Regular and Adaptive Meshing Algorithms for Modeling of Spherical Inclusions by Finite Element Method

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Abstract - **This paper consider and compare algorithms of generation finite-element meshes for nanocomposites with spherical inclusions.**

Keywords - **FEM, nanocomposite, meshing, Delaunay triangulation, Delaunay refinement.**

I. INTRODUCTION

Nowadays increasingly using composite materials with different kinds of inclusions. FEM is well suitable for modeling of heterogeneous bodies [1]. In this paper we consider composite materials with spherical inclusions. The described method is also suitable for modeling of powder composites (when concentration of inclusions in the body more than half of the volume). The main difficulty here is the creation of finite element mesh, which should describe the structure of the material. For creation of geometric model uses a special generator that creates a model by given parameters for the main material and inclusions [2]. Data of geometry used by finite mesh generator. Regular and adaptive mesh generators are described in this article.

II. REGULAR MESHING ALGORITHM

The algorithm builds a regular mesh describes inclusions in the body. Number of nodes in each mesh

line is the same, but the distance between them may vary.

Regular meshing algorithm (Fig. 1):

1. Numbering of inclusions. Creating nodes referenced to body corners.
2. Creating of nodes, referenced to borders of inclusions in *X* and *Y* directions. For *X* axis left border node denote *xsn*, right – *xfn*, for inclusion with number *n*. For *Y* axis denote analogous.
3. Insetrion of additional nodes between the adjacent nodes, distance between them over the maximum allowable.
4. Merging or nodes, distance between them less of minimum allowable.
5. Calculating of inclusions elements numbers by the ordering numbers of nodes, referenced to inclusions borders.
6. For inclusions elements set reject elements that out of inclusion area.

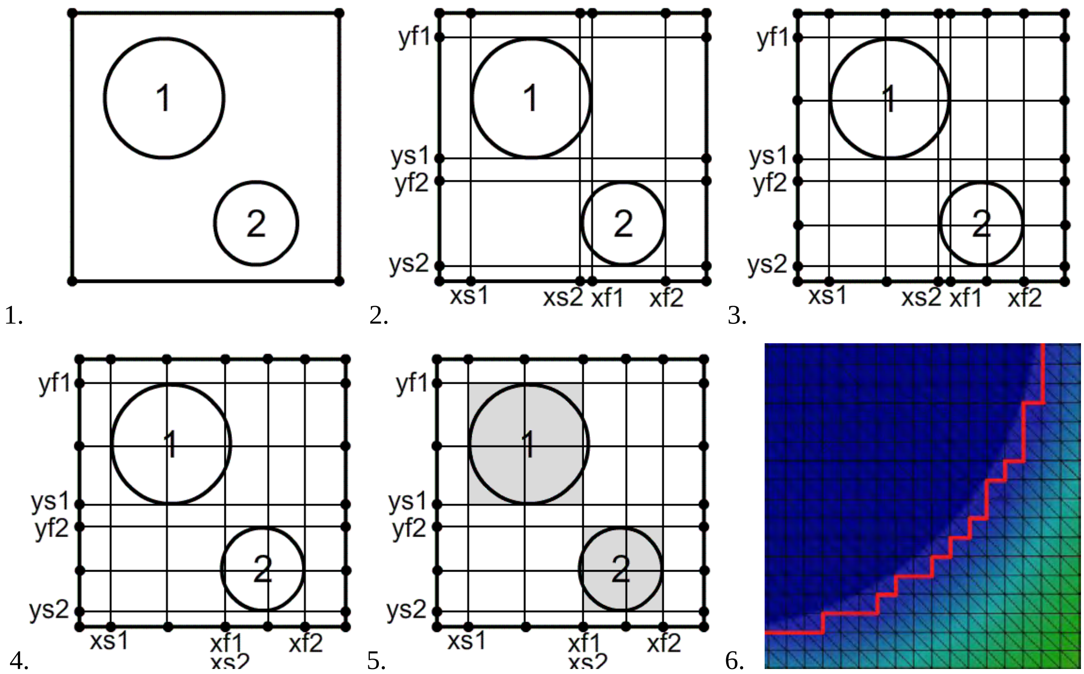

Fig. 1 Regular meshing algorithm steps.

