Renormalization the Tensor Model of the Info-Communication Network

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Abstract - **Renormalization technique provided for the tensor model of the object interaction in the open info-communication network based on the least action principle.**

Keywords – **Object interaction, info-communication network, tensor renormalization.**

The tensor math modeling is an actual issue in the sphere of info-communications [1], [2]. However, the network tensor analysis needs further development. *The objective of this work is to determine renormalization technique for the tensor model of the open info-communication network based on the least action principle*.

Let $P(n,m)=\{p_{nm}\}$, $n=0,1,2,...,N$ – non-directed matrix graph of the open network built on the open set of vertices $X=\{x_n\}$; $p_{nm}=p(x_n,x_m)=p_{mn}=p(x_m,x_n)$ – non-negative real numbers; x_0 is open zero element of X. The non-diagonal elements $p_{nm}=p_{mn}$, $n\neq m$; $n,m=0,1,...,N$ are internal and the diagonal elements $p_{nn}=p_n=p(x_n)$, $n=0,1,2,...,N$ are external interaction edges of vertices.

We want to determine a Riemann metric tensor $R(P)=\{r_{nm}\}$ as a function of P, where R is a real symmetric positively defined matrix, that meets the following requirements: all the non-diagonal elements of R and P are equal: $r_{nm}=p_{nm}$, $n\neq m$; all the diagonal elements of R satisfy the inequality $r_{nn}\geq p_{nn}$. Let $x=\{x_n\}$, $n=0,1,2,...,N$; $r(x)=r(x_n)=r_n$ is unknown vector of a posterior estimation for the matrix R diagonal elements; $\zeta(x)=\zeta(x_n)=\zeta_n$ – a prior defined value of $r(x)$; $q(x)=q(x_n)=q_n$ – the measured value of $r(x)$ obtained from the matrix P as following: $q_n=q(P)=\sum_{m=0}^{N}p_{nm}$.

We formulate the principle of least action due to the following lagrangian $L(x,r(x),a)$:

$$L(x,\rho(x),\alpha)=[\rho(x)-\vartheta_\alpha(x)]^2;$$
$$\rho(x)=\sqrt{r(x)};$$
$$\vartheta_\alpha(x)=\sqrt{g_\alpha(x)};$$
$$g_\alpha(x)=\frac{\tilde g_\alpha(x)}{\int\tilde g_\alpha(x)};$$
$$\tilde g_\alpha(x)=\frac{q(x)+\alpha\zeta(x)}{1+\alpha\zeta(x)};$$
$$0<\alpha<\infty.$$

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The variable factor α generates a family of $\tilde g(x)_\alpha$ performing the smooth transition of $\tilde g_\alpha(x)$ from the $q(x)$ function (if $\alpha\to 0$) to the $\zeta(x)$ function (if $\alpha\to\infty$).

The least action principle for the lagrangian L means [3]:

$$S(\rho,\alpha)=\int L(x,\rho(x),\alpha)dx=\min. \quad (1)$$

This turns to the variance optimization task [3]:

$$\delta S(\rho,\alpha)=0. \quad (2)$$

The variance task Eq. (2) results the Euler equation that formulates the necessary condition of an extrema for the Eq.(1), [3]:

$$\begin{cases}\dfrac{\partial L}{\partial\rho}-\dfrac{d}{dx}\cdot\dfrac{\partial L}{\partial\rho'}=0,\\ \rho'=\dfrac{d\rho(x)}{dx}.\end{cases}$$

In our case $\frac{\partial L}{\partial\rho'}=0$; $\frac{\partial L}{\partial\rho}=2\rho(x)-2\vartheta_\alpha(x)$. Therefore, $\rho(x)=\vartheta_\alpha(x)$, or $r(x)=g_\alpha(x)$. Now, we put the $r(x)=g_\alpha(x)$ vector into the main diagonal of matrix R. It results a family $\{R_\alpha\}$ of tensors $R(P,\alpha)$. To select a subset $(R_\alpha)\subseteq\{R_\alpha\}$ of "good candidates" for the final solution of the variance task Eq. (1) we inspect the spectrum family λ_α of Eigen values of R_α tensor's family. The decision criteria is $(\lambda_\alpha)_{\min}\geq\varepsilon$, where ε is predefined critically minimal positive value of λ_α. The subset $(R_\alpha)\subseteq\{R_\alpha\}$ presents the limited number of tensors that consider "good candidates" for the final choice. The tensors $R_\alpha\in(R_\alpha)$ satisfy the requirements of Riemann metric tensor [3].

REFERENCES

[1] A.E. Petrov, "Dual Network Models of Large Systems", Large System Control, vol.30.1 *Network Models in the Control*, pp.76–90, Nov. 2010. Available: *http://ubs.mtas.ru/upload/library/UBS30106.pdf.*

[2] V.V. Popovsky, A.V. Lemeshko, O.U. Evseeva, "Mathematic Models of the Functional Properties of the Tekecommunication Systems", *Telecommunication Problems*, № 2 (4), pp.3–41, 2011. Available: http://pt.journal.kh.ua/2011/2/1/112_popovsky_functional.pdf.

[3] G.A. Korn, T.M. Korn, "Mathematical Handbook for Scientists and Engineers: Definitions, Theorems and Formulas for Reference and Review, Dover Publicatio, 2000, 1152 p.