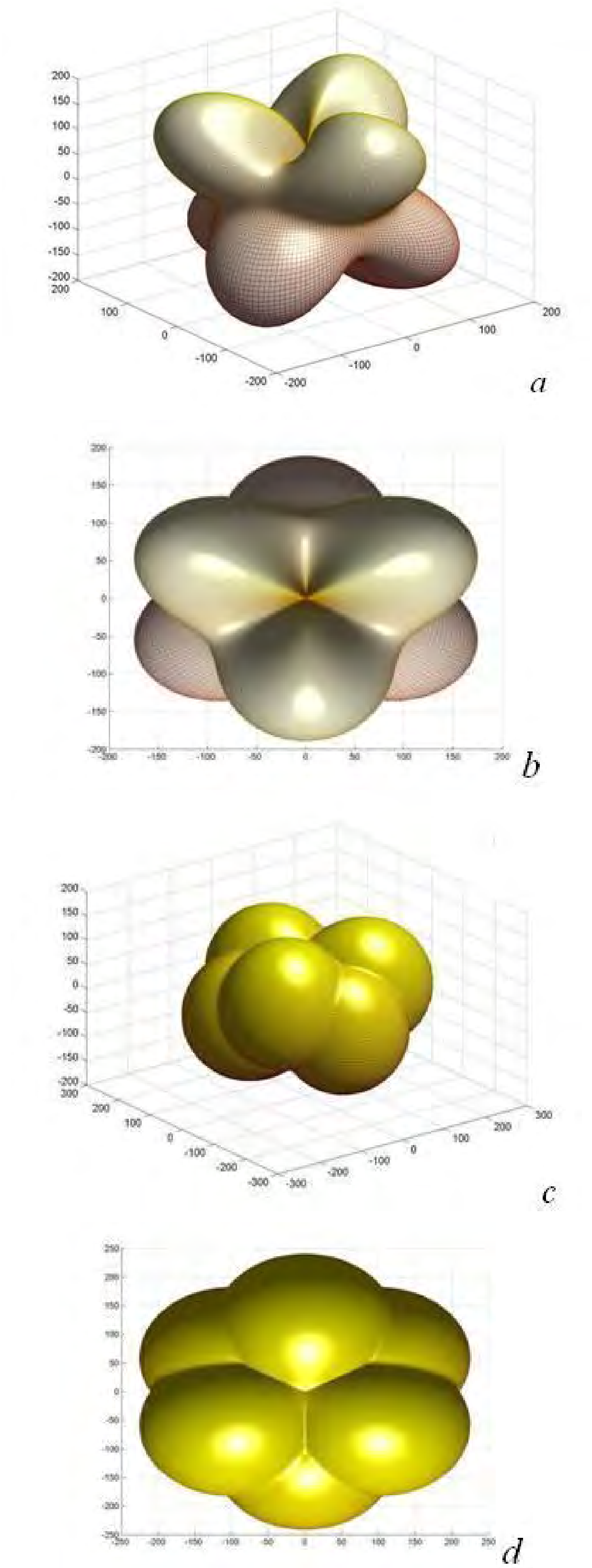

Fig.1 The extremal surfaces of the wave normal (*a*–*b*) and the electrical field (*c*–*d*) (in 10^{-12} m·V^{-1}) for the linear electro-optic effect in $LiNbO_3$ crystal at the wavelength of 633 nm. Figures *a*, *c* – the isometric projections, figures *b*, *d* – the top views

The form of the extremal surfaces practically does not change at the increase of the wavelength in the interval 488 ÷ 1440 nm, and the angles θ_i, φ_i, θ_E, φ_E corresponding to the maximal values of $\delta\tilde{\Delta}_k$ also remain unchanged. However, the maximal value of the normalized path difference decreases (Fig. 2) because of decreasing of the refractive indexes n_o, n_e and the birefringence $n_o - n_e$ of $LiNbO_3$ at higher wavelengths. Indeed, the change of the refractive index n caused by electro-optic effect is proportional to n^3 [1], so it decreases with increasing of the wavelength. The contribution of the inverse piezoelectrical effect in the value of $\delta\tilde{\Delta}_k$ is directly proportional to the birefringence of the crystal (see Eq. (2)) and also decreases with increasing of the wavelength. As it follows from our calculations, decrease of $\delta\Delta_i$ due to increasing of the wavelength from 488 to 1440 nm is not significant (about 15 % from its value at 488 nm).