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III. ADAPTIVE MESHING ALGORITHM

Algorithm is based on the direct methods of Delaunay triangulation (method of active edges) and Ruppert's refinement algorithm with Chew's modifications [3], [4], [5]. Also included here Cuthill–McKee indexing algorithm [6].

They say that the triangulation satisfies the Delaunay condition if there is no any of the given points of triangulation inside of the circumscribed circle of any triangle [3].

Delaunay triangulation has the largest sum of the minimum angles of its triangles, and the least sum radii of circumscribing circles of the triangles of all possible meshes on the same system of nodes.

However, triangulation algorithms based on Delaunay criterion cannot be used for finite element method, without some modification. This modification - is the Jim Ruppert's algorithm with Paul Chew's refinement. This algorithm allows to improve the form of simplex of finite-element mesh, reducing it to an equilateral. The basic idea of the algorithm (and all its modifications) is that: when the form of triangle is not equilateral, then the new node is added to triangulation – the center of the circumscribed circle of that triangle, as a consequence - Delaunay criterion is not fulfilled and the mesh reconstructed. Thus, the newly created triangles gradually approximate its shape to equilateral. The feature of the algorithm is the fact that the size of triangles is automatically reduced as it approaches the boundary of area. Flowchart of the algorithm is shown in Fig. 2. Example of the algorithm is shown in figure 3.

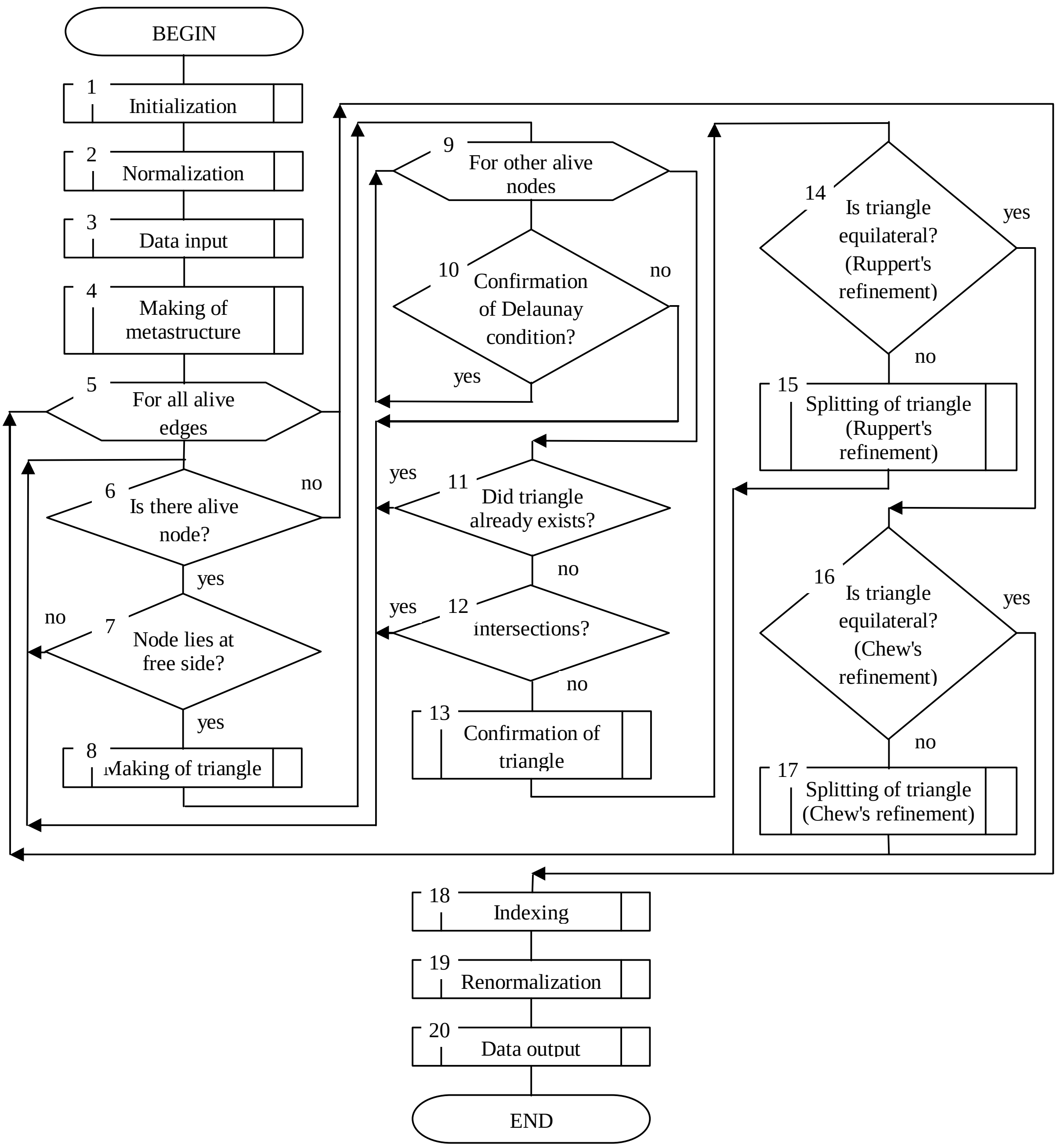

Fig. 2 Flowchart of the adaptive grids generator algorithm.

IV. Comparison of Regular and Adaptive Meshing

Regular Meshing

Advantages:

- High performance (linear algorithm complexity).
- Easy to configure.

Disadvantages:

- Low accuracy.
- Not available mesh density adaptation.

Adaptive Meshing

Advantages:

- High accuracy.
- Mesh adaptation.

Disadvantages:

- High algorithm complexity (more than $O(N^2)$) [3].
- Difficult to configure.

Fig. 3 shows the example of regular and adaptive mesh and results of analysis. Both meshes has almost same band width of taken banded matrix, so the complexity of analysis is also similar, because matrix band width is most significant in FEM computational complexity [7]. Despite the larger number of nodes in a regular mesh, the accuracy of the results is higher for adaptive.

V. Conclusion

Regular mesh generator is better to use when fast results is required, because it has high performance and it is easy to configure. Especially will be tangible benefits at series of analysis with various configurations of the body. To obtain more accurate results are better suitable adaptive mesh, but it is difficult to configure and for its generation spent more resources.

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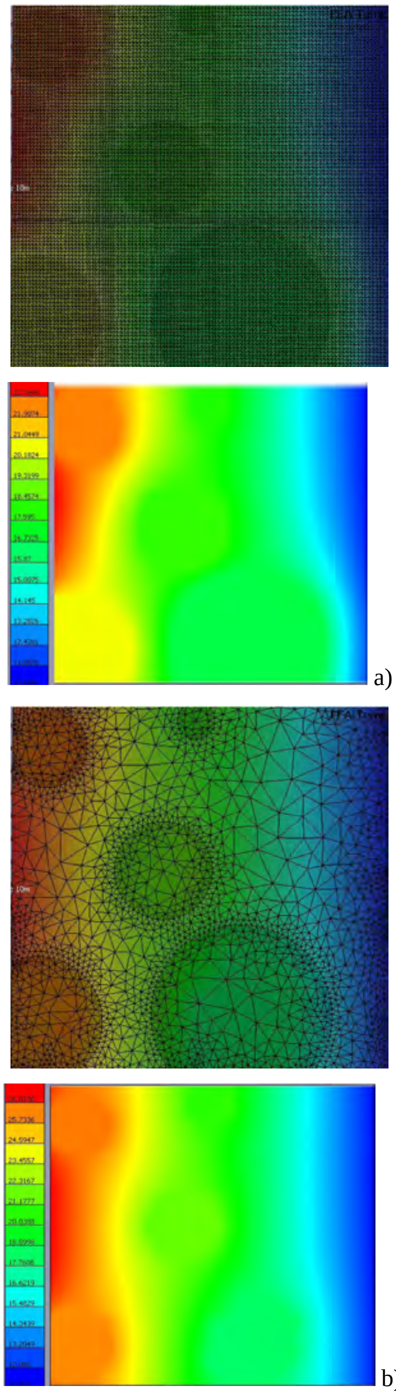

a) b)

Fig. 3 Examples of regular and adaptive meshes and analysis results. a) regular b) adaptive.

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