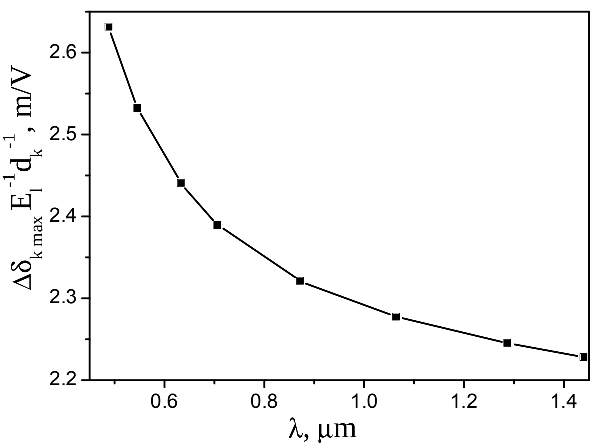

Fig.2 Dependence of the maximal value of the normalized path difference $\delta\tilde{\Delta}_{k\,max}$ on the wavelength for $LiNbO_3$ crystal

V. Conclusion

New method for determination of most efficient application of crystalline materials as active elements of electro-optic devices is proposed. The determination of the most efficient geometry of the linear electro-optic effect in $LiNbO_3$ crystal is carried out by the extremal surfaces method. At that the target function of the optimization is the normalized path difference $\delta\tilde{\Delta}_k$, i.e. the path difference for the orthogonally polarized waves divided on the absolute value of the electrical field and the length of the crystal. The parameters of the optimization are the angles determining the orientation of the vector of the electrical field (for the surface of the wave normal) or the wave normal (for the surface of the electrical field).

As it is shown, for $LiNbO_3$ crystal the maximal achievable value of the path difference is equal to $244 \cdot 10^{-12}$ m·V^{-1} at the wavelength of 633 nm and at the angle of ~ 61.2° between the directions of the light propagation and the electrical field. The form of the extremal surfaces as well as the angles endured the maximal value of the path difference remains practically unchanged at increasing of the wavelength in the interval 488 ÷ 1440 nm but the maximal value of $\delta\tilde{\Delta}_k$ decreases in accordance with decreasing of the refractive indexes and the birefringence of $LiNbO_3$ crystals.

This research was supported by a Marie Curie International Incoming Fellowship within the 7th European Community Framework Programme (project No. 272715).

References

[1] A. Yariv and P. Yeh, "Optical waves in crystals," Moscow: „Mir", 1987.

[2] O.A. Buryy, D.M. Vinnik, M.V. Kaidan and A.S. Andrushchak, "New method of optimization for acoustooptic interaction geometry in crystalline materials of the arbitrary symmetry class," *The bulletine of the Lviv Polytechnic National University, series "Electronics"*, No. 708, pp.184-194, 2011.

[3] Yu. Sirotin and M. Shaskolskaja, "Fundamentals of crystal physics," Moscow: "Nauka", 1975.

[4] A.S. Andrushchak, B.G. Mytsyk, N.M. Demyanyshyn, M.V. Kaidan, O.V. Yurkevych, S.S. Dumych, A.V. Kityk and W. Shranz, "Spatial anisotropy of linear electro-optic effect for crystal materials: I. Experimental determination of electro-optic tensor by means of interferometric technique," *Optics and Lasers Eng.,* Vol. 47, pp. 31–38, 2009.

[5] G.P. Laba, O.V. Yurkevych, I.D. Carbovnik, M.V. Kaidan, S.S. Dumych, I.M. Solskii and A.S. Andrushchak, "Spatial anisotropy of electro-, piezo- and acousto-optical effects in crystalline materials of solid state electronics. Approbation on example of $LiNbO_3$ and $LiNbO_3$:MgO crystals," *The bulletine of the Lviv Polytechnic National University, series "Electronics"*, No. 619, pp. 172-180, 2008.

[6] W. H. Press, B. P. Flannery, S. A. Teukolsky and W. T. Vetterling, "Numerical Recipes in Pascal. The Art of Scientific Computing," Cambridge University Press, England, 1989.

[7] A.S. Andrushchak, B.G. Mytsyk, N.M. Demyanyshyn, M.V. Kaidan, O.V. Yurkevych, S.S. Dumych, A.V. Kityk and W. Shranz, "Spatial anisotropy of linear electro-optic effect in crystal materials: II. Indicative surfaces as efficient tool for electro-optic coupling optimization in $LiNbO_3$," *Optics and Lasers Eng.,* Vol. 47, pp. 24–30, 2009